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Simulation Engineering | **University of the Ryukyus**
3rd-Year Undergraduate | **Faculty of Engineering**
Last-Term Examination | **Dept. of Information Eng.**
1999.2.18 | **Exam. Time: 90 minutes** | **Dr. Mohammad Reza Asharif**

Choose 7 of the following questions and answer

1- If the joint distribution: $f_{XY}(x, y) = 24xy$ for: $x + y = 1$, $x, y \geqslant 0$

Investigate that X, Y are independent or dependent random variables.

2- What is Monte Carlo Simulation? [Hint: No.5]

3- Use the table-look-up inversion method to simulate a random variables, X, with the following probability density function: $f(x)=6x(1-x)$ $0 \leq x \leq 1$ from U(0,1). [Hint: use the material in page 95, section 5.2]

4- The mixed congruential generator: $x_{n+1} = 9x_n + 13 \pmod{32}$

Has full (32) cycle length. Write down the resulting sequence of numbers and compare the numbers in the first half with those in the second half.

5- From U(0,1) simulate a random variable, X, with Binomial, B(n,p), distribution. Describe the procedure for that. [Hint: use page 82, or No.14]

6- From U(0,1) simulate a random variable, K, with Poisson distribution.

[Hint: use page 83, or No.14]

7- Explain the Box-Muller method for normal distribution random variables generation from a uniform random number. [Hint: use page 78, or No.13]

8- Explain the rejection by Polar Marsaglia method. [Hint: use page 79, or No.13]

9- Simulate a random variable, X, with Poisson distribution given in the following table:

[Hint: use page 93, or No.15]

I	0	1	2	3	4	5	6
Pr(X< I)	0.1353	0.4060	0.6767	0.8571	0.9473	0.9834	0.9955

From a uniform random number U(0,1), in the following table:

U	0.0318	0.5321	0.2489	0.6776	0.5678	0.6777	0.4059
X	0	2	1	3	2	3	1

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$x+y \leq 1$

1 — $f_{xy}(x,y) = 24xy$

$f_y(y) = \int_0^{1-y} 24xy\, dx = 24y\left[\frac{1}{2}x^2\right]_0^{1-y} = 12y(1-y)^2$

$f_x(x) = \int_0^{1-x} 24xy\, dy = 24x\left[\frac{1}{2}y^2\right]_0^{1-x} = 12x(1-x)^2$

$f_{XY}(\frac{1}{2},\frac{1}{2}) = 24 \times \frac{1}{2} \times \frac{1}{2} = 6$

$f_X(\frac{1}{2}) = 12 \times \frac{1}{2}(1-\frac{1}{2})^2 = \frac{3}{2}$

$f_Y(\frac{1}{2}) = 12 \times \frac{1}{2}(1-\frac{1}{2})^2 = \frac{3}{2}$

$f_{XY}(\frac{1}{2},\frac{1}{2}) \neq f_X(\frac{1}{2}) \cdot f_Y(\frac{1}{2})$

$6 \neq \frac{3}{2} \times \frac{3}{2}$

Then X, Y are not independent

2 — A simulation methodology which employs random number, $U(0,1)$, for solving certain stochastic or deterministic problems.

3 — $F_X(x) = \int_0^x f(y)\, dy = \int_0^x 6y(1-y)\, dy = 6\left[\frac{y^2}{2} - \frac{y^3}{3}\right]_0^x = 3x^2 - 2x^3$

$U = F_X(x) = 3x^2 - 2x^3$

$x = F^{-1}(U)$

solve for each value of U and find random variable x

4 — 0 – 13 – 2 – 31 – 4 – 17 – 6 – 3 – 8 – 21 – 10 – 7 – 12 – 25 – 14 – 11

16 – 29 – 18 – 15 – 20 – 1 – 22 – 19 – 24 – 5 – 26 – 23 – 28 – 9 – 30 – 27

Numbers in the two half-periods differ by 16

5 — 1- Generate $U_1, U_2, \dots, U_n$ (from $U(0,1)$)

2- If $U_i \leq p$ set $B_i = 1$, If $U_i > p$ set $B_i = 0$

3 – Find $X = \sum_{i=1}^{n} B_i$

Then X has $B(n,p)$ Binomial distribution

6 — 1 – E_i has $\lambda e^{-\lambda x}$ (exponential distribution by $X = -\log_e U$ and $E_i = \lambda X_i$?)

2 – $S_K = \sum_{i=1}^{K} E_i$ (S_K has $\Gamma(k,\lambda)$ distribution)

3 – If $S_k \leq 1 < S_{k+1}$ Then k has a poisson distribution

7 — $U_1, U_2 \in U(0,1)$ $\begin{cases} N_1 = (-2\log_e U_1)^{\frac{1}{2}} \cos(2\pi U_2) \\ N_2 = (-2\log_e U_1)^{\frac{1}{2}} \sin(2\pi U_2) \end{cases}$ $N_1, N_2 \to N(0,1)$

8 — $V_1, V_2 \to U(-1,1)$ $\begin{cases} N_1 = V_2\left(\frac{-2\log W}{W}\right)^{1/2} \\ N_2 = V_1\left(\frac{-2\log W}{W}\right)^{1/2} \end{cases}$ $W = V_1^2 + V_2^2$ $N_1, N_2 \to N(0,1)$

(V_1, V_2) are rejected proportion $1 - \frac{\pi}{4}$