Simulation Exam **Name:** **University of the Ryukyus**
3-rd year undergraduate **No:** **Faculty of Engineering**
2015-2-6 Last Term Examination **Department of Information Eng.**
Time: 90 minutes (write answers in boxes) **Prof. M.R. Asharif**

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1- In Poisson distribution, if $\lambda=2$, i=4, find: Pr[x=4]=? , E[x]=? and Var[x]=?
(Hint: See page 18, Chp. 2.7) *10%*

Pr[x=4]= 0.09, E[x]= 2, Var[x]= 2

$$\Pr(X=i)=\frac{e^{-\lambda}\lambda^{i}}{i!}$$, E[X]= λ =2, Var[X]= λ = 2

$$\Pr(X=5)=\frac{e^{-2}2^{4}}{4!}=\frac{16}{24*7.39}=0.09$$

2- In Randomised Response Technique (RRT), if we have:
Pr[Yes|N]=0.4 (answering probability to non-embarrassing question).
Pr[Yes]=0.4 (total probability from survey).
p_0 =0.4 (probability for answering to non-embarrassing question).
Find Pr[Yes|E]=? (answering probability to embarrassing question).
(Hint: See page 51) *10%*

Pr[Yes|E]= ***0.4***

Pr[Yes]= Pr[Yes|N] p_0 + Pr[Yes|E] (1- p_0)
0.4 = 0.4* 0.4+ Pr[Yes|E] * 0.6
Pr[Yes|E] = 0.24/0.6 = 0.4

3- In the mixed congruential generator:

$$x_{n+1}=21x_n+11,(\mathrm{mod}10)$$

Simulate the first ten numbers with seed $x_0=1$.
Does it have full cycle length?
(Hint: See page 58-61) *10%*

x(0)=1 , ***x(1)=2 , x(2)=3 , x(3)=4 , x(4)=5 ,x(5)=6 x(6)=7 , x(7)=8 , x(8)=9 , x(9)= 0 , There is a clear rule for successive sequences and the full 10 lengths will be generated.***

4- The first pseudo-random number generator proposed by Von Neuman (1951) was the "mid-square (MS)" such that:
x(0)=6666 , x(1) = MS(6666)=Mid(6666)^2 =44<u>4355</u>56, x(2)=9660.
Find the first ten x(0)~x(10) numbers.
(Hint: See page 71) *10%*

x(0)= 6666, x(1)= 4355, x(2)= 9660, x(3)= 3156, x(4)= 9603, x(5)= 2176, x(6)= 7349, x(7)= 78, x(8)= 6084, x(9)= 150, x(10)= 225

5- Simulate the normal distributed random variables (N1, N2) by using The Box-Muller method from the following U1, U2 uniform distributed random variables: U1=0.6, U2=0.25
(Hint: See page 78 Eq. 4.1) *10%*

N1= 0, N2= 1

$N1=(-2\log_e U1)^{0.5}\cos(2\pi U2)= (-2\log_e 0.6)^{0.5}\cos(2*\pi*0.25)= 0*[-2*(-0.5)]^{0.5}$

$N2=(-2\log_e U1)^{0.5}\sin(2\pi U2)= (-2\log_e 0.6)^{0.5}\sin(2*\pi*0.25)= 1*[-2*(-0.5)]^{0.5}$

N1= 0, N2=1

6- *Two independent uniform random numbers with U(0,1) are given in the binary form as below:* *U1=0.010111010*
U2=0.111110111

Simulate the binomial distribution B(9,1/2) random variables, X1, from U1 and X2, with B(9,1/4) from U1 and U2. *10%*

(*Hint: See page 83*)

X1= **5**
X2= **4**

B(9,1/2) → X1 = 5

U1 ○U2 = 0.010110010

B(9,1/4) → X2= 4

7- ***Simulate a Poisson distribution random variable, K, with parameter $\lambda = 2$ from the following uniform random variables:***
U1= 0.8, U2= 0.9, U3= 0.7, U4= 0.6, U5=0.8, U6= 0.3, U7=0.5 *10%*

(Hint: See page 84)

K=6

$$\Pi_{i=1}^{K} U_i \le e^{-\lambda} = e^{-2} = 0.135$$

K=1 : 0.8 > 0.135 →K=1 is not Poisson
K=2 : 0.8*0.9 =0.72 > 0.135 →K=2 is not Poisson
K=3 : 0.8*0.9*0.7 =0.504 > 0.135 →K=3 is not Poisson
K=4 : 0.8*0.9*0.7*0.6 =0.3024 > 0.135 →K=4 is not Poisson
K=5 : 0.8*0.9*0.7*0.6*0.8 =0.242 > 0.135 →K=5 is not Poisson
K=6 : 0.8*0.9*0.7*0.6*0.8*0.3 =0.0.0726 < 0.135 →K=6 is Poisson

8- ***Simulate random variable X with geometric distribution and p=0.433from U(0,1)=0.567***

(Hint: See page 93 Eq. 5.4) *10%*

x= 2

$$X = 1 + \left\lceil \frac{\log_e U}{\log_e (1-p)} = 1 + \frac{\log_e 0.567}{\log_e (1-0.433)} = 1 + \frac{-0.567}{-0.567} = 1 + 1 \right\rceil$$

X=2

9- ***If the sequence x(n) has the following properties:***
x(0)=0, x(1)=x(2)=1
where: x(n)=x(n-1)+x(n-2)-x(n-3) for n>=3
Then, find x(100), by regression or simulation method. *10%*
We call it Plus-Plus-Minus sequence: PPM(n)

x(100) =**100/2=50**

x(3)=2, x(4)=2, x(5)=3, x(6)=3, x(7)=4, x(8)=4

X(n)=n/2 for n:even, x(n)=(n+1)/2 for n: odd

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