Simulation Exam **Name:**
3-rd year undergraduate **No:**
2014-1-31 Last Term Examination
Time: 90 minutes (write answers in boxes)

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1- *In the mixed congruential generator:*

$$x_{n+1} = 201x_n + 11, (\text{mod}\,100)$$

Simulate the first ten numbers with seed $x_0 = 1$ **.**

Can you find a rule in sequence, and estimate the cyle length? *10%*

x(0)=1 , ***x(1)=12 , x(2)=23 , x(3)=34 , x(4)=45 ,x(5)=56 x(6)=67 , x(7)=78 , x(8)=89 , x(9)= 0 , There is a clear rule for successive sequences and the full 100 lengths will be generated.***

(Hint: See page 58-61)

2- *If* ***y=arctan(x)*** ***and x is a random variable with the standard Cauchy p.d.f , that is:*** *10%*

$$f(x) = \frac{1}{\pi(1+x^2)}$$

Then, find the probability density function (p.d.f) of random variable, y, f(y).

(Hint: See page 33)

f(y) =***1/π***

f(y) = f(x) |dx/dy|, y = arctan(x), x = tan(y), dx/dy = 1/cos²(y)

f(y) = [1/π(1+x²)][1/cos²(y)] = [1/π(1+tan²(y))][1/cos²(y)]

f(y) = 1/π

3- *In Randomised Response Technique (RRT), if we have:*

Pr[Yes|N]=0.8 (answering probability to non-embarrassing question).

Pr[Yes]=0.8 (total probability from survey).

p_0 =0.5 (probability for answering to non-embarrassing question).

Find Pr[Yes|E]=? (answering probability to embarrassing question).

(Hint: See page 51) *10%*

Pr[Yes]= Pr[Yes|N] p_0 + Pr[Yes|E] (1- p_0)

0.8 = 0.8 * 0.5 + Pr[Yes|E] * 0.5

Pr[Yes|E] = 0.4/0.5 = 0.8

Pr[Yes|E]= **0.8**

4- *Simulate random variable X with exponential distribution from U(0,1)=0.5*

(Hint: See page 81 or page 24) *10%*

X= **0.69**

X= - $\log_e U$ = - $\log_e 0.5$ = 0.69

5- *In the following iterative function (Julia Sets):* *10%*

$$a(n+1)=[a(n)]^2-0.39$$

If a(0)=0.2i, where i=$\sqrt{-1}$, find the attractor of this function, a(∞).

(Hint: See Julia Sets)

a(∞)= - 0.3

a(1) = $(0.2i)^2$-0.39 = - 0.04- 0.39 = - 0.43

a(2) = $(- 0.43)^2$- 0.39 = 0.1849 - 0.39 = - 0.2051

a(3) = $(- 0.2051)^2$ – 0.39 = - 0.3479

n= ∞, a(∞) = x, x^2-x-0.39=0, x = (1- 1.6)/2 = - 0.3

6- ***Simulate a Binomial random variable X with B(8,0.54) from a set of uniform random variables U (0,1), by using Bernouli random variable, where:***
U1=0.8, U2=0.3, U3=0.5, U4=0.1, U5=0.4, U6=0.1, U7=0.6, U8=0.7 *10%*
(Hint: See page 82)

$X = \Sigma B_i = 5$

U6,U4 U2 U5 U3 U7 U8 U1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
p=0.54

7- ***Simulate the normal distributed random variables (N1, N2) by using Polar-Marsaglia method (rejection method) from each pair of the following uniform distributed random variables:*** *(Hint: See page 80)*
(V1,V2)=(0.3,0.2) , (V1,V2)=(- 0.6, - 0.9), (V1,V2)=(0.6,- 0.8) *10%*

(N1,N2) = ***(1.12, 1.68)*** , (N1,N2) = ***Rejected*** , (N1,N2) = ***(0, 0)***

$W=V1^2+V2^2=0.09+0.04=0.13$
$N1=V2[(-2\log_e W)/W]^{0.5}=0.2[(-2\log_e 0.13)/0.13]^{0.5}= 0.2[(-2*-2.04)/0.13]^{0.5}=0.2*5.6=1.12$
$N2=V1[(-2\log_e W)/W]^{0.5}= 0.3[(-2\log_e 0.13)/0.13]^{0.5}= 0.3[(-2*-2.04)/0.13]^{0.5}=0.3*5.6=1.68$

$W=V1^2+V2^2=0.36+0.81=1.17 >1$ ➔ Rejected

$W=V1^2+V2^2=0.36+0.64=1$
$N1=V2[(-2\log_e W)/W]^{0.5}= - 0.8[(-2\log_e 1)/1]^{0.5}=0$ & $N2=0$

8- ***Use the table-look-up method to simulate random variables X from U(0,1). Where the p.d.f of X has logistic distribution as follows:***
$f(x)=e^{-x}/(1+e^{-x})^2$, $-\infty < x < \infty$ *10%*
Also, find the value of X when U=0.5
(Hint: see page 95-96)

X= $\log_e [U/(1-U)]$
X|u=0.5 = ***0***

$$U=F(x)=\int_{-\infty}^{x} f(x)dx= \int_{-\infty}^{x} e^{-x}/(1+e^{-x})^2 dx= [1/(1+e^{-x})]^{x}_{-\infty}$$
$$1+e^{-x}= 1/U, \quad X=\log_e [U/(1-U)], \quad for: U=0.5, \quad X=0$$

9- ***Simulate the random variable X with the following probabilities:***
(Hint: see page 93, 94) *10%*

I	0	1	2	3	4	5	6
Pr [X<I]	0.143	0.256	0.472	0.614	0.721	0.843	0.976

From a U(0,1) in the following table:

U	0.632	0.934	0.811	0.148	0.201	0.480	0.971
X	***4***	***6***	***5***	***1***	***1***	***3***	***6***

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