حل تمرین ۱۰۰
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Simulation
3-rd year undergraduate
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Time: 90 minutes (write answers in boxes)

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1- Use the table-look-up method to simulate random variables X from U(0,1).
Where the p.d.f of X is: $f(x)=2x/(1+x^2)$, $0 =< x =< (e-1)^{1/2}$ *10%*
(Hint: see page 95)

$X=\sqrt{e^U-1}$

2- Simulate the random variable X with the following probabilities:
(Hint: see page 93)

I	0	1	2	3	4	5	6
Pr [X<I]	0.15	0.24	0.37	0.58	0.75	0.95	0.99

From a U(0,1) in the following table: *10%*

U	0.94	0.85	0.16	0.68	0.35	0.56	0.97
X	5	5	1	4	2	3	6

3- The mixed congruential generator: $x_{n+1} = 17x_n + 3$ (mod 8)
has full 8 cycle-length. With seed $x_0 = 1$, simulate all cycles, one after each.

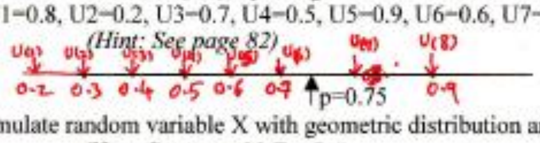
x(0)=1 ,x(1)= 4 ,x(2)= 7 ,x(3)= 2 ,x(4)= 5 , x(5)= 0 ,x(6)= 3 ,x(7)= 6 *15%*

(Hint: See page 61)

4- Simulate the normal distributed random variables (N1, N2) by using The Box-Muller method from the following U1, U2 uniform distributed random variables:
U1=0.4, U2=0.6 *15%*
(Hint: See page 78 Eq. 4.1)

(N1= −1.095 ,N2= −0.795

5- Simulate a Binomial random variable X with B(8,0.75) from a set of uniform random variables U (0,1), by using Bernouli random variable, where:
U1=0.8, U2=0.2, U3=0.7, U4=0.5, U5=0.9, U6=0.6, U7=0.3, U8=0.4 *10%*
(Hint: See page 82)



$X=\sum_{i=1}^{6} B_i = 6$

6-Simulate random variable X with geometric distribution and p=0.5 from U(0,1)=0.2
(Hint: See page 93 Eq. 5.4) *10%*

X= 3

7- Simulate a Poisson distribution random variable, K, with parameter $\lambda=1$ from the following uniform random variables: U={0.7, 0.8, 0.9, 0.5}
(Hint: See page 84) *10%*

K= 4

8- In randomized response technique (RRT), if we have P0=0.4, and Pr[N | Yes]=0.8, and total probability from survey is: Pr[Yes]=0.9, find the Pr[E | Yes] =?
(Hint: See page 51) *10%*

Pr[E | Yes] = 0.966

1- $U = F(x) = \int_0^x \frac{2x}{1+x^2}\,dx = \log_e(1+x^2)\Big]_0^x = \log_e(1+x^2)$

$$1+x^2 = e^U \rightarrow X = \sqrt{e^U - 1}$$

4- $U_1 = 0.4$, $U_2 = 0.6$

$N_1 = (-2\log_e U_1)^{1/2}\cos(2\pi U_2)$

$N_1 = (-2\log_e 0.4)^{1/2}\cos(2\pi \times 0.6)$

$N_1 = (1.3537)\,(-0.809) = -1.095$

$N_2 = (-2\log_e U_1)^{1/2}\sin(2\pi U_2)$

$N_2 = (1.3537)(-0.588)$

$N_2 = -0.795$

6- $X = 1 + \left\lfloor \frac{\log_e U}{\log_e(1-p)} \right\rfloor = 1 + \left\lfloor \frac{\log_e 0.2}{\log_e 0.5} \right\rfloor = 1 + \left\lfloor \frac{-1.6}{-0.69} \right\rfloor$

$X = 1 + \lfloor 2.3 \rfloor = 3$

7- $U = \{0.7, 0.8, 0.9, 0.5\}$

$$\prod_{i=1}^{K} U_i < e^{-1} = 0.368$$

$K=1$ $\quad 0.7 > 0.368$ Not poisson

$K=2$ $\quad 0.7 \times 0.8 = 0.56 > 0.368$ Not Poisson

$K=3$ $\quad 0.7 \times 0.8 \times 0.9 = 0.504 > 0.368$ Not poisson

$K=4$ $\quad 0.7 \times 0.8 \times 0.9 \times 0.5 = 0.252 < 0.368 \rightarrow$ $\boxed{K=4 \text{ Poisson}}$

8- $P_r[\text{Yes}] = P_r[N \mid \text{Yes}]\,p_0 + P_r[E \mid \text{Yes}]\,(1-p_0)$

$0.9 = 0.8 \times 0.4 + P_r[E \mid \text{Yes}] \times 0.6$

$P_r[E \mid \text{Yes}] = \frac{0.58}{0.6} \boxed{= 0.966}$