4 years (2005~2008) Last exams. for Copy to be submitted to students

Original with my Solution M.R.Asharif 2008/2/13

**Simulation Exam** **Name:**
**3-rd year undergraduate No:**
**2008-2-18 Last Term Examination**
**Time: 90 minutes (write answers in boxes)**

**University of the Ryukyus**
**Faculty of Engineering**
**Department of Information Eng.**
**Prof. M.R. Asharif**

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**1- If the sequence x(n) has the following properties:**

$x(0)=0,\ x(1)=x(2)=1$

**where:** $x(n)=x(n-1)+x(n-2)-x(n-3)$ *for n>=3*

**Then, find x(100), by regression or simulation method.** *10%*

$X(3)=2,\ x(4)=2,\ x(5)=3,\ x(6)=3$

$x(7)=4,\ x(8)=4, \ldots$

$x(n)=\frac{n}{2}$, for n: even – $x(n)=\frac{n+1}{2}$ for n. odd

$\boxed{x(100) = \frac{100}{2} = 50}$

**2- In randomised response technique (RRT), if we have:**

***Pr[Yes]=0.7*** (total probability from survey).
***Pr[Yes|N]=0.8*** (answering probability to non-embarrassing question).
***Pr[Yes| E]=0.3*** (answering probability to embarrassing question).
***Find: 1-p₀*** (condition for answering to embarrassing question).

*(Hint: See page 51)* *10%*

$P_r[yes] = P_r[yes|N]\,p_0 + P_r[yes|E]\,(1-p_0)$

$P_0 = \dfrac{P_r[yes] - P_r[yes|E]}{P_r[yes|N] - P_r[yes|E]}$

$P_0 = \dfrac{0.7-0.3}{0.8-0.3} = \dfrac{0.4}{0.5} = 0.8 \rightarrow 1-P_0 = 0.2$

$\boxed{1-P_0 = 0.2}$

**3- In the following chaotic system:** *10%*

$x(n+1)=4\,r\,x(n)\,[1-x(n)]$

**If** ***r=0.7*****, find the attractor of this chaotic system by simulation or direct computation.**

*(Hint: See chap. 6, page 136)*

$\boxed{x(\infty) = 0.6429}$

$y = 4rx(1-x)$

$y' = 4r - 8rx = 0 \rightarrow x = 0.5$ Max-location

$y_{max} = 4r \times 0.5 \times 0.5 = r$ Max-value

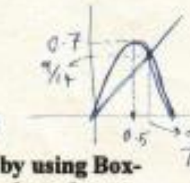
$\begin{cases} y = x \\ y = 4rx(1-x) \end{cases} \Rightarrow x = 4rx(1-x)$

$x = \frac{4r-1}{4r}$

$r = 0.7$

$x = \frac{1.8}{2.8} = \frac{9}{14}$

$y = \frac{9}{14} = 0.6429$




**4- Simulate the normal distributed random variables** ***(N1, N2)*** **by using Box-Muller method from the following pair of uniform distributed random variables:** ***(U1,U2)=(0.9,0.2)*** *(Hint: See page 78 use Eq. 4.1)*

*10%*

Box-Muller $\begin{cases} N_1 = (-2\log_e U_1)^{1/2} \cos 2\pi U_2 \\ N_2 = (-2\log_e U_1)^{1/2} \sin 2\pi U_2 \end{cases}$

$\boxed{(N1,N2) = (0.142,\ 0.437)}$

$\begin{cases} N_1 = (-2\log_e 0.9)^{1/2} \cos(2\pi \times 0.2) \\ N_2 = \underbrace{(-2\log_e 0.9)^{1/2}}_{0.46} \sin(2\pi \times 0.2) \end{cases}$

$\log_e 0.9 = -0.105$

$\cos(2\pi \times 0.2) = 0.309$

$\sin(2\pi \times 0.2) = 0.951$

$N_1 = 0.46 \times 0.309 = 0.142$

$N_2 = 0.46 \times 0.951 = 0.437$

2008/2/18
Last-Term Exam
Simulation.

**5- Simulate the Gamma distributed random variables, *G*, with $\Gamma(n,\lambda)$ for *n=4*, $\lambda = 0.25$ from the following uniform distributed random variables, *U(0,1)*:**
***U1=0.80, U2=0.90, U3=0.71, U3=0.72*** *10%*

*(Hint: See page 82)*

G= 4

$$G = -\frac{1}{\lambda}\ln\prod_{i=1}^{4}U_i = -\frac{1}{0.25}\ln(0.8\times0.9\times0.71\times0.72)$$

$$G = -4\log_e 0.368 = -4\log_e e^{-1} = 4 \qquad 0.368 \cong e^{-1}$$

**6- Two independent uniform random numbers with U(0,1) are given in the binary form as below: U1=0.01010110**
**U2=0.11010101**

**Simulate the binomial distribution B(8,1/2) random variables, X1, from U1 and X2, with B(8,1/4) from U1 and U2.** *10%*

*(Hint: See page 83)*

X1= 4
X2= 3

$B(8,\frac{1}{2}) \rightarrow X_1 = 4$

$U_1 \oplus U_2 = 0.01010100$

$B(8,\frac{1}{4}) \longrightarrow X_2 = 3$

**7- Simulate a Poisson distribution random variable, K, with parameter $\lambda = 0.9$ from the following uniform random variables:**
**U1= 0.9, U2= 0.7, U3= 0.8, U4= 0.4** *(Hint: See page 84)* *10%*

$\lambda = 0.9 \rightarrow e^{-\lambda} = e^{-0.9} = 0.406$

$\prod_{i=1}^{K} U_i < e^{-\lambda} = 0.406$

K= 4

for 1: $U_1 = 0.9 > 0.406 \rightarrow K \neq 1$

for 2: $U_1 \times U_2 = 0.9\times0.7 = 0.63 > 0.406 \rightarrow K \neq 2$

for 3: $U_1 \times U_2 \times U_3 = 0.9\times0.7\times0.8 = 0.504 > 0.406 \rightarrow K \neq 3$

for 4: $U_1\times U_2\times U_3\times U_4 = 0.9\times0.7\times0.8\times0.4 = 0.202 < 0.406 \rightarrow K = 4$ is poisson

**8- Simulate random variable *X* with geometric distribution and *p=0.3* from *U(0,1)=0.7***

*(Hint: See page 93 Eq. 5.4)* *10%*

X= 2

$$X = 1 + \left\lfloor \frac{\log_e U}{\log_e(1-p)} \right\rfloor = 1 + \left\lfloor \frac{\log_e 0.7}{\log_e(1-0.3)} \right\rfloor = 1 + 1 = 2$$

**9- Use the table-look-up method to simulate random variables *X* from *U(0,1)*. Where the p.d.f of *X* has Caushy distribution as follows:**

***f(x)=1 / π(1+x²),*** $-\infty < x < \infty$ *10%*

**Also, find the value of *X* when *U=0.75***

*(Hint: see page 95-96)*

X= $\tan[\pi(U-0.5)]$
X|u=0.75 = 1

$$U = F(x) = \int_{-\infty}^{x} f(x)\,dx = \frac{1}{\pi}\int_{-\infty}^{x}\frac{1}{1+x^2}\,dx$$

$I = \int \frac{1}{1+x^2}\,dx$

$x = \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\frac{dx}{d\theta} = \frac{1}{\cos^2\theta}$

$$I = \int \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}} \times \frac{1}{\cos^2\theta}\,d\theta = \int d\theta = \theta = \arctan x$$

$$U = \frac{1}{\pi}\int_{-\infty}^{x}\frac{1}{1+x^2}\,dx = \frac{1}{\pi}\left[\arctan(x)\right]_{-\infty}^{x} = \frac{1}{\pi}\left(\arctan(x) + \frac{\pi}{2}\right)$$

$$U = \frac{1}{\pi}\arctan(x) + \frac{1}{2} \rightarrow \arctan(x) = \pi(U-0.5)$$

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$\boxed{x = \tan(\pi(U-0.5))}$, for: $U = 0.75 \rightarrow x = \tan(\pi \times 0.25)$

$x = \tan\frac{\pi}{4} = \boxed{1}$