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My Solution
for 2011/2/1

Simulation Exam **Name:**	**University of the Ryukyus**
3-rd year undergraduate No:	**Faculty of Engineering**
2011-2-4 Last Term Examination	**Department of Information Eng.**
Time: 90 minutes (write answers in boxes)	**Prof. M.R. Asharif**

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1- **If $y=x^2$ and x is a random variable with the normal p.d.f , that is:**

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

Then, find the probability density function (p.d.f) of random variable, y: f(y).

10%

(Hint: See page 33)

$f(y) = f(x)\frac{dx}{dy}$

$y = x^2 \rightarrow \frac{dy}{dx} = 2x \rightarrow \frac{dx}{dy} = \frac{1}{2x}$

$f(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \times \frac{1}{2x} = \frac{1}{2\sqrt{2\pi y}} e^{-\frac{1}{2}y}$

$\boxed{f(y) = \frac{1}{2\sqrt{2\pi y}} e^{-\frac{1}{2}y}}$

2- Find the probability of S=k, if we have the following relation:

S=X+Y

Where both random variables X and Y have the Geometric distribution:

$Pr[X=i]=q^{i-1}p$ and $Pr[Y=i]=q^{i-1}p$

(Hint: See page 38)

10%

$P_r[S=k] = \sum_{i=0}^{k} P_r[X=i] \cdot P_r[Y=k-i]$

$P_r[S=k] = \sum_{i=0}^{k} q^{i-1} \cdot p \cdot q^{k-i-1} \cdot p = \sum_{i=0}^{k} p^2 q^{k-2}$

$P_r[s=k] = p^2 q^{k-2} \sum_{i=0}^{k} 1 = p^2 q^{k-2}(k+1)$

$\boxed{Pr[S=K] = p^2 q^{k-2}(k+1)}$

3- In Randomised Response Technique (RRT), if we have:

Pr[Yes|N]=0.2 (answering probability to non-embarrassing question).
Pr[Yes]=0.6 (total probability from survey).
p_0=0.5 (probability for answering to non-embarrassing question).
Find Pr[Yes|E]=*?* (answering probability to embarrassing question).

(Hint: See page 51)

10%

$P_r[Yes] = P_r[Y|N]P_0 + P_r[Y|E](1-P_0)$

$0.6 = 0.2 \times 0.5 + P_r[Y|E]\, 0.5$

$P_r[Y|E] = \frac{0.6 - 0.1}{0.5} = 1$

$\boxed{Pr[Yes|E] = 1}$

4-**The first pseudo-random number generator proposed by Von Neuman (1951) was the "mid-square (MS)" such that:**

$X(0)=7777$, $x(1) = MS(7777) = Mid(7777)^2 = 60\underline{4817}29 = 4817$

Find the first five numbers.

x(0)=7777 , x(1)= 4817 ,x(2)=2034 ,x(3)=1371 ,x(4)= 8796 ,x(5)= 3696

10%

(Hint: See page 71)

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5- In the following iterative function: *10%*

$$a(n+1)=[a(n)]^2-0.75$$

If $a(0)=0.5i$, where $i=\sqrt{-1}$, find the attractor of this function, $a(\infty)$.

(Hint: See Julia Sets)

$a(1)=(0.5i)^2-0.75=-0.25-0.75=-1$

according to Julia set $\rightarrow a(\infty)=-0.5$

or if $\lim_{n\to\infty} a(n+1)=a(n)=x$

$x=x^2-0.75$

$x^2-x-0.75=0$

$x=\frac{1\pm\sqrt{1+3}}{2}=1.5,\ -0.5$

$x=-0.5=a(\infty)$

$a(\infty)=-0.5$

6- Simulate the normal distributed random variables *(N1, N2)* by using Box-Muller method from the following pair of uniform distributed random variables: *(U1,U2)=(0.8825,0.5)* *(Hint: See page 78 use Eq. 4.1)* *10%*

$$\begin{cases} N_1=(-2\log_e U_1)^{1/2}\cos(2\pi U_2) \\ N_2=(-2\log_e U_1)^{1/2}\sin(2\pi U_2) \end{cases}$$

(N1,N2)= $(-0.5,\ 0)$

$N_1=(-2\log_e 0.8825)^{1/2}\cos\pi=[-2\times(-0.125)]^{1/2}\cos\pi=(0.25)^{1/2}\cos\pi=0.5(-1)=-0.5$

$N_2=(-2\log_e 0.8825)^{1/2}\sin\pi=0$

7- Simulate the Gamma distributed random variables, G, with $\Gamma(n,\lambda)$ for n=5, $\lambda=0.25$ from the following uniform distributed random variables, U(0,1): *U1=0.9, U2=0.8, U3=0.7, U4=0.81, U5=0.9* *10%*

(Hint: See page 82)

$G=-\frac{1}{\lambda}\log_e\prod_{i=1}^{n}U_i=-\frac{1}{0.25}\log_e(0.9\times0.8\times0.7\times0.81\times0.9)$

$G=-4\log_e(0.3674)=-4(-1.0012)\simeq 4$

G= 4

8- Two independent uniform random numbers with U(0,1) are given in the binary form as below: **U1=0.10101010**
U2=0.01111011

Simulate the binomial distribution B(8,1/2) random variables, X1, from U1 and X2, with B(8,1/4) from U1 and U2. *10%*

(Hint: See page 83)

X1= 4
X2= 3

$B(8,\frac{1}{2})\rightarrow X_1=4$

$U_1\otimes U_2=0.00101010$

$B(8,\frac{1}{4})\rightarrow X_2=3$

9- Simulate random variable *X* with geometric distribution and *p=0.2*from *U(0,1)=0.74* *10%*

(Hint: See page 93 Eq. 5.4)

X= 2

$X=1+\left\lfloor\frac{\log_e U}{\log_e(1-p)}\right\rfloor=1+\left\lfloor\frac{\log_e 0.74}{\log_e 0.8}\right\rfloor$

$X=1+\left\lfloor\frac{-0.301}{-0.223}\right\rfloor=1+\lfloor 1.3498\rfloor=1+1=2$

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