My solution 2005/2/7

Simulation Exam **Name:**
3-rd year undergraduate No:
2005-2-14
Time: 90 minutes (write answers in boxes)

University of the Ryukyus
Faculty of Engineering
Department of Information Eng.
Prof. M.R. Asharif

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1- In randomised response technique (RRT), if we have p_0=0.4, Pr[Yes|N]=0.7, and total probability from survey is: Pr[Yes]=0.7, find the Pr[Yes| E] =?

(Hint: See page 51) *10%*

$P_r[yes] = P_r[yes|N]\cdot P_0 + P_r[yes|E]\cdot(1-P_0)$

$P_r[yes|E] = \frac{0.7 - 0.7\times 0.4}{0.6} = \frac{0.42}{0.6} = 0.7$

Pr[Yes | E] = 0.7

2- For the following two dimensional transformation:

w=x-y $x = \frac{w+z}{2}$, $y = \frac{z-w}{2}$

z=x+y

Find the joint pdf of f(w,z), if the the joint pdf of f(x,y), has the following Normal distribution:

$$f(x,y) = \frac{1}{2\pi} e^{-\frac{x^2+y^2}{2}}$$

$$J^{-1} = \begin{vmatrix} \frac{\partial x}{\partial w} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial w} & \frac{\partial y}{\partial z} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

(Hint: See page 35) *10%*

$f(w,z) = f(x,y)\cdot|J^{-1}|$

$f(w,z) = \frac{1}{2\pi} e^{-\frac{x^2+y^2}{2}} |\frac{1}{2}|$

$x^2+y^2 = (\frac{w+z}{2})^2 + (\frac{z-w}{2})^2 = \frac{w^2+z^2}{2}$

$f(w,z) = \frac{1}{4\pi} e^{-\frac{w^2+z^2}{4}}$

f(w,z)= $\frac{1}{4\pi} e^{-\frac{w^2+z^2}{4}}$

3- In the mixed congruential generator: $x_{n+1} = 781\, x_n + 387$, (mod 1000)
Simulate the first five numbers with seed $x_0 = 1$. Then find the correlation between two successive numbers. $\rho = \pm 0.781$ 5% (See back)

x(0)=1, x(1)=168, x(2)=595, x(3)=82, x(4)=429, x(5)=436 5%

(Hint: See page 60-61)

4- Simulate the normal distributed random variables (N1, N2) by using The Box-Muller method from the following U1, U2 uniform distributed random variables: U1=0.3, U2=0.5 *10%*

(Hint: See page 78 Eq. 4.1)

$N_1 = (-2\log_e U_1)^{1/2} \cos(2\pi U_2) = (-2\log_e 0.3)^{1/2}\cos(2\pi\times 0.5)$

$N_1 = -1.55$, $N_2 = (-2\log_e 0.3)^{1/2} \sin(2\pi\times 0.5) = 0$

N1= −1.55, N2= 0

5- Simulate the Gamma distributed random variables, G, with $\Gamma(n,\lambda)$ for n=5, $\lambda = 0.5$ from the following uniform distributed random variables, U(0,1): U1=0.9, U2=0.7, U3=0.6, U4=0.2, U5=0.4 *10%*

(Hint: See page 82)

G= 7

$G = -\frac{1}{\lambda} \log_e \prod_{i=1}^{n} U_i = -\frac{1}{0.5}\log_e(0.9\times 0.7\times 0.6\times 0.2\times 0.4)$

$G = -2\log_e(0.03024) \simeq 7$

6- **Two independent uniform random numbers with U(0,1) are given in the binary form as below:** **U1=0.10110110**
U2=0.10111110

Simulate the binomial distribution B(8,1/2) random variables, X1, from U1 and X2, with B(8,1/4) from U1 and U2. *10%*

(Hint: See page 83)

$U_1 \otimes U_2 = 0.10110110$

X1= 5
X2= 5

7- **Simulate a Poisson distribution random variable, K, with parameter** $\lambda = 0.5$ **from the following uniform random variables: U1= 0.8, U2= 0.8, U3= 0.6, U4= 0.5**

10%

(Hint: See page 84)

K= 3

$\prod_{i=1}^{K} U_i < e^{-\lambda} = e^{-0.5} = 0.6065$

$K=1 \rightarrow 0.8 > 0.6065$ Not poisson

$K=2 \rightarrow 0.8\times0.8 = 0.64 > 0.6065$ Not poisson

$K=3 \rightarrow 0.8\times0.8\times0.6 = 0.384 < 0.6$

$\boxed{K=3}$

Poisson

8-**Simulate the random variable X with the following probabilities:**

(Hint: see page 93)

I	0	1	2	3	4	5	6
Pr [X<I]	0.2	0.3	0.6	0.7	0.9	0.92	0.95

From a U(0,1) in the following table: *10%*

U	0.15	0.55	0.35	0.65	0.75	0.85	0.93
X	0	2	2	3	4	4	6

9-**Simulate random variable X with geometric distribution and p=0.5 from U(0,1)=0.3**

(Hint: See page 93 Eq. 5.4)

X= 2 *10%*

$$X = 1 + \left\lfloor \frac{\log_e U}{\log_e (1-p)} \right\rfloor = 1 + \left\lfloor \frac{\log_e 0.3}{\log_e 0.5} \right\rfloor$$

$$X = 1 + \left\lfloor \frac{-1.2}{-0.69} \right\rfloor = 1 + \lfloor 1.74 \rfloor = 2$$

3) $x_{n+1} = a x_n + b \pmod{m}$

$a = 781,\ b = 387,\ m = 1000$

$x(0) = 1$

$x(1) = 781 \times 1 + 387 \pmod{1000} = 168$

$x(2) = 781 \times 168 + 387 \pmod{1000} = 131595 \pmod{1000} = 595$

$x(3) = 781 \times 595 + 387 \pmod{1000} = 465082 \pmod{1000} = 82$

$x(4) = 781 \times 82 + 387 \pmod{1000} = 64429 \pmod{1000} = 429$

$x(5) = 781 \times 429 + 387 \pmod{1000} = 335436 \pmod{100} = 436$

$$\rho = \frac{1}{a} - \frac{6 \times b}{a \cdot m}\left(1 - \frac{b}{m}\right) \pm \frac{a}{m}$$

$$\rho = \frac{1}{781} - \frac{6 \times 387}{781 \times 1000}\left(1 - \frac{387}{1000}\right) \pm \frac{781}{1000}$$

$$\rho = 0.0013 - 0.003 \times 0.613 \pm 0.781 = -0.00054 \pm 0.781$$

$$\rho \simeq \pm 0.781$$