myself Original with my solution
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Simulation Exam **Name:**
3-rd year undergraduate **No:**
2006-2-13
Time: 90 minutes (write answers in boxes)

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1- Use the table-look-up method to simulate random variables X from U(0,1). Where the p.d.f of X is: $f(x)=3x^2/(1+x^3)$, $0=<x=<(e-1)^{1/3}$ *10%*
Also, find the value of X when U=0.1
(Hint: see page 95)

$\boxed{X=\sqrt[3]{e^U-1} \quad X|_{U=0.1}=0.472}$

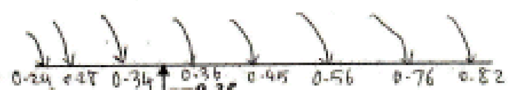
$U=F(x)=\int_0^x f(x)dx=\int_0^x \frac{3x^2}{1+x^3}dx=\left[\ln(1+x^3)\right]_0^x=\ln(1+x^3)$

$1+x^3=e^U \rightarrow x=\sqrt[3]{e^U-1}$, $x|_{U=0.1}=\sqrt[3]{e^{0.1}-1}=\sqrt[3]{1.105-1}=0.472$

2- Simulate a Binomial random variable X with B(8,0.35) from a set of uniform random variables U (0,1), by using Bernoulli random variable, where: U1=0.82, U2=0.24, U3=0.36, U4=0.45, U5=0.34, U6=0.76, U7=0.28, U8=0.56 *10%*

(Hint: See page 82)

$\boxed{X= 3}$



$X=\sum_{i=1}^{3} B_i=3$

3- In randomised response technique (RRT), if we have p_0 for answering [N], $(1-p_0)$ for answering [E] and Pr[Yes|N]=0.9, and total probability from survey is: Pr[Yes]=0.9, find the Pr[Yes| E] =?
(Hint: See page 51) *10%*

$P_r[Yes]=P_r[Yes|N]\times P_0+P_r[Yes|E]\times(1-P_0)$
$0.9=0.9\times P_0+P_r[Yes|E]\times(1-P_0)$
$P_r[Yes|E]=\frac{0.9-0.9\times P_0}{1-P_0}=0.9$

$\boxed{Pr[Yes \mid E]=0.9}$

4- In the mixed congruential generator: $x_{n+1}=47x_n+37$, (mod 500) Simulate the first five numbers with seed $x_0=1$. Then find the correlation between two successive numbers. $\boxed{\rho = 0.1053}$ *5%*

$\rho = 0.02-0.0087 \pm 0.094$
$\rho = 0.1053$
$\rho = -0.0827$

$\boxed{x(0)=1,\ x(1)=84,\ x(2)=485,\ x(3)=332,\ x(4)=141,\ x(5)=164}$ *5%*

$\rho=\frac{1}{a}-\frac{6b}{a\cdot m}\left(1-\frac{b}{m}\right)\pm\frac{a}{m}$ *(Hint: See page 60-61)*
$a=47,\ b=37,\ m=500,$ $\rho=\frac{1}{47}-\frac{6\times37}{47\times500}\left(1-\frac{37}{500}\right)\pm\frac{47}{500}=\frac{1}{47}-\frac{6\times37\times463}{47\times500\times500}\pm\frac{47}{500}=\frac{1}{47}-8.7\times10^{-3}\pm0.094$

5- Simulate the normal distributed random variables (N1, N2) by using Polar-Marsaglia method (rejection method) from each pair of the following uniform distributed random variables: *(Hint: See page 80)*
(V1,V2)=(0.8,0.7) , (V1,V2)=(0.6,0.8), (V1,V2)=(0.3,-0.4) *10%*

$\boxed{(N1,N2)=\text{rejected},\ (N1,N2)=(0,0),\ (N1,N2)=(-1.328,\ 0.996)}$

$(V_1,V_2)=(0.8,0.7)$
$W=V_1^2+V_2^2=0.8^2+0.7^2=0.64+0.49=1.13>1 \rightarrow$ rejected

$(V_1,V_2)=(0.6,0.8)$ $W=0.36+0.64=1$
$N_1=V_2\left(\frac{-2\log_e W}{W}\right)^{1/2}=0.8\left(\frac{-2\log_e 1}{1}\right)=0$
$N_2=V_1\left(\frac{-2\log_e W}{W}\right)^{1/2}=0.6\left(\frac{-2\log_e 1}{1}\right)=0$

$(V_1,V_2)=(0.3,-0.4)$
$W=0.09+0.16=0.25$
$N_1=(-0.4)\left(\frac{-2\log_e 0.25}{0.25}\right)^{1/2}=\frac{(-0.4)(2.77)^{1/2}}{0.5}$
$N_1=\frac{(-0.4)(1.66)}{0.5}=-1.328$
$N_2=\frac{(0.3)(1.66)}{0.5}=0.996$

6- If $y=\exp(-x)$ and x is a random variable with the exponential p.d.f $f(x)=\exp(-x)$, then find the probability density function (p.d.f) of random variable, $f(y)$.

10%

(Hint: See page 33)

$\boxed{f(y) = 1}$

$$f(y)=f(x)\left|\frac{dx}{dy}\right| \qquad \frac{dx}{dy}=-e^{x}$$

$$\frac{dy}{dx}=-e^{-x} \qquad f(y)=e^{-x}\left|-e^{x}\right|=1$$

7- Simulate the Gamma distributed random variables, G, with $\Gamma(n,\lambda)$ for n=5, $\lambda = 0.2$ from the following uniform distributed random variables, U(0,1):
U1=0.453, U2=0.906, U3=0.543, U4=0.679, U5=0.271

10%

(Hint: See page 82)

$\boxed{G = 16}$

$$G=-\frac{1}{\lambda}\log_e \prod_{i=1}^{n} U_i = -\frac{1}{0.2}\log_e\left[0.453\times 0.906\times 0.543\times 0.679\times 0.271\right]$$

$$G=(-5)\log_e[0.041]=(-5)(-3.194)\cong 15.97\simeq \boxed{16}$$

8- Simulate a Poisson distribution random variable, K, with parameter $\lambda = 0.8$ from the following uniform random variables:
U1= 0.95, U2= 0.89, U3= 0.78, U4= 0.69, U5=0.72

10%

(Hint: See page 84)

$\boxed{K = 5}$

$$\prod_{i=1}^{K} U_i < e^{-\lambda} = e^{-0.8} = 0.449$$

K=1 $0.95 > 0.449$: k=1 is not Poisson

K=2 $0.95\times 0.89 = 0.8455 > 0.449$: K=2 is not Poisson

K=3 $0.95\times 0.89\times 0.78 = 0.659 > 0.449$: K=3 is not Poisson

K=4 $0.95\times 0.89\times 0.78\times 0.69 = 0.455 > 0.449$: k=4 is not Poisson

K=5 $0.95\times 0.89\times 0.78\times 0.69\times 0.72 = 0.327 < 0.449$: $\boxed{K=5 \text{ is Poisson}}$

9- Simulate the random variable X with the following probabilities:

(Hint: see page 93)

10%

I	0	1	2	3	4	5	6
Pr [X<I]	0.212	0.327	0.687	0.917	0.923	0.924	0.956

From a U(0,1) in the following table:

U	0.954	0.945	0.329	0.689	0.678	0.326	0.211
X	6	6	2	3	2	1	0