| Simulation Exam Name: | University of the Ryukyus |
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| 3-rd year undergraduate No: | Faculty of Engineering |
| 2011-2-4 Last Term Examination | Department of Information Eng. |
| Time: 90 minutes | (write answers in boxes) |

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1- If $y=x^{2}$ and $x$ is a random variable with the normal p.d.f, that is:

$$
f(x)=\frac{1}{\sqrt{2 \pi}} e^{-\frac{1}{2} x^{2}}
$$

Then, find the probability density function (p.d.f) of random variable, $y: f(y)$.
(Hint: See page 33)
$10 \%$

2- Find the probability of $S=k$, if we have the following relation:

$$
\mathbf{S}=\mathbf{X}+\mathbf{Y}
$$

Where both random variables $X$ and $Y$ have the Geometric distribution:
$\operatorname{Pr}[X=i]=q^{i-1} p$ and $\operatorname{Pr}[Y=i]=q^{i-1} p$
(Hint: See page 38)
10\%
$\operatorname{Pr}[\mathrm{S}=\mathrm{K}]=$

3- In Randomised Response Technique (RRT), if we have:
$\operatorname{Pr}[\mathbf{Y e s} \mid \mathbf{N}]=\mathbf{0 . 2}$ (answering probability to non-embarrassing question).
$\operatorname{Pr}[Y e s]=\mathbf{0 . 6}$ (total probability from survey).
$\mathbf{p}_{\mathbf{0}}=\mathbf{0 . 5} \quad$ (probability for answering to non-embarrassing question). Find $\operatorname{Pr}[Y e s \mid E]=$ ? (answering probability to embarrassing question).
(Hint: See page 51)
10\%

$$
\operatorname{Pr}[\mathrm{Yes} \mid \mathrm{E}]=
$$

4-The first pseudo-random number generator proposed by Von Neuman (1951) was the "mid-square (MS)" such that:
$\mathbf{X}(0)=7777, \mathbf{x}(1)=\operatorname{MS}(7777)=\operatorname{Mid}(7777)^{2}=60481729=4817$
Find the first five numbers.

$$
\mathrm{x}(0)=7777, \mathrm{x}(1)=4817 \quad, \mathrm{x}(2)=\quad, \mathrm{x}(3)=\quad, \mathrm{x}(4)=\quad, \mathrm{x}(5)=
$$

$$
a(n+1)=[a(n)]^{2}-0.75
$$

If $a(0)=0.5 i$, where $i=\sqrt{-1}$, find the attractor of this function, $a(\infty)$.
(Hint: See Julia Sets)

$$
a(\infty)=
$$

6- Simulate the normal distributed random variables (N1, N2) by using BoxMuller method from the following pair of uniform distributed random variables: $(\boldsymbol{U} 1, \boldsymbol{U} 2)=(\mathbf{0} .8825,0.5) \quad$ (Hint: See page 78 use Eq. 4.1)

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(N1,N2)=
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7- Simulate the Gamma distributed random variables, $\mathbf{G}$, with $\Gamma(n, \lambda)$ for $\mathbf{n}=\mathbf{5}$, $\lambda=0.25$ from the following uniform distributed random variables, $\mathbf{U}(\mathbf{0}, \mathbf{1})$ : $U 1=0.9, U 2=0.8, U 3=0.7, U 4=0.81, U 5=0.9$
(Hint: See page 82)


8 - Two independent uniform random numbers with $\mathrm{U}(0,1)$ are given in the binary form as below: $\quad \mathrm{U} 1=\mathbf{0 . 1 0 1 0 1 0 1 0}$ $\mathrm{U} 2=\mathbf{0 . 0 1 1 1 1 0 1 1}$
Simulate the binomial distribution $B(8,1 / 2)$ random variables, X 1 , from U 1 and X 2 , with $\mathrm{B}(8,1 / 4)$ from U 1 and U 2 .

10\%
(Hint: See page 83)

| $\mathrm{X} 1=$ |
| :--- |
| $\mathrm{X} 2=$ |

9- Simulate random variable $X$ with geometric distribution and $p=0.2$ from $U(0,1)=0.74$
(Hint: See page 93 Eq. 5.4)


9- Use the table-look-up method to simulate random variables $X$ from $U(0,1)$. Where the p.d.f of $X$ has logistic distribution as follows:

$$
f(x)=e^{-x} /\left(1+e^{-x}\right)^{2}, \quad-\infty<x<\infty \quad 10 \%
$$

Also, find the value of $X$ when $U=0.5$
(Hint: see page 95-96)
$\mathrm{X}=$
$\mathrm{X} \mid \mathrm{u}=0.5=$

