

Original
with solution

Simulation
3-rd year undergraduate
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Time: 90 minutes

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- 1- Use the table-look-up method to simulate random variables X from U(0,1).
Where the p.d.f of X is: $f(x)=2x$ ($0 \leq x \leq 1$) 10%
(Hint: see page 95)

$$X = \sqrt{U}$$

- 2- Simulate the random variable X with the following probabilities:
(Hint: see page 93)

I	0	1	2	3	4	5	6
Pr [X<I]	0.1	0.2	0.5	0.6	0.8	0.9	0.95

From a U(0,1) in the following table: 15%

U	0.05	0.65	0.25	0.92	0.55	0.15	0.85
X	0	4	2	6	3	1	5

- 3- The mixed congruential generator: $x_{n+1} = 17x_n + 3 \pmod{8}$
has full (8) cycle length. With seed $x_0 = 1$ generate all cycle one after each.

$$x(0)=1, x(1)=4, x(2)=7, x(3)=2, x(4)=5, x(5)=0, x(6)=3, x(7)=6$$
 15%

(Hint: See page 61)

- 4- Find the normal distributed random variables (N1, N2) by using Polar- Marsaglia method (rejection method) from each pair of the following uniform distributed random variables: (Hint: See page 80)
(V1, V2)=(0.1, 0.3), (V1, V2)=(0.9, 0.8), (V1, V2)=(0.6, 0.8) 20%

$$(N1, N2) = 2.034, 0.676 \text{ (N1, N2) = rejected, (N1, N2) = (0, 0)}$$

- 5- Two independent uniform random numbers with U(0,1) are given in the binary form as below:
U1=0.10110010
U2=0.01101010

Find the binomial distribution B(8, 1/2) random variables X1 from U1 and X2 with B(8, 1/4) from U1 and U2. (Hint: See page 83)

$$U_1 \otimes U_2 = 0.00100010$$

$$\begin{matrix} X1 = 4 \\ X2 = 2 \end{matrix}$$
 20%

- 6- Find random variable X with geometric distribution and $p=0.5$ from U(0,1)=0.3
(Hint: See page 93 Eq. 5.4) 20%

$$X = 2$$

$$1 - F_X(x) = \int_0^x f(y) dy = \int_0^x 2y dy = x^2$$

$$U = F_X(x) = x^2 \rightarrow \boxed{X = \sqrt{U}}$$

$$4 - W = V_1^2 + V_2^2 = 0.01 + 0.09 = 0.1 < 1 \quad (V_1, V_2) = (0.1, 0.3)$$

$$N_1 = V_2 \left(\frac{-2 \log_e W}{W} \right)^{1/2}$$

$$N_1 = 0.3 \left(\frac{-2 \log_e 0.1}{0.1} \right)^{1/2} = 0.3 \left(\frac{-2 \times (-2.3)}{0.1} \right)^{1/2} = 0.3 \times 6.78$$

$$N_1 = \boxed{2.034}$$

$$N_2 = V_1 \left(\frac{-2 \log_e W}{W} \right)^{1/2} = 0.1 \times 6.78 = \boxed{0.678}$$

$$(V_1, V_2) = (0.9, 0.8)$$

$$W = 0.9^2 + 0.8^2 = 1.45 > 1 \quad \text{rejected}$$

$$(V_1, V_2) = (0.6, 0.8)$$

$$W = 0.36 + 0.64 = 1$$

$$N_1 = 0.8 \left(\frac{-2 \log_e 1}{1} \right) = \boxed{0}$$

$$N_2 = \boxed{0}$$

$$6 - X = 1 + \left[\frac{\log_e U}{\log_e (1-p)} \right]$$

$$X = 1 + \left[\frac{\log_e 0.3}{\log_e 0.5} \right] = 1 + \left[\frac{-1.2}{-0.7} \right] = 1 + [1.714]$$

$$\boxed{X = 2}$$