Original with solution

Feb. 14, 2004

Simulation Exam(A) **Name:**
3-rd year undergraduate No:
2004-2-16
Time: 90 minutes (write answers in boxes)

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1- In the mixed congruential generator: $x_{n+1} = 5x_n + 1, (\bmod\ 2^{10})$ simulate the first seven numbers with seed $x_0 = 0$. Then find the correlation between two successive numbers. $\boxed{\rho = 0.2}$ *5%*

$\boxed{x(0)=0, x(1)=1, x(2)=6, x(3)=31, x(4)=156, x(5)=781, x(6)=834, x(7)=75}$ *10%*

(Hint: See page 60-61)

2- Simulate the normal distributed random variables (N1, N2) by using Polar-Marsaglia method (rejection method) from each pair of the following uniform distributed random variables: *(Hint: See page 80)*

(V1,V2)=(0.4,0.6), (V1,V2)=(0.5,0.9), (V1,V2)=(0.6,-0.8)

$\boxed{(N1,N2)=(0.95, 0.63),\ (N1,N2)= \text{rejected},\ (N1,N2)=(0,0)}$ *10%*

3- Use the table-look-up method to simulate random variables X from U(0,1). Where the p.d.f of X is: $f(x)=\log_e x$ (only complete equation) $\boxed{X = X\log_e X - u}$ *10%*

(Hint: see page 95)

4- Simulate the random variable X with the following probabilities:

(Hint: see page 93)

I	0	1	2	3	4	5	6
Pr [X<I]	0.01	0.21	0.31	0.48	0.56	0.58	0.62

From a U(0,1) in the following table: *10%*

U	0.55	~~0.65~~ 0.48	0.28	0.18	~~0.88~~ 0.38	~~0.66~~ 0.46	0.02
X	4	~~5~~ 4	2	1	3	3	1

5- Simulate a Binomial random variable X with B(9,0.72) from a set of uniform random variables U (0,1), by using Bernouli random variable, where:

U1=0.9, U2=0.7, U3=0.6, U4=0.2, U5=0.4, U6=0.5, U7=0.3, U8=0.8 $U_9=0.1$ *10%*

(Hint: See page 82)

0.1 0.2 0.3 0.4 0.5 0.6 0.7 p=0.72 0.8 0.9

$\boxed{X = \sum_{i=1}^{9} B_i = 7}$

6- Simulate random variable X with geometric distribution and p=0.8 from U(0,1)=0.9

(Hint: See page 93 Eq. 5.4) $\boxed{X = 1}$ *10%*

7- Simulate a Poisson distribution random variable, K, from the following exponential random variables: E1=0.1, E2=0.7, E3=0.3, E4=0.2 $\boxed{K = 2}$ *10%*

(Hint: See page 84)

8- In randomized response technique (RRT), if we have P0=0.5, and Pr[N | Yes]=0.8, and total probability from survey is: Pr[Yes]=0.6, find the Pr[E | Yes] =?

(Hint: See page 51) $\boxed{Pr[E \mid Yes] = 0.4}$ *10%*

9- Describe ~~one of the~~ variance reduction techniques. *5%*
(Control variates in

171

1 - $x_{n+1} = a x_n + b \pmod{m}$

$\rho \simeq \frac{1}{a} - \frac{6b}{am}\left(1 - \frac{b}{m}\right) \pm \frac{a}{m}$

$x_{n+1} = 5x_n + 1 \pmod{2^{10}}$

$a = 5,\ b = 1,\ m = 2^{10}$

$\rho \simeq \frac{1}{5} - \frac{6}{5 \times 2^{10}}\left(1 - \frac{1}{2^{10}}\right) \pm \frac{5}{2^{10}}$

$\boxed{\rho \simeq 0.2}$

$x_0 = \boxed{0}$

$x_1 = \boxed{}$

$x_2 = \boxed{6}$

$x_3 = \boxed{31}$

$x_4 = \boxed{156}$

$x_5 = \boxed{781}$

$x_6 = 3906 \pmod{1024} = \boxed{834}$

$\begin{array}{r} 3906 \\ 3072 \\ \hline 834 \end{array}$ 3

$x_7 = 4171 \pmod{1024} = \boxed{75}$

$\begin{array}{r} 4171 \\ 4096 \\ \hline 75 \end{array}$ 4

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2 - $(V_1, V_2) = (0.4, 0.6)$

$W = V_1^2 + V_2^2 = 0.16 + 0.36 = 0.52$

$N_1 = V_2\left(\frac{-2\log_e W}{W}\right)^{1/2} = 0.6\left(-\frac{2\log_e 0.52}{0.52}\right)^{1/2} = 0.6\left(\frac{-2\times(-0.65)}{0.52}\right)^{1/2}$

$N_1 = 0.6 \times (2.5)^{1/2} = 0.6 \times 1.58 \simeq \boxed{0.95}$

$N_2 = V_1\left(\frac{-2\log_e W}{W}\right)^{1/2} = 0.4 \times 1.58 \simeq \boxed{0.63}$

$(V_1, V_2) = (0.5, 0.9)$

$W = 0.25 + 0.81 = 1.06 > 1$ rejected

$(V_1, V_2) = (0.6, -0.8)$

$W = 0.36 + 0.64 = 1$

$\left\{ \begin{array}{l} N_1 = (-0.8)\left(\frac{2\log_e 1}{1}\right) = 0 \\ N_2 = 0 \end{array} \right.$

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3 - $U = F_X(x) = \int_0^x f(y)\,dy = \int_0^x \log_e y\,dy = y\log_e y - \int_0^x \frac{y}{y}\,dy = \left[y\log_e y - y\right]_0^x$

$\boxed{U = x\log_e x - x}$ $\left(\lim_{x\to 0} x\log_e x = 0\right)$

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6 - $X = 1 + \left\lfloor \frac{\log_e U}{\log_e(1-p)} \right\rfloor = 1 + \left\lfloor \frac{\log_e 0.9}{\log_e(1-0.8)} \right\rfloor = 1 + \left\lfloor \frac{-0.1}{-1.6} \right\rfloor = 1 + \lfloor 0.0625 \rfloor$

$\boxed{X = 1}$

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7 - $S_K = \sum_{i=1}^{k} E_i$, $S_1 = E_1 = 0.1 < 1$, $S_2 = E_1 + E_2 = 0.1 + 0.7 = 0.8 < 1$

$S_3 = E_1 + E_2 + E_3 = 0.8 + 0.3 = 1.1 > 1$

$S_2 < 1 < S_3 \implies K = 2$

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8 - $P_r[\text{Yes}] = P_r[\text{Yes}|N] \times P_0 + P_r[\text{Yes}|E] \times (1 - P_0)$

$0.6 = 0.8 \times 0.5 + P_r[\text{Yes}|E] \times 0.5$

$P_r[\text{yes}|E] = \frac{0.6 - 0.4}{0.5} = 0.4$