Original My Solution M.R. Asharif 2009/2/9

Simulation Exam. **Name:**
3-rd year undergraduate No:
2009-2-9 Last Term Examination
Time: 90 minutes (write answers in boxes)

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1-In the mixed congruential generator:

$$x_{n+1} = 101\, x_n + 11 \,(\mathrm{mod}\ 100)$$

Simulate the first five numbers with seed $x_0 = 1$. Can you estimate a rule for x(8)?

(Hint: See page 58-61) 10%

x(0)=1, x(1)=12, x(2)=23, x(3)=34, x(4)=45, x(5)=56, x(8)=89

2- In randomised response technique (RRT), if we have:

Pr[Yes|N]=0.1 (answering probability to non-embarrassing question).
Pr[Yes| E]=0.9 (answering probability to embarrassing question).
p_0=0.2 (probability for answering to non-embarrassing question).
Find Pr[Yes]=? (total probability from survey).

(Hint: See page 51) 10%

$Pr[Yes] = Pr[Yes|N]\,P_0 + Pr[Yes|E](1-P_0)$
$Pr[Yes] = 0.1\times 0.2 + 0.9\times 0.8$
$Pr[Yes] = 0.02 + 0.72 = 0.74$

***Pr[Yes]*= 0.74**

3- In the following chaotic system:

x(n+1)=4 r x(n) [1-x(n)]

If the attractor of this chaotic system will be x(∞)=0.6, find "r" by simulation or direct computation [for any value of x(0)]. 10%

(*Hint: See chap. 6, page 136*)

$y(\infty) = x(\infty) = 4r\, x(\infty)[1-x(\infty)]$
$4r[1-0.6] = 1$
$r = \frac{1}{1.6} = \frac{5}{8} = 0.625$

r= 0.625

4- Find the probability of S=k, if we have the following relation:

S=X+Y

Where both random variables X and Y have the Geometric distribution:

$Pr[X=i]= q^{i-1} p$ and $Pr[Y=i]= q^{i-1} p$ *(see page 16)*

(Hint: See page 38) 10%

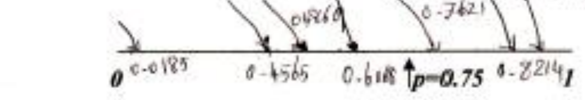
$P_r[S=k] = \sum_{i=0}^{k} P_r[X=i]\cdot P_r[Y=k-i]$
$P_r[S=k] = \sum_{i=0}^{k} q^{i-1}\cdot p\cdot q^{k-i-1}\cdot p$
$P_r[S=k] = p^2 q^{k-2} \sum_{i=0}^{k} 1 = p^2 q^{k-2}(k+1)$

Pr[S=K]= $p^2 q^{k-2}(k+1)$

5- Simulate a Binomial random variable *X* with *B(7,0.75)* from a set of uniform random variables *U (0,1)*, by using Bernouli random variable, where:

U1=0.6068, U2=0.4860, U3=0.8913, U4=0.7621, U5=0.4565, U6=0.0185, U7=0.8214

(Hint: See page 82) 10%



X= 4

2009/2/9
Last-term Exam.
Simulation.

6- Use the table-look-up method to simulate random variables *X* from *U(0,1)*. Where the p.d.f of *X* has logistic distribution *(see page 28)* **as follows:**

$f(x)=e^{-x}/(1+e^{-x})^2$, $\quad -\infty< x<\infty$ **10%**

Also, find the value of *X* when *U=0.5*

(Hint: see page 95-96)

$X= \log_e \frac{U}{1-U}$

$X|_{u=0.5} = 0$

$U = F(x) = \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{x} \frac{e^{-x}}{(1+e^{-x})^2}dx$

$U = F(x) = \left[\frac{1}{1+e^{-x}}\right]_{-\infty}^{+x} = \frac{1}{1+e^{-x}}$

$1+e^{-x} = \frac{1}{U}$

$e^{-x} = \frac{1}{U} - 1 = \frac{1-U}{U}$

$-x = \log_e \frac{1-U}{U}$

$x = \log_e \frac{U}{1-U}$

$U=0.5 \Rightarrow x|_{U=0.5} = \log_e \frac{0.5}{0.5} = 0$

7- Simulate the normal distributed random variables *(N1, N2)* by using Box-Muller method from the following pair of uniform distributed random variables: *(U1,U2)=(0.606,0.25)* *(Hint: See page 78 use Eq. 4.1)*

10%

(N1,N2)= $(0, 1)$

$\begin{cases} N_1 = (-2\log_e U_1)^{1/2} \cos(2\pi U_2) \\ N_2 = (-2\log_e U_1)^{1/2} \sin(2\pi U_2) \end{cases}$

$\begin{cases} N_1 = (-2\log_e 0.606)^{1/2} \cos(\frac{\pi}{2}) = 0 \\ N_2 = (-2\log_e 0.606)^{1/2} \sin(\frac{\pi}{2}) = (-2\log_e 0.606)^{1/2} = 1 \end{cases}$

8- Simulate the random variable X with the following probabilities:

(Hint: see page 93-94) **10%**

I	0	1	2	3	4	5	6
Pr [X<I]	0.2311	0.4860	0.6068	0.8913	0.9218	0.9568	0.9797

From a U(0,1) in the following table:

U	0.2523	0.8757	0.7373	0.1365	0.2987	0.8939	0.4692
X	1	3	3	0	1	4	1

9- Simulate random variable *X* with geometric distribution and *p=0.1* from *U(0,1)=0.5*

(Hint: See page 93 Eq. 5.4) **10%**

X= 7

$X = 1 + \left\lfloor \frac{\log_e U}{\log_e(1-p)} \right\rfloor = 1 + \left\lfloor \frac{\log_e 0.5}{\log_e 0.9} \right\rfloor$

$X = 1 + \left\lfloor \frac{-0.69}{-0.105} \right\rfloor = 1 + \lfloor 6.57 \rfloor = 1 + 6 = 7$

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