(Solution) M.R.Asharif 2002/2/16

Simulation
3-rd year undergraduate
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Time: 90 minutes (write answers in boxes)

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1- Use the table-look-up method to simulate random variables X from U(0,1).
Where the p.d.f of X is: f(x)=1/(1+x)
(Hint: see page 95)

X= $e^U - 1$ *10%*

2- Simulate the random variable X with the following probabilities:
(Hint: see page 93)

I	0	1	2	3	4	5	6
Pr [X<I]	0.2	0.3	0.6	0.7	0.9	0.92	0.95

From a U(0,1) in the following table: *15%*

U	0.15	0.55	0.35	0.65	0.75	0.85	0.93
X	0	2	2	3	4	4	6

3- The mixed congruential generator: $x_{n+1} = 5x_n + 3 \pmod 4$
has full 4 cycle-length. With seed $x_0 = 1$, simulate 8 numbers, one after each.

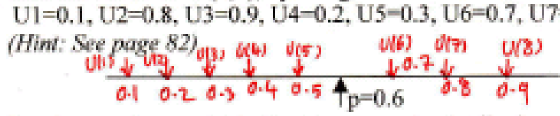
x(0)=1 ,x(1)= 0 ,x(2)= 3 ,x(3)= 2 ,x(4)= 1 , x(5)= 0 ,x(6)= 3 ,x(7)= 2 *15%*

(Hint: See page 61)

4- Simulate the normal distributed random variables (N1, N2) by using Polar-Marsaglia method (rejection method) from each pair of the following uniform distributed random variables: *(Hint: See page 80)*
(V1,V2)=(-0.7,0.9) , (V1,V2)=(-0.2,0.4), (V1,V2)=(-0.6,-0.8)

(N1,N2)= rejected, (N1,N2)=(1.6,-0.8) , (N1,N2)=(0,0) *15%*

5- Simulate a Binomial random variable X with B(8,0.6) from a set of uniform random variables U (0,1), by using Bernouli random variable, where:
U1=0.1, U2=0.8, U3=0.9, U4=0.2, U5=0.3, U6=0.7, U7=0.5, U8=0.4 *15%*
(Hint: See page 82)



X= $\sum_{k=1}^{5} B_k = 5$

6- Simulate random variable X with geometric distribution and p=0.2 from U(0,1)=0.5
(Hint: See page 93 Eq. 5.4)

X= 4 *15%*

7- Suppose that we have a set of uniform random variables: U={0.7, 0.8, 0.9, 0.5}
simulate the exponential p.d.f. random variables, Ei, by using: Ei = - $\log_e$ U. *5%*

E1= 0.36 , E2= 0.22 , E3= 0.1 , E4= 0.7

Then, from this set {Ei}, simulate a random number, K, with Poisson distribution.
(Hint: See page 83)

K= 3 *10%*

1- $F_X(x) = \int_0^x f(y)\,dy = \int_0^x \frac{1}{1+y}\,dy = \left[\ln(1+y)\right]_0^x = \ln(1+x)$

$U = F_X(x) = \ln(1+X) \rightarrow 1+X = e^U \rightarrow \boxed{X = e^U - 1}$

4- $W = V_1^2 + V_2^2$

$(V_1, V_2) = (-0.7, 0.9)$

$W = 0.49 + 0.81 > 1$ rejected

$(V_1, V_2) = (-0.2, 0.4)$

$W = 0.04 + 0.16 = 0.2 < 1$

$N_1 = V_2\left(\frac{-2\log_e W}{W}\right)^{1/2} = 0.4\left(\frac{-2\log_e 0.2}{0.2}\right)^{1/2}$

$N_1 = 0.4\left(\frac{-2\times(-1.6)}{0.2}\right)^{1/2} = 1.6$

$N_2 = V_1\left(\frac{-2\log_e W}{W}\right)^{1/2} = (-0.2)\times 4 = -0.8$

$(V_1, V_2) = (-0.6, 0.8)$

$W = V_1^2 + V_2^2 = 1$

$N_1 = V_2\left(\frac{-2\log_e W}{W}\right)^{1/2} = V_2 \times 0 = 0$

$N_2 = 0$

6- $X = 1 + \left[\frac{\log_e U}{\log_e(1-p)}\right] = 1 + \left[\frac{\log_e 0.5}{\log_e(1-0.2)}\right] = 1 + \left[\frac{-0.7}{-0.2}\right] = 1 + [3.5] = \boxed{4}$

7- $E_1 = -\log_e U = -\log_e 0.7 = 0.36$, $E_2 = 0.22$, $E_3 = 0.1$, $E_4 = 0.7$

$S_0 = 0$

$S_K = \sum_{i=1}^{k} E_i$

$S_1 = E_1 = 0.36 < 1$

$S_2 = E_1 + E_2 = 0.36 + 0.22 = 0.58 < 1$

$S_3 = E_1 + E_2 + E_3 = 0.58 + 0.1 = 0.68 < 1$

$S_4 = E_1 + E_2 + E_3 + E_4 = 0.68 + 0.7 = 1.38 > 1$

Then $K = \cancel{4}\,3$