
1- In the mixed congruential generator:

$$x_{n+1} = 1001x_n + 111, (\text{mod } 100)$$

Simulate the first five numbers with seed $x_0 = 11$.

$x(0)=11, x(1)= 22, x(2)= 33, x(3)= 44, x(4)= 55, x(5)= 66$	10%
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(Hint: See page 58-61)

2- Derive the probability density function (pdf, $f(x)$) of the random variable X where:

$$X = -\log_e U \quad \text{with } U(0,1) \quad 10\%$$

(Hint: See page 33-34)

$f(x) = e^{-x}$

$f(x) = f(u) |du/dx|, dx/du = -1/u, |du/dx| = u,$
 $f(x) = 1 * u = e^{-x}$
 $f(u) = 1, x = -\log_e(u), u = e^{-x}$

3- Simulate the normal distributed random variables ($N1, N2$) by using The Box-Muller method from the following $U1, U2$ uniform distributed random variables: $U1=0.4, U2=0.5$

$(N1 = -1.34, N2 = 0)$	10%
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(Hint: See page 78 Eq. 4.1)

$N1 = (-2 \log_e U1)^{0.5} \cos(2 \pi U2) = (-2 \log_e 0.4)^{0.5} \cos(2 * \pi * 0.5) = -[-2 * (-0.9)]^{0.5}$
 $N1 = (-2 \log_e U1)^{0.5} \sin(2 \pi U2) = (-2 \log_e 0.4)^{0.5} \sin(2 * \pi * 0.5) = 0 * [-2 * (-0.9)]^{0.5}$
 $N1 = -1.34$
 $N2 = 0$

4- In Poisson distribution, if $\lambda = 5$, find: $Pr[x=5]=?, E[x]=?$ and $Var[x]=?$

$Pr[x=5] = 0.174, E[x] = 5, Var[x] = 5$

(Hint: See page 18, Chp. 2.7) 10%

$$Pr(X = i) = \frac{e^{-\lambda} \lambda^i}{i!}, E[X] = \lambda = 5, Var[X] = \lambda = 5$$

$$Pr(X = 5) = \frac{e^{-5} 5^5}{5!} = \frac{0.0067 * 3125}{120} = 0.174$$

5- In Randomised Response Technique (RRT), if we have:

$Pr[Yes|N]=0.8$ (answering probability to non-embarrassing question).

$Pr[Yes]=0.8$ (total probability from survey).

$p_0=0.6$ (probability for answering to non-embarrassing question).

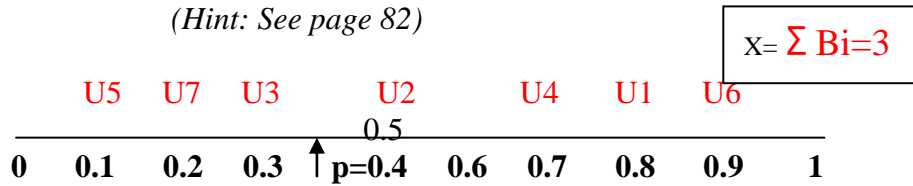
Find $Pr[Yes|E]=?$ (answering probability to embarrassing question).

(Hint: See page 51) 10%

$Pr[Yes] = Pr[Yes|N] p_0 + Pr[Yes|E] (1 - p_0)$
 $0.8 = 0.8 * 0.6 + Pr[Yes|E] * 0.4$
 $Pr[Yes|E] = (0.8 - 0.48) / 0.4 = 0.8$

$Pr[Yes E] = 0.8$

- 6- Simulate a Binomial random variable X with $B(7,0.4)$ from a set of uniform random variables $U(0,1)$, by using Bernouli random variable, where:
 $U1=0.8, U2=0.5, U3=0.3, U4=0.7, U5=0.1, U6=0.9, U7=0.2$ 10%
 (Hint: See page 82)



- 7- Simulate a Poisson distribution random variable, K , with parameter $\lambda=0.5$ from the following uniform random variables:
 $U1=0.8, U2=0.9, U3=0.5, U4=0.2$
 (Hint: See page 84) 10%

K is Poisson if: $\prod_{i=1}^k U_i < e^{-\lambda} = e^{-0.5} = 0.6$ K=3

K=1 ? $U1=0.8 > 0.6$ then not Poisson

K=2 ? $U1*U2=0.8*0.9=0.72 > 0.6$ then not Poisson

K=3 ? $U1*U2*U3=0.8*0.9*0.5=0.36 < 0.6$ then K=3 is Poisson

- 8- Simulate the random variable X with the following probabilities:
 (Hint: see page 93,94) 10%

I	0	1	2	3	4	5	6
Pr [X<I]	0.22	0.32	0.52	0.76	0.88	0.96	0.99

From a $U(0,1)$ in the following table:

U	0.87	0.44	0.95	0.25	0.97	0.65	0.75
X	4	2	5	1	6	3	3

- 9- Simulate random variable X with geometric distribution and $p=0.3$ from $U(0,1)=0.34$
 (Hint: See page 93 Eq. 5.4) 10%

$X = 4$

$$X = 1 + \left[\frac{\log_e U}{\log_e (1 - p)} = 1 + \frac{\log_e 0.34}{\log_e (1 - 0.3)} = 1 + \frac{-1.08}{-0.356} = 1 + 3.02 \right]$$

X=4
