

1- *What is the Monte Carlo simulation.* (10)

A simulation methodology which employs random numbers, $U(0,1)$, for solving certain stochastic or deterministic problems

2- *Classify simulation models into three different dimensions.* (10)

- 1 - **Static vs. dynamic simulation models.**
- 2 - **Deterministic vs. stochastic simulation models.**
- 3 - **Continuous vs. discrete simulation models.**

3- *Explain about kind of problems that exist with simulation method?* (10)

- 1 - **complexity of writing computer programs.**
- 2 - **Large amount of computer time.**
- 3 - **Not considering of all aspects of real model.**

4- *What is state of a system? Give an example.* (10)

State of a system is the collection of variables necessary to describe a system at a particular time. EX: In bank system : The number of busy tellers, the number of customers in the bank, the time of arrival of each customer in the bank.

5- *Write the differential equations for predator-prey problem.* (10)

$$\frac{dx}{dt} = rx(t) - ax(t)y(t)$$

$$\frac{dy}{dt} = -sy(t) + bx(t)y(t)$$

6- *Name two approaches for the simulation clock advancing.* (10)

- 1 - **Next-event time advance.**
- 2 - **Fixed-increment time advance.**

7- Find the value of the following integral by using the Monte-Carlo method (use 6 points). (15)

$$I = -\int_1^e \cos[\pi \log_e(x)] dx = \int_1^e g(x) dx$$

- a) Use the following uniform distributed random number $U(0,1)$:
 $U_i = 0.09, 0.16, 0.48, 0.84, 0.65, 0.79$
 b) Find $1 < x_i < e$ from: $x_i = (e-1)u_i + 1$, $e = 2.72$
 c) Find $\log_e(x_i)$
 d) Then find $g(x_i) = \cos[\pi \log_e(x_i)]$, and fill the following table:

i	1	2	3	4	5	6
x_i	1.15	1.28	1.83	2.44	2.12	2.36
$\log_e(x_i)$	0.14	0.25	0.60	0.89	0.75	0.86
$g(x_i)$	0.9	0.71	-0.31	-0.94	-0.71	-0.9

Using Monte-Carlo with 6 points: $I=0.36$

Use the following equation:

$$I = (b-a) \left(\frac{\sum_{i=1}^6 g(x_i)}{6} \right)$$

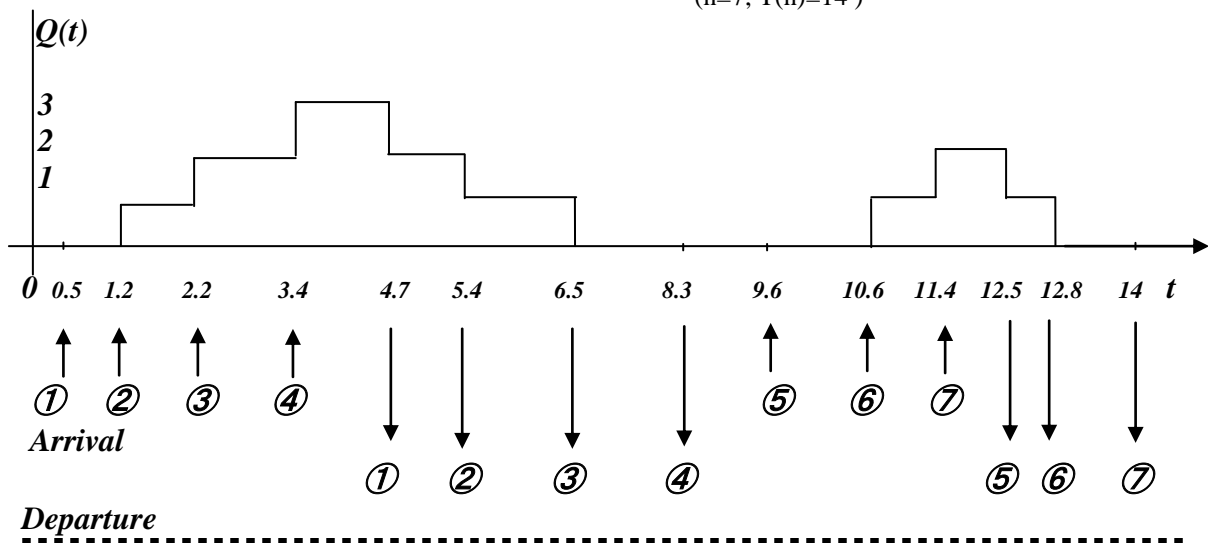
$$I = -(e-1) (0.9+0.71-0.31-0.94-0.71-0.9)/6$$

$$I = -(1.72)(-1.25)/6 = 2.15/6 = 0.36$$

8- In the following single server queuing MMI system, find: (15)

- a) Average delay in queue ($d(n)$: ADQ).
 b) Average number of customers in the queue ($q(n)$: ANCQ).
 c) Efficiency of utilization of the server ($u(n)$: %).

($\uparrow i$ means i^{th} arrival and $\downarrow i$ means i^{th} departure)
 ($n=7, T(n)=14$)



a)

$$D1=0, D2=4.7-1.2=3.5, D3=5.4-2.2=3.2, D4=6.5-3.4=3.1, D5=0, D6=12.5-10.6=1.9, D7=12.8-11.4=1.4$$

$$d(n) = \sum_{i=1}^n D_i/n = (0+3.5+3.2+3.1+0+1.9+1.4)/7 = 13.1/7 = 1.87 \text{ ADQ (time)}$$

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b)

$$T0 = 1.2 + (10.6 - 6.5) + (14 - 12.8) = 1.2 + 4.1 + 1.2 = 6.5$$

$$T1 = (2.2 - 1.2) + (6.5 - 5.4) + (11.4 - 10.6) + (12.8 - 12.5) = 1 + 1.1 + 0.8 + 0.3 = 3.2$$

$$T2 = (3.4 - 2.2) + (5.4 - 4.7) + (12.5 - 11.4) = 1.2 + 0.7 + 1.1 = 3$$

$$T3 = (4.7 - 3.4) = 1.3$$

$$q(n) = \sum_{i=0}^{\infty} i T_i/T(n) = (0 \times 6.5 + 1 \times 3.2 + 2 \times 3 + 3 \times 1.3)/14 = (3.2 + 6 + 3.9)/14 = 13.1/14 = 0.94 \text{ ANCQ (men)}$$

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c) $u(n) = \sum_{t=0}^{14} B(t)$

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 $u(n) = [(8.3 - 0.5) + (14 - 9.6)]/14 = (7.8 + 4.4)/14 = 12.2/14 = 0.87 = 87\% \text{ server utility (busy)\%}$
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