Simulation Exam   Name: University of the Ryukyus
3-rd year undergraduate   No: Faculty of Engineering
2012-2-10 Last Term Examination   Department of Information Eng.
Time: 90 minutes   (write answers in boxes)   Prof. M.R. Asharif

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1- ***In the mixed congruential generator:***

$$x_{n+1} = 1001x_n + 111, (\text{mod } 100)$$

Simulate the first five numbers with seed $x_0 = 11$ **.**

x(0)=11, x(1)= 22 ,x(2)= 33 ,x(3)= 44 ,x(4)= 55 , x(5)= 66

10%

(Hint: See page 58-61)

2- ***Derive the probability density function (pdf, f(x)) of the random variable X where:***

X= $-\log_e U$ ***with U(0,1)***   *10%*

(Hint: See page 33-34)

f(x)= e^{-x}

f(x)=f(u)|du/dx|, dx/du=-1/u, |du/dx|=u,
f(x)=1*u=u= e^{-x}
f(u)=1,   x= - loge(u),   u=e^{-x}

3- ***Simulate the normal distributed random variables (N1, N2) by using The Box-Muller method from the following U1, U2 uniform distributed random variables: U1=0.4, U2=0.5***

(N1= -1.34 ,N2= 0

10%

(Hint: See page 78 Eq. 4.1)

N1=$(-2\log_e U1)^{0.5}\cos(2\pi U2)$ = $(-2\log_e 0.4)^{0.5}\cos(2*\pi*0.5)$= - $[-2*(-0.9)]^{0.5}$
N1=$(-2\log_e U1)^{0.5}\sin(2\pi U2)$ = $(-2\log_e 0.4)^{0.5}\sin(2*\pi*0.5)$= 0* $[-2*(-0.9)]^{0.5}$
N1= - 1.34
N2=0

4- ***In Poisson distribution, if*** λ***=5, find: Pr[x=5]=? , E[x]=? and Var[x]=?***

Pr[x=5]= 0.174, E[x]= 5, Var[x]= 5

(Hint: See page 18, Chp. 2.7)   *10%*

$$\Pr(X = i) = \frac{e^{-\lambda}\lambda^i}{i!}$$, E[X]= λ =5,   Var[X]= λ = 5

$$\Pr(X = 5) = \frac{e^{-5}\lambda 5}{5!} = \frac{0.0067 * 3125}{120} = 0.174$$

5- ***In Randomised Response Technique (RRT), if we have:***

Pr[Yes|N]=0.8   (answering probability to non-embarrassing question).
Pr[Yes]=0.8   (total probability from survey).
p_0 =0.6   (probability for answering to non-embarrassing question).
Find Pr[Yes|E]=? (answering probability to embarrassing question).

(Hint: See page 51)   *10%*

Pr[Yes|E]= 0.8

Pr[Yes]= Pr[Yes|N] p_0 + Pr[Yes|E] (1- p_0)
0.8=0.8*0.6+ Pr[Yes|E]*0.4
Pr[Yes|E]=(0.8-0.48)/0.4 = 0.8

6- ***Simulate a Binomial random variable X with B(7,0.4) from a set of uniform random variables U (0,1), by using Bernouli random variable, where:***
U1=0.8, U2=0.5, U3=0.3, U4=0.7, U5=0.1, U6=0.9, U7=0.2 *10%*
(Hint: See page 82)

X= Σ Bi=3

U5 U7 U3 U2 U4 U1 U6

0.5

0 0.1 0.2 0.3 ↑p=0.4 0.6 0.7 0.8 0.9 1

7- ***Simulate a Poisson distribution random variable, K, with parameter λ=0.5 from the following uniform random variables:***
U1=0.8, U2=0.9, U3= 0.5, U4= 0.2
(Hint: See page 84) *10%*

K= 3

K is Poisson if: $\prod_{i=1}^{k} U_i < e^{-\lambda} = e^{-0.5} = 0.6$

K=1 ? U1=0.8>0.6 then not Poisson

K=2 ? U1*U2=0.8*0.9=0.72>0.6 then not Poisson

K=3 ? U1*U2*U3=0.8*0.9*0.5=0.36<0.6 then K=3 is Poisson

8- ***Simulate the random variable X with the following probabilities:***
(Hint: see page 93,94) *10%*

I	0	1	2	3	4	5	6
Pr [X<I]	0.22	0.32	0.52	0.76	0.88	0.96	0.99

From a U(0,1) in the following table:

U	0.87	0.44	0.95	0.25	0.97	0.65	0.75
X	4	2	5	1	6	3	3

9- ***Simulate random variable X with geometric distribution and p=0.3from U(0,1)=0.34***
(Hint: See page 93 Eq. 5.4) *10%*

X= 4

$$X = 1 + \left[\frac{\log_e U}{\log_e (1-p)} = 1 + \frac{\log_e 0.34}{\log_e (1-0.3)} = 1 + \frac{-1.08}{-0.356} = 1 + 3.02 \right]$$

X=4

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