Simulation Exam **Name:** **University of the Ryukyus**
3-rd year undergraduate No: **Faculty of Engineering**
2016-2-5 Last Term Examination **Department of Information Eng.**
Time: 90 minutes (write answers in boxes) **Prof. M.R. Asharif**

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1- In Poisson distribution, if λ=3, i=6, find: Pr[x=6]=? , E[x]=? and Var[x]=?

(Hint: See page 18, Chp. 2.7) *10%*

Pr[x=6]= 0.05, E[x]= 3, Var[x]= 3

$$\Pr(X = i) = \frac{e^{-\lambda}\lambda^i}{i!}$$, E[X]= λ =3, Var[X]= λ = 3

$$\Pr(X = 6) = \frac{e^{-3}3^6}{6!} = \frac{729}{720 * 20} = 0.05$$

2- In the mixed congruential generator: $x_{n+1} = 53x_n + 7, (\bmod 13)$

Simulate the following cycles with seed $x_0 = 1$***. Notice, it has full cycle length.***

x(0)= 1	**x(1)= 8**	**x(2)= 2**	**x(3)= 9**	**x(4) = 3**	**x(5) = 10**	**x(6)= 4**
x(7) = 11	**x(8)= 5**	**x(9)= 12**	**x(10)= 6**	**x(11)= 0**	**x(12)= 7**	**x(13)= 1**

10%

(Hint: See page 60-61)

3- In Randomised Response Technique (RRT), if we have:

Pr[Yes|N]=0.5 ***(answering probability to non-embarrassing question).***
Pr[Yes]=0.8 ***(total probability from survey).***
p_0 =0.4 ***(probability for answering to non-embarrassing question).***
Find Pr[Yes|E]=? (answering probability to embarrassing question).

(Hint: See page 51) *10%*

Pr[Yes|E]= 1

Pr[Yes]= Pr[Yes|N] p_0 + Pr[Yes|E] (1- p_0)

Pr[Yes]= Pr[Yes|N] p_0 + Pr[Yes|E] (1- p_0)

0.8 = 0.5* 0.4+ Pr[Yes|E] * 0.6

Pr[Yes|E] = (0.8-0.2)/0.6 = 1

4- If y=arctan(x) and x is a random variable with the standard Cauchy p.d.f, that is: *10%*

$$f(x) = \frac{1}{\pi(1 + x^2)}$$

Then, find the probability density function (p.d.f) of random variable, y, f(y).

(Hint: See page 33)

f(y) = *1/π*

f(y) = f(x) |dx/dy|, y = arctan(x), x = tan(y), dx/dy = 1/cos^2(y)

f(y) = [1/π(1+x^2)][1/cos^2(y)] = [1/π(1+tan^2(y))][1/cos^2(y)]

f(y) = 1/π

5- In the following iterative function (Julia Mandelbrot Sets): *10%*

$$x(n+1)=[x(n)]^2-0.56$$

If $x(0)=0.1i$***, where*** $i=\sqrt{-1}$***, find the attractor of this function,*** $x(\infty)$***.***

(Hint: See Julia Mandelbrot Sets)

$x(\infty)= -0.4$

$x(1) = (0.1i)^2-0.39 = -0.01-0.56 = -0.57$

$x(2) = (-0.57)^2-0.56 = 0.3249-0.56 = -0.2351$

$x(3) = (-0.2351)^2 - 0.56 = -0.5047$

$n=\infty,\ x(\infty) = x,\ x^2-x-0.56=0,\ x = (1-1.8)/2 = -0.4$

6- Simulate the normal distributed random variables (N1, N2) by using Box-Muller method from the following pair of uniform distributed random variables: (U1,U2)=(0.8,0.25) *(Hint: See page 78 use Eq. 4.1)*

(N1,N2)= **(0, 0.63)**	*10%*

$$N1=(-2\log_e U1)^{0.5}\cos(2\pi U2)= (-2\log_e 0.8)^{0.5}\cos(2*\pi*0.25)= 0*[-2*(-0.2)]^{0.5}$$

$$N2=(-2\log_e U1)^{0.5}\sin(2\pi U2)= (-2\log_e 0.8)^{0.5}\sin(2*\pi*0.25)= 1*[-2*(-0.2)]^{0.5}$$

N1= 0, N2=0.63

7- Suppose that we have a set of uniform random variables: U={0.5, 0.8, 0.7, 0.3} simulate the exponential p.d.f. random variables, Ei, by using: Ei = - loge U_i***.*** *10%*

E1= 0.69	, E2= 0.22	, E3= 0.35	, E4= 1.2
S1= 0.69	, S2= 0.91	, S3= 1.26	, S4= 2.46

Then, from this set {Ei}, find $S_k = \sum_{i=1}^{k} E_i$ ***with*** $S_0=0$ ***and then simulate a random number, K, with Poisson distribution (for*** $S_k<1<S_{k+1}$***).***

(Hint: See page 83)

Since: $S_2<1<S_3$ ***Then: K-2***

K=**2**

8- Simulate a Binomial random variable X with B(8,0.83) from a set of uniform random variables U (0,1), by using Bernouli random variable, where: *10%*

U1=0.7, U2=0.6, U3=0.9, U4=0.5, U5=0.3, U6=0.4, U7=0.1, U8=0.2

(Hint: See page 82)

$X = \Sigma B_i = 7$

U7 U8 U5 U6 U4 U2 U1 U3

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

p=0.83

9- Use the table-look-up method to simulate random variables X from U(0,1). Where the p.d.f of X has logistic distribution as follows:

$$f(x)=e^{-x}/(1+e^{-x})^2, \quad -\infty< x<\infty$$ *10%*

Also, find the value of X when U=0.6

(Hint: see page 95-96)

X= $\log_e [U/(1-U)]$
X|u=0.6 = **0.4**

$$U=F(x)=\int_{-\infty}^{x} f(x)dx= \int_{-\infty}^{x} e^{-x}/(1+e^{-x})^2dx= [1/(1+e^{-x})]_{-\infty}^{x}$$

$$1+e^{-x}= 1/U,\quad X=\log_e [U/(1-U)],\ \text{for: } U=0.6,\ X=0.4$$