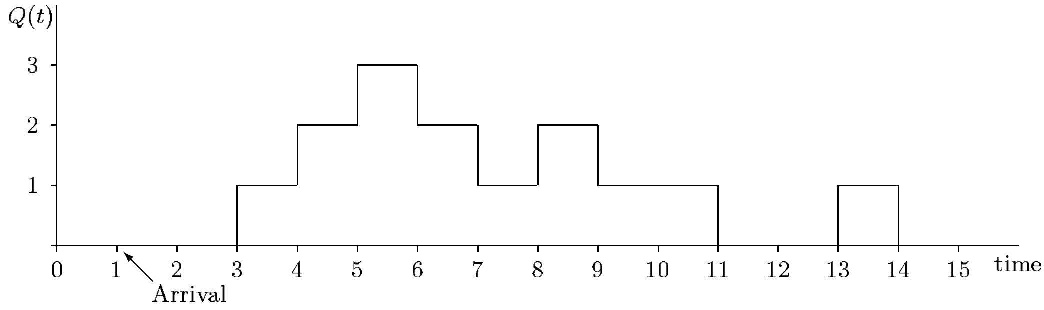
- Q1 : What is a system?
- A1 : The facility or process of interest is usually called a system.
- Q2 : What is a model?
- A2 : Assumptions which usually take the form of mathematical of logical relationships, constitute a model.
- Q3 : What is simulation?
- A3 : In a simulation, we use computer to imitate or simulate the operations of various kinds of real-world system by using its numerical model.
- Q4 : What kind of problems are with simulations?
- A4 :
  1 - complexity of writing computer programs.
  2 - Large amount of computer time.
  3 - Not considering of all aspects of real model.
- Q5 : What is state of a system?
- A5 : State of a system is the collection of variables necessary to describe a system at a particular time. EX: In bank system : The number of busy tellers, the number of customers in the bank, the time of arrival of each customer in the bank.
- Q6 : Explain the discrete and continuous systems.
- A6 : In discrete system, the state variable change instantaneously at separated points in time. EX: the number of customers in the bank.
  In continuous system, the state variables change continuously with respect to time. EX: the position or velocity of an airplane in air.
- Q7 : Classify simulation models in to these different dimensions.
- A7 :
  1 - Static vs. dynamic simulation models.
  2 - Deterministic vs. stochastic simulation models.
  3 - Continuous vs. discrete simulation models.

- Q1 : What is iconic model?
- A1 : A physical model which represents actual system is called iconic model. EX: a cockpit disconnected from airplane.
- Q2 : In which simulation models time is considered? A) Static B) Dynamic
- A2 : In (B) Dynamic models.
- Q3 : Which model use random number? A) Deterministic B) Stochastic
- A3 : B) Stochastic model
- Q4 : In a single server, what are the ”state variables”?
- A4 :
  1 - The status of the server $\begin{cases} idle \\ busy \end{cases}$
  2 - The number of customers waiting in queue.
  3 - The time of arrival of each customers waiting in queue.
- Q5 : What are the ”events” in a single server model?
- A5 : Events $\begin{cases} \textit{1} \text{ - The arrival time of customer.} \\ \textit{2} \text{ - The departure time of customer after being served.} \end{cases}$
- Q6 : Does events in a model ”always” change the state of the system?
- A6 : No, some time it changes but not always.

- Q1 : What is simulation clock?
- A1 : A variable or a mechanism that keeps track of the current time in a simulation, is called simulation clock.
- Q2 : Is there any relation between simulation time (clock) and the time needed to run a simulation on a computer?
- A2 : No. There is no relation.
- Q3 : Name two approach for the simulation clock advancing.
- A3 :
  1 - Next-event time advance.
  2 - Fixed-increment time advance.
- Q4 : What are the three measures of the system performance in a single server queueing system?
- A4 :
  1 - The average delay in queue $\hat{d}(n)$.
  2 - The time-average number of customer in queue $\hat{q}(n)$.
  3 - The proportion of time the server is busy $\hat{u}(n)$.
- Q5 : Explain the average delay in queue and the time average number of customers and the proportion of time the server is busy.
- A5 :

$$
\begin{aligned}
\hat{d}(n) &= \frac{\sum_{i=1}^{n} D_i}{n}, \\
\hat{q}(n) &= \frac{\sum_{i=0}^{\infty} iT_i}{T(n)}, \\
\hat{u}(n) &= \frac{\int_0^{T(n)} B(t)dt}{T(n)}. \neg
\end{aligned}
$$



- Q1 : In the above single-server queuing system, find

  1 - $\hat{d}(n)$, average delay in queue.

  2 - Average number of customers in the queue, $\hat{q}(n)$ and

  3 - Efficiency of utilization of the server, $\hat{u}(t)$. ($n = 6$ number of customers to finish program.)

- A1 -1 :

$$\hat{d}(n) = \frac{\sum_{i=1}^{n} D_i}{n} = \frac{\sum_{i=1}^{6} D_i}{6},$$

where D_i is time delay in queue for i^{th} customer.

$$\begin{array}{lll} D_1 = 0, & D_2 = 6 - 3 = 3, & D_3 = 7 - 4 = 3, \\ D_4 = 9 - 5 = 4, & D_5 = 11 - 8 = 3, & D_6 = 14 - 13 = 1 \end{array}$$

$$\hat{d}(n) = \frac{0 + 3 + 3 + 4 + 3 + 1}{6} = 2.33 \quad \text{Average Delay in Queue}$$

- A1 -2 :

$$\hat{q}(n) = \frac{\sum_{i=0}^{\infty} iT_i}{T(n)} = \frac{\sum_{i=0}^{\infty} iT_i}{14},$$

where T_i is duration of time for having i customer in the queue.

$$\begin{aligned} T_0 &= 3 + (13 - 11) = 5, \\ T_1 &= (4 - 3) + (8 - 7) + (11 - 9) + (14 - 13) = 1 + 1 + 2 + 1 = 5, \\ T_2 &= (5 - 4) + (7 - 6) + (9 - 8) = 1 + 1 + 1 = 3, \\ T_3 &= 6 - 5 = 1, \end{aligned}$$

$$\hat{q}(n) = \frac{0 \times 5 + 1 \times 5 + 2 \times 3 + 3 \times 1}{14} = \frac{14}{14} = 1 \quad \text{Average number of customers in queue}$$

- A1 -3 :

$$\hat{u}(n) = \frac{\sum_{t=0}^{15} B(t)}{T(n)} = \frac{15 - 1}{15} = \frac{14}{15} = 0.93 \Rightarrow 93\% \text{ the server was busy.}$$

When server is busy $B(t) = 1$.

- Q1 : Write the equation for predictor-prey problem :
- A1 :

$$\begin{cases} \dfrac{dx}{dt} = rx(t) - ax(t)y(t) \\ \\ \dfrac{dy}{dt} = -sy(t) + bx(t)y(t) \end{cases} \tag{1}$$

- Q2 : What is Monte Carlo simulation?
- A2 : A simulation methodology which employs random numbers, $U(0,1)$, for solving certain stochastic or deterministic problems.
- 3 - Cumulative distribution function : (CDF)
  $F(x) = Pr(X \leq x)$.
- 4 - Probability Density Function ; (PDF)
  $f(x) = \frac{dF(x)}{dx}$

  properties :

$$f(x) \geq 0$$
$$\int_{-\infty}^{+\infty} f(x)dx = 1$$
$$Pr(a \leq x \leq b) = \int_a^b f(x)dx$$

- 1 - Joint c.d.f :

$$F(x,y) = Pr(X \leq x \quad \text{and} \quad Y \leq y)$$
$$f(x,y) = \frac{\partial^2 F(x,y)}{\partial x \partial y} \tag{2}$$

- 2 - Marginal p.d.f :

$$f_X(x) = \int_{-\infty}^{+\infty} f(x,y)dy$$
$$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)dx \tag{3}$$

- 3 - Conditional p.d.f :

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} \tag{4}$$

- 4 - Independent random variables :

$$f_{X,Y}(x,y) = f_X(x) f_Y(y) \tag{5}$$

- 5 - Discrete marginal p.d.f :

$$P_r[X = x] = \sum_y Pr.[X = x, Y = y] \tag{6}$$