Simulation Exam **Name:**
3-rd year undergraduate No:
2013-2-8 Last Term Examination
Time: 90 minutes (write answers in boxes)

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1- In the mixed congruential generator:

$$x_{n+1} = 43x_n + 11, (\text{mod}\,100)$$

Simulate the first five numbers with seed $x_0 = 1$ **.**

x(0)=1, **x(1)= 54, x(2)= 33, x(3)= 30, x(4)= 1 , x(5)= 54**

10%

(Hint: See page 58-61)

2- If y=e $^{-x}$ and x is a random variable with the normal p.d.f , that is:

$$f(x) = e^{-x}$$

Then, find the probability density function (p.d.f) of random variable, y; f(y).

10%

(Hint: See page 33)

f(y) =**1**

f(y)=f(x) |dx/dy|, dy/dx= - e $^{-x}$
f(y)= 1

3- In Randomised Response Technique (RRT), if we have:

Pr[Yes|N]=0.8 (answering probability to non-embarrassing question).
Pr[Yes]=0.9 (total probability from survey).
p_0 =0.5 (probability for answering to non-embarrassing question).
Find Pr[Yes|E]=? (answering probability to embarrassing question).

(Hint: See page 51)

Pr[Yes]= Pr[Yes|N] p_0 + Pr[Yes|E] (1- p_0)
0.9=0.8 * 0.5 + Pr[Yes|E] * 0.5

Pr[Yes|E]=**1**

4- Simulate the Gamma distributed random variables, G, with $\Gamma(n,\lambda)$ **for n=6,** $\lambda = 0.7$
from the following uniform distributed random variables, U(0,1):
U1=0.4, U2=0.9, U3=0.6, U4=0.8, U5=0.7, U6=0.5

10%

(Hint: See page 82)

G= **4**

***G*= - (1/λ)log$_e$ ($\Pi^6_{i=1}$ U_i)**
***G*= - (1/0.7) log$_e$ (0.4* 0.9* 0.6* 0.8* 0.7* 0.5)**
***G*= - (1/0.7) log$_e$ (0.06048) = - (1/0.7) (-2.805)= 4**

5- Find the probability of S=k, if we have the following relation:

S=X+Y

Where both random variables X and Y have the Geometric distribution:
Pr[X=i]= q^{i-1} p and Pr[Y=i]= q^{i-1} p (where: i >= 1, p=q= 0.5, k=3)

(Hint: See page 16 & 38) *10%*

Pr[S = k] = $\sum^k_{i=1}$ Pr[X=i]*Pr[Y=k-i]

Pr[S = k] = $\sum^k_{i=1} q^{i-1}\, p\, q^{k-i-1}\, p = p^2 q^{k-2} \sum^k_{i=1} (1)$

Pr[S = k] = $p^2 q^{k-2}$ k = 0.5^3 * 3 = 0.375

Pr[S=K]= **0.375**

6- **Simulate the normal distributed random variables *(N1, N2)* by using Polar-Marsaglia method (rejection method) from each pair of the following uniform distributed random variables:** *(Hint: See page 80)*

(V1,V2)=(- 0.3,0.4) , (V1,V2)=(0.6, - 0.8), (V1,V2)=(- 0.7,0.8)

10%

(N1,N2)= **(1.33, -0.999)** , (N1,N2)= **(0, 0)** , (N1,N2)= **Rejected**

$W=V1^2+V2^2=0.09+0.16=0.25$

$N1=V2[(-2\log_e W)/W]^{0.5}=0.4[(-2\log_e 0.25)/0.25]^{0.5}=0.4[(-2*-1.39)/0.25]^{0.5}=0.4*3.33=1.33$

$N2=V1[(-2\log_e W)/W]^{0.5}= - 0.3[(-2\log_e 0.25)/0.25]^{0.5}= - 0.3 [(-2*1.39)/0.25]^{0.5}$

$N2 = - 0.3*3.33= -0.999$

$W=V1^2+V2^2=0.36+0.64=1$

$N1=V2[(-2\log_e W)/W]^{0.5}= - 0.8[(-2\log_e 1)/1]^{0.5}=0$ & $N2=0$

$W=V1^2+V2^2=0.49+0.64=1.13 >1$ ➔ Rejected

7- **Simulate the random variable X with the following probabilities:**

(Hint: see page 93, 94) *10%*

I	0	1	2	3	4	5	6
Pr [X<I]	0.334	0.456	0.675	0.735	0.894	0.982	0.998

From a U(0,1) in the following table:

U	0.8901	0.6764	0.5802	0.7394	0.4523	0.3381	0.6771
X	4	3	2	4	1	1	3

8- **In the following chaotic system:** *10%*

$$x(n+1)=4\,r\,x(n)\,[1-x(n)]$$

If *r=0.7*, find the attractor of this chaotic system.

x(∞)= *0.642857*

(*Hint: See chap. 6, page 136*)

If *r=0.91* show that there is no attractor and system is chaotic.

For r= 0.7:

$y=2.8 x-2.8 x^2$

$y=x$

$2.8 x^2- 1.8 x=0$

$X = 1.8/2.8 = 0.642857$ (attractor) $= y$

For r= 0.91 : $y=3.64 x- 3.64 x^2$, $y=x$, ==➔ $x= 2.64/3.64 = 0.725$

$y'=3.64-7.28 x =0$ ==➔ $x=3.64/7.28 = 0.5$

$y_{max}= 3.64 X 0.5 – 3.64 X 0.25 = 0.91$

9- ***In Geometric distribution, if p=0.3, find: Pr[x=4]=? , E[x]=? and Var[x]=?***

For: i=4

$Pr(X=i)= q^{i-1}p=0.7^3*0.3=0.103$

$E(X)=1/p=1/0.3=3.33$

$Var(X)= q/p^2 =0.7/0.09 = 7.77$

Pr[x=4]= **0.103** E[x]= **3.33** Var[x]= **7.77**

(Hint: See page 16, Chp. 2.7) *10%*

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