Simulation Exam. Name:
3-rd year undergraduate No:
2009-2-9 Last Term Examination
Eng.
Time: 90 minutes (write answers in boxes) Prof. M.R. Asharif
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1-In the mixed congruential generator:

$$
x_{n+1}=101 x_{n}+11(\bmod 100)
$$

Simulate the first five numbers with seed $x_{0}=1$. Can you estimate a rule for $\mathbf{x}(\mathbf{8})$ ?
(Hint:
See page 58-61) $10 \%$

$$
x(0)=1, \quad x(1)=\quad, x(2)=\quad, x(3)=\quad, x(4)=\quad, x(5)=\quad x(8)=
$$

2- In randomised response technique (RRT), if we have:
$\operatorname{Pr}[\mathbf{Y e s} \mid \mathbf{N}]=\mathbf{0 . 1} \quad$ (answering probability to non-embarrassing question).
$\operatorname{Pr}[\mathbf{Y e s} \mid \mathbf{E}]=\mathbf{0 . 9} \quad$ (answering probability to embarrassing question).
$\mathbf{p}_{\mathbf{0}}=\mathbf{0 . 2}$ (probability for answering to non-embarrassing question).
Find $\operatorname{Pr}[\mathrm{Yes}]=$ ? (total probability from survey).
(Hint: See page 51)
$10 \%$

$$
\operatorname{Pr}[Y e s]=
$$

3- In the following chaotic system:

$$
x(n+1)=4 r x(n)[1-x(n)]
$$

If the attractor of this chaotic system will be $x(\infty)=0.6$, find " $r$ " by simulation or direct computation [for any value of $x(0)$ ].
(Hint: See chap $\square$

4- Find the probability of $S=k$, if we have the following relation:

$$
\mathbf{S}=\mathbf{X}+\mathbf{Y}
$$

Where both random variables $X$ and $Y$ have the Geometric distribution:

$$
\operatorname{Pr}[X=i]=\boldsymbol{q}^{i-1} \boldsymbol{p} \quad \text { and } \quad \operatorname{Pr}[\boldsymbol{Y}=i]=\boldsymbol{q}^{i-1} \boldsymbol{p} \quad \text { (see page } 16 \text { ) }
$$

(Hint: See page 38)


5- Simulate a Binomial random variable $X$ with $B(7,0.75)$ from a set of uniform random variables $\boldsymbol{U}(0,1)$, by using Bernouli random variable, where:
$U 1=0.6068, U 2=0.4860, U 3=0.8913, U 4=0.7621, U 5=0.4565$, $U 6=0.0185, U 7=0.8214$
(Hint: See page 82)
$10 \%$

6- Use the table-look-up method to simulate random variables $X$ from $U(0,1)$. Where the p.d.f of $\boldsymbol{X}$ has logistic distribution (see page 28) as follows:

$$
f(x)=e^{-x} /\left(1+e^{-x}\right)^{2}
$$

$10 \%$
Also, find the value of $\boldsymbol{X}$ when $\boldsymbol{U}=0.5$
(Hint: see page 95-96)

$$
\begin{aligned}
& \mathrm{X}= \\
& \mathrm{X} \mid \mathrm{u}=0.5=
\end{aligned}
$$

7- Simulate the normal distributed random variables (N1, N2) by using BoxMuller method from the following pair of uniform distributed random variables: $(\boldsymbol{U} 1, \boldsymbol{U} 2)=(\mathbf{0 . 6 0 6}, 0.25) \quad$ (Hint: See page 78 use Eq. 4.1) $10 \%$

```
(N1,N2)=
```

8-Simulate the random variable $X$ with the following probabilities:
(Hint: see page 93-94)
$10 \%$

| I | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Pr}[\mathrm{X}<\mathrm{I}]$ | 0.2311 | 0.4860 | 0.6068 | 0.8913 | 0.9218 | 0.9568 | 0.9797 |

From a $\mathbf{U}(0,1)$ in the following table:

| U | Q.2523 | 0.8757 | 0.7373 | 0.1365 | 0.2987 | 0.8939 | ф.4692 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| X |  |  |  |  |  |  |  |

9- Simulate random variable $X$ with geometric distribution and $p=0.1$ from $U(0,1)=0.5$
(Hint: See page 93 Eq. 5.4)
$10 \%$

```
X=
```

