Simulation Exam Name: University of the Ryukyus
3-rd year undergraduate No: Faculty of Engineering
2012-2-10 Last Term Examination
Time: $\mathbf{9 0}$ minutes (write answers in boxes)
Department of Information Eng.
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1- In the mixed congruential generator:

$$
x_{n+1}=1001 x_{n}+111,(\bmod 100)
$$

Simulate the first five numbers with seed $x_{0}=11$.

$$
\mathrm{x}(0)=1, \quad \mathrm{x}(1)=\quad, \mathrm{x}(2)=\quad, \mathrm{x}(3)=\quad, \mathrm{x}(4)=\quad, \mathrm{x}(5)=
$$

(Hint: See page 58-61)
2- Derive the probability density function $(p d f, f(x))$ of the random variable $X$ where:

$$
X=-\log _{e} U \quad \text { with } U(0,1)
$$

(Hint: See page 33-34)
10\%
$f(x)=$

3- $\quad$ Simulate the normal distributed random variables (N1, N2) by using The BoxMuller method from the following U1, U2 uniform distributed random variables: $U 1=0.4, U 2=0.5$
( $\mathrm{N} 1=\quad$, $2=$

10\%
(Hint: See page 78 Eq. 4.1)

4- In Poisson distribution, if $\lambda=5$, find: $\operatorname{Pr}[x=5]=$ ?, $E[x]=$ ? and $\operatorname{Var}[x]=$ ?

$$
\operatorname{Pr}[\mathrm{x}=5]=\quad \mathrm{E}[\mathrm{x}]=\quad \operatorname{Var}[\mathrm{x}]=
$$

(Hint: See page 18, Chp. 2.7)

5- In Randomised Response Technique (RRT), if we have:
$\operatorname{Pr}[$ Yes $\mid N]=0.8$ (answering probability to non-embarrassing question).
$\operatorname{Pr}[$ Yes $]=0.8$ (total probability from survey).
$p_{0}=0.6 \quad$ (probability for answering to non-embarrassing question).
Find $\operatorname{Pr}[Y e s \mid E]=$ ? (answering probability to embarrassing question).
(Hint: See page 51)

6- Simulate a Binomial random variable $X$ with $B(7,0.4)$ from a set of uniform random variables $U(0,1)$, by using Bernouli random variable, where: $\boldsymbol{U} 1=0.8, \boldsymbol{U} 2=0.5, \boldsymbol{U} 3=0.3, \boldsymbol{U} 4=0.7, \boldsymbol{U} 5=0.1, \boldsymbol{U} 6=0.9, \boldsymbol{U} 7=0.2 \quad 10 \%$ (Hint: See page 82)


7- Simulate a Poisson distribution random variable, $K$, with parameter $\lambda=0.5$ from the following uniform random variables:

$$
U 1=0.8, U 2=0.9, U 3=0.5, U 4=0.2
$$

(Hint: See page 84)


8- Simulate the random variable $X$ with the following probabilities:
(Hint: see page 93 )
10\%

| I | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Pr}[\mathrm{X}<\mathrm{I}]$ | 0.22 | 0.32 | 0.52 | 0.76 | 0.88 | 0.96 | 0.99 |

From a $\mathbf{U}(0,1)$ in the following table:

| U | 0.87 | 0.44 | 0.95 | 0.25 | 0.97 | 0.65 | 0.75 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| X |  |  |  |  |  |  |  |

9- Simulate random variable $X$ with geometric distribution and $p=0.3$ from $\boldsymbol{U}(0,1)=0.34$
(Hint: See page 93 Eq. 5.4)


