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My solution for 2010/2/2

Simulation Exam **Name:**
3-rd year undergraduate No:
2010-2-8 Last Term Examination
Time: 90 minutes (write answers in boxes)

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5) **1-In the mixed congruential generator:** $x_{n+1} = 17\,x_n + 3, (\mathrm{mod}\ 8)$
Simulate ~~the first five numbers~~ all cycles (8) with seed $x_0 = 1$ **. ~~Then find the correlation between two successive numbers.~~**

x(0)=1 ,x(1)= 4 ,x(2)= 7 ,x(3)= 2 ,x(4)= 5 , x(5)= 0 ,x(6)= 3 ,x(7)= 6

10%

(Hint: See page 60-61)

2- In randomised response technique (RRT), if we have:
Pr[Yes|N]=0.8 (answering probability to non-embarrassing question).
Pr[Yes]=0.9 (total probability from survey).
p_0=0.4 (probability for answering to non-embarrassing question).
Find Pr[Yes|E]=? (answering probability to embarrassing question).
(Hint: See page 51) *10%*

$P_r[Yes] = P_r[Yes|N]P_0 + P_r[Yes|E](1-P_0)$

$0.9 = 0.8 \times 0.4 + P_r[Yes|E] \times 0.6$

$P_r[Yes|E] = \frac{0.9-0.32}{0.6} = 0.97$

Pr[Yes|E]= 0.97

5) **3- In the following chaotic system:** *10%*

x(n+1)=4 r x(n) [1-x(n)]

If *r*= 0.75 **, find the attractor of this chaotic system.**
(Hint: See chap. 6, page 136)

x(∞)= 0.66

$y = 4 \times 0.75\, x(1-x)$

$y = 3x(1-x)$

$y' = 3 - 6x = 0 \rightarrow x = 0.5$

$y_{max} = 3 \times 0.5 \times 0.5 = 0.75 = r$

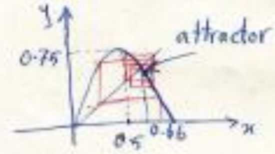
$y = x$

$y = 3x(1-x)$

$x = 3x(1-x)$

$1 = 3 - 3x$

$x = \frac{2}{3} = \boxed{0.66}$ attractor.



4) **4- The Fibonacci sequence is defined as follows:**
Fib(1)=Fib(2)=1
Fib(n)=Fib(n-1)+Fib(n-2) ***for n>=3***
It can be shown that:
Fib(n)={ [(1+√5)/2]ⁿ-[(1-√5)/2]ⁿ }/√5
Find Fib(10), both by direct method and using the above equation. *10%*

Fib(3) = 2 , 3 , 5 , 8 , 13 , 21 , 34 , 55 ← Fib(10)

Fib(10) = 55

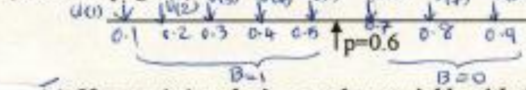
$fib(10) = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{10} - \left(\frac{1-\sqrt{5}}{2}\right)^{10}\right] = 55$

My solution 2010/2/2

(2002)5- Simulate a Binomial random variable X with B(8,0.6) from a set of uniform random variables U (0,1), by using Bernouli random variable, where:

U1=0.1, U2=0.8, U3=0.9, U4=0.2, U5=0.3, U6=0.7, U7=0.5, U8=0.4 *10%*

(Hint: See page 82)



$\boxed{X=\sum_{i=1}^{5} B_i = 5}$

6- If y=exp(-x) and x is a random variable with the exponential p.d.f f(x)=exp(-x), then find the probability density function (p.d.f) of random variable, f(y). *10%*

(Hint: See page 33)

$f(y) = f(x)\left|\frac{dx}{dy}\right|$

$y = e^{-x}$

$\frac{dy}{dx} = -e^{-x}$

$\frac{dx}{dy} = -e^{x}$

$f(y) = e^{-x}\left|-e^{x}\right| = 1$

$\boxed{f(y) = 1}$ Uniform Dis.

7- Simulate the Gamma distributed random variables, G, with $\Gamma(n,\lambda)$ for n=5, $\lambda = 0.5$ from the following uniform distributed random variables, U(0,1): U1=0.9, U2=0.7, U3=0.6, U4=0.2, U5=0.4 *10%*

(Hint: See page 82)

$G = -\frac{1}{\lambda}\log_e \prod_{i=1}^{n} U_i = -\frac{1}{0.5}\log_e(0.9\times0.7\times0.6\times0.2\times0.4)$

$G = -2\log_e(0.03024) \simeq 7$

$\boxed{G = 7}$

8- Simulate a Poisson distribution random variable, K, with parameter λ=1 from the following uniform random variables: U={0.7, 0.8, 0.9, 0.5} *10%*

(Hint: See page 84)

$\prod_{i=1}^{K} U_i < e^{-\lambda} = e^{-1} = 0.368$

K=1 $0.7 > 0.368$ Not poisson

K=2 $0.7\times0.8 = 0.56 > 0.368$ Not poisson

K=3 $0.7\times0.8\times0.9 = 0.504 > 0.368$ Not poisson

K=4 $0.7\times0.8\times0.9\times0.5 = 0.252 < 0.368 \rightarrow$ K=4 Poisson

$\boxed{K= 4}$

9- Simulate the normal distributed random variables (N1, N2) by using rolar-Marsaglia method (rejection method) from each pair of the following uniform distributed random variables: *(Hint: See page 80)*

(V1,V2)=(-0.7,0.9), (V1,V2)=(-0.2,0.4), (V1,V2)=(-0.6,-0.8) *10%*

(N1,N2)= rejected, (N1,N2)= (1.6, -0.8), (N1,N2)= (0, 0)

$W = V_1^2 + V_2^2$

$(V_1, V_2) = (-0.7, 0.9)$

$W = 0.49 + 0.81 > 1$ rejected

$(V_1, V_2) = (-0.2, 0.4)$

$W = 0.04 + 0.16 = 0.2 < 1$

$N_1 = V_2\left(\frac{-2\log_e W}{W}\right)^{1/2} = 0.4\left(\frac{-2\log_e 0.2}{0.2}\right)^{1/2}$

$(V_1, V_2) = (-0.6, -0.8)$

$W = 0.36 + 0.64 = 1$

$N_1 = V_2 \times 0 = 0$

$N_2 = 0$

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$N_1 = 0.4\left(\frac{-2(-1.6)}{0.2}\right)^{1/2} = 1.6$

$N_2 = V_1\left(\frac{-2\log_e W}{W}\right)^{1/2} = -0.2\times4 = -0.8$