My solution
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Digital Signal Processing
Undergraduate Course **Student's Name:**
Mid-Term Examination **Sudent's No.**
2008.6.13 [write your answer in the blocks, each one 10 score]

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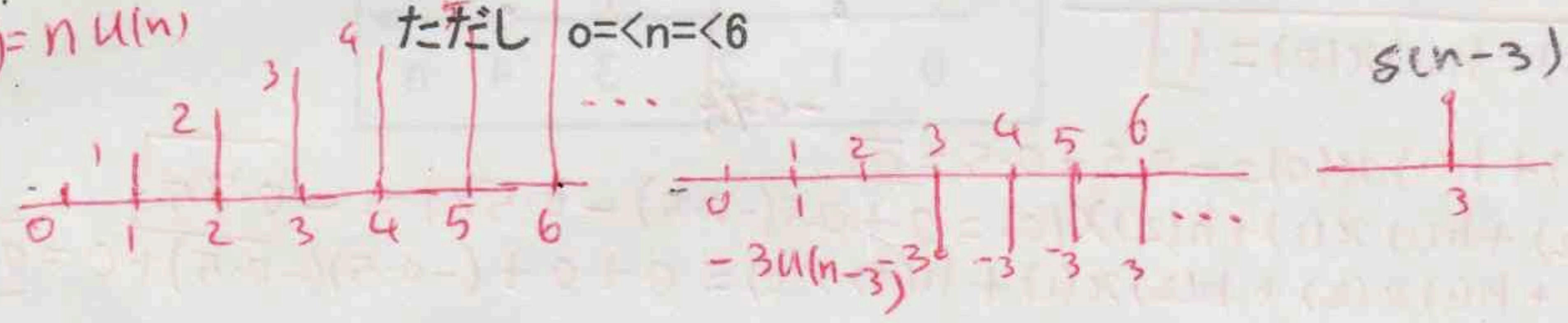
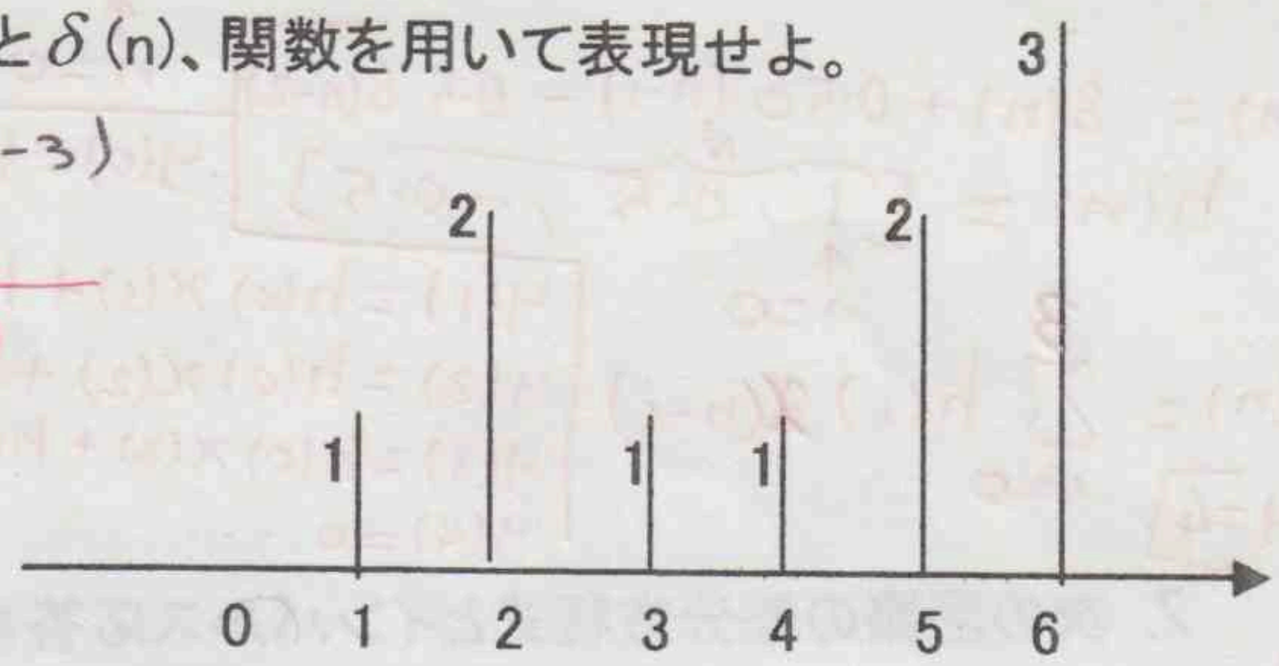
1. 図で信号、x(n)、を r(n)=nu(n)、と unit step (u(n))、とδ (n)、関数を用いて表現せよ。
ただし o=<n=<6

$r(n) = n\,u(n)$





x(n)= $n\,u(n) - 3\,u(n-3) + \delta(n-3)$

2・次の信号をポロットせよ。

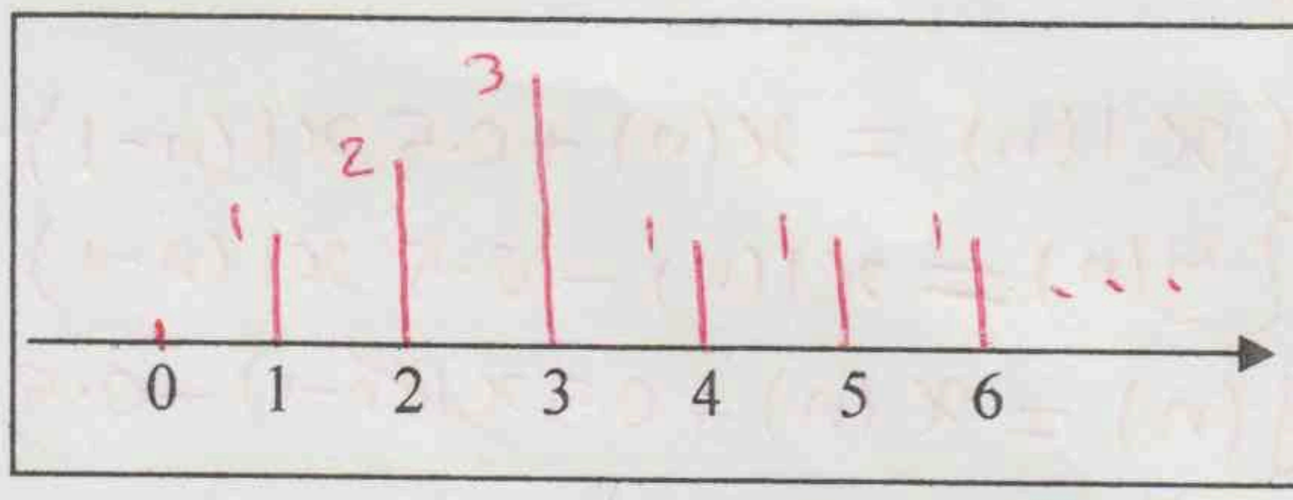
$x(n) = [\underset{n=0}{0}\ 1\ 2\ 3\ 1\ 1\ 1 \ldots]$

$$x(n) = nu(n) - nu(n-4) + u(n-4)$$

$n\,u(n) = [0\ 1\ 2\ 3\ \cancel{4}\ \cancel{5}\ \cancel{6} \ldots]$

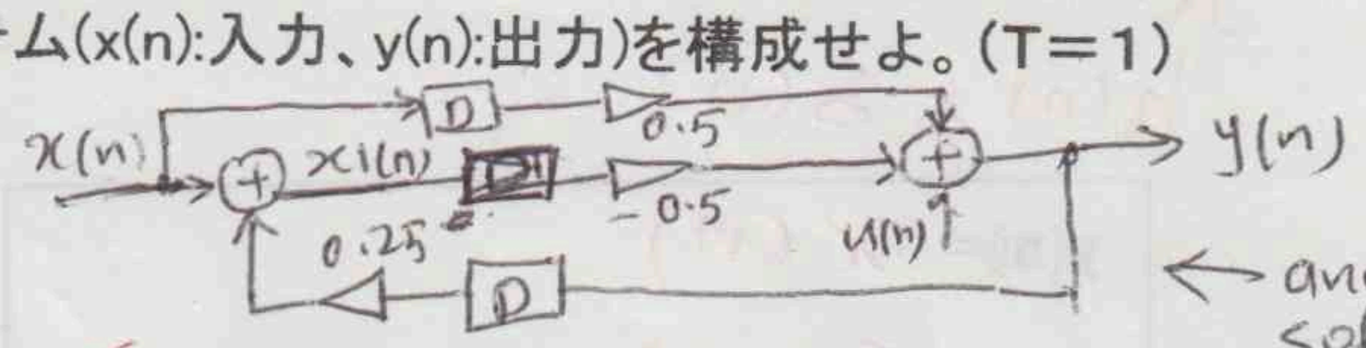
$-n\,u(n-4) = [0\ 0\ 0\ 0\ \cancel{-4}\ \cancel{-5}\ \cancel{-6} \ldots]$

$u(n-4) = [0\ 0\ 0\ 0\ 1\ 1\ 1 \ldots]$



3・以下の二つ差分方程式を満足する離散時間システム(x(n):入力、y(n):出力)を構成せよ。(T=1)

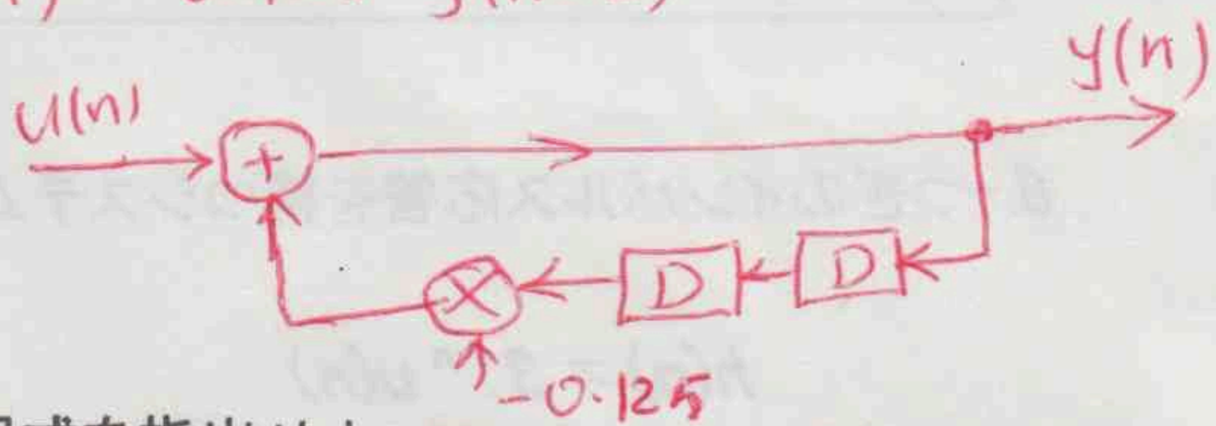
$$\begin{cases} x1(n) = x(n) + 0.25\,y(n-1) \\ y(n) = u(n) + 0.5x(n-1) - 0.5x1(n-1) \end{cases}$$



another solution

$y(n) = u(n) + 0.5\,\cancel{x(n-1)} - 0.5\,\cancel{x(n-1)} - 0.125\,y(n-2)$

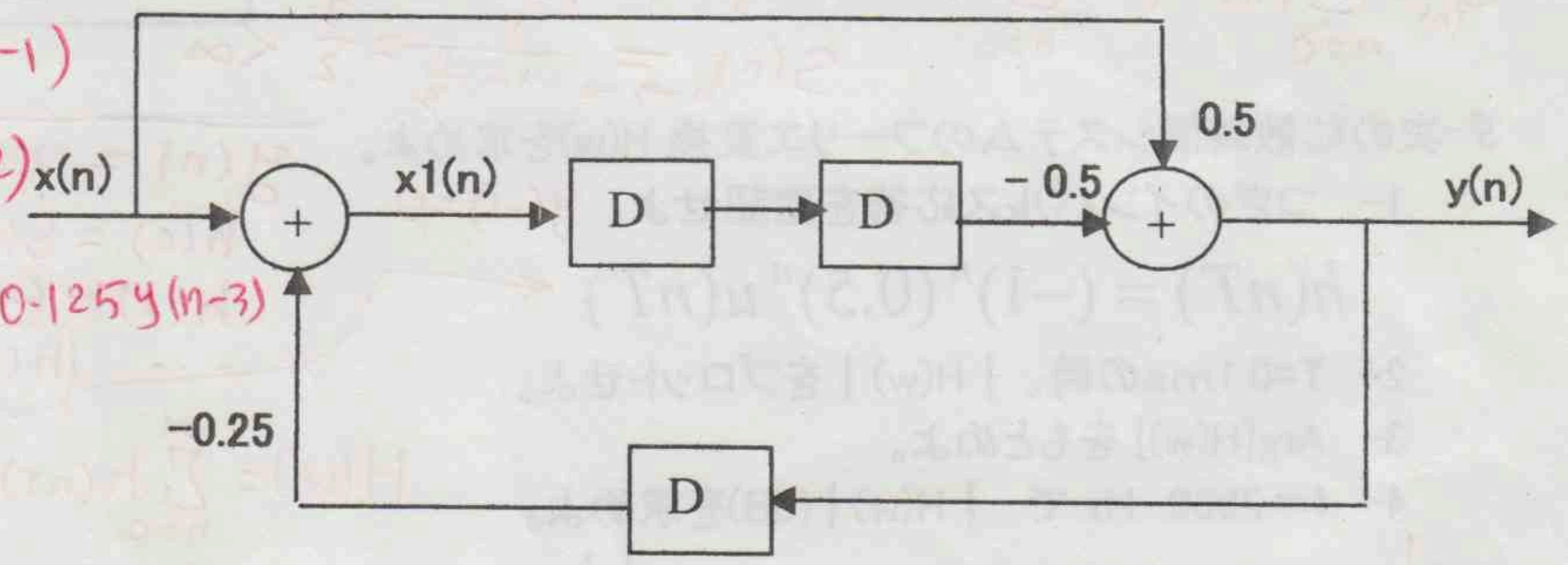
$\boxed{y(n) = u(n) - 0.125\,y(n-2)}$



4—図に示す離散時間システム(IIR Digital Filter) の差分方程式を指出せよ。
T=1

$$\begin{cases} x1(n) = x(n) - 0.25\,y(n-1) \\ y(n) = 0.5x(n) - 0.5x1(n-2) \end{cases}$$

$y(n) = 0.5x(n) - 0.5\,x(n-2) + 0.125\,y(n-3)$



y(n)= $0.5x(n) - 0.5x(n-2) + 0.125\,y(n-3)$

5・以下3個の入出力を示すシステムの線形性、時不変性、因果性、安定性を判定し、○か×で示せよ。

1) $y(nT) = e^{n}\,x(nT-T)$
2) $y(nT) = 2 + x(nT)$
3) $y(nT) = x^2(nT) + x(nT+T)$

	Linearity,	Shift Invariance,	Causality,	Stability	
1)	○	×	○	×	$y(n)\to\infty$, $n\to\infty$
2)	×	○	○	○	FIFO
3)	×	○	×	○	FIFO
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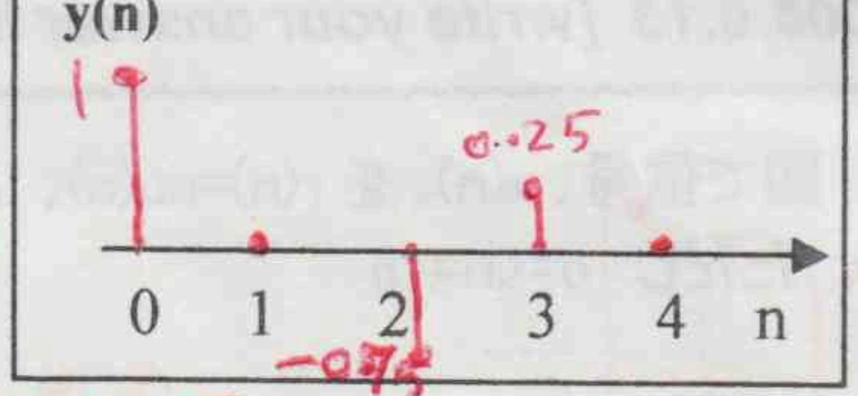
6 ・次の LSI システムでは、x(n) は入力で、出力 y(n)を計算せよ。

$y(n) = \left[\underset{\uparrow\, n=0}{1}, 0, -0.75, 0.25 \right]$

y(n) = x(n) + 0.5 x(n−1) − 0.5 x(n−2)

x(n) = δ (n) − 0.5δ (n−1) $= \left[\underset{\uparrow\, n=0}{1}, -0.5 \right]^{M}$



System impulse response → $h(n) = \delta(n) + 0.5\delta(n-1) - 0.5\,\delta(n-2)$

$h(n) = \left[\underset{\uparrow\, n=0}{1}, 0.5, -0.5 \right]^{N}$

$\boxed{y(0) = h(0)x(0) = 1}$

Convolution → $y(n) = \sum_{i=0}^{3} h(i)\,x(n-i)$

$M+N-1 = 2+3-1 = \boxed{4}$ length of convolution

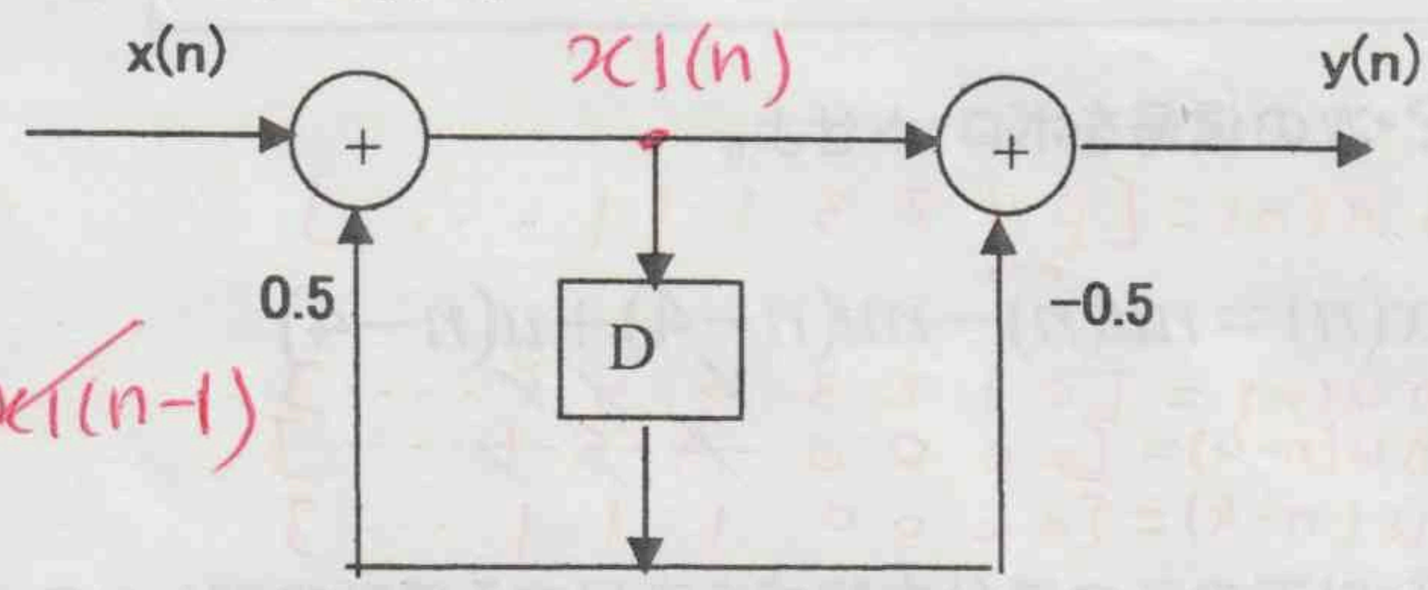
$y(1) = h(0)x(1) + h(1)x(0) = -0.5 + 0.5 = \boxed{0}$

$y(2) = h(0)x(2) + h(1)x(1) + h(2)x(0) = 0 + 0.5(-0.5) - 0.5\times 1 = \boxed{-0.75}$

$y(3) = h(0)x(3) + h(1)x(2) + h(2)x(1) + h(3)x(0) = 0 + 0 + (-0.5)(-0.5) + 0 = \boxed{0.25}$

$y(4) = 0 \ldots$

7. 次の回路の差分方程式とインパルス応答を求めよ。 y(−1)=0



$$\begin{cases} x1(n) = x(n) + 0.5\,x1(n-1) \\ y(n) = x1(n) - 0.5\,x1(n-1) \end{cases}$$

$y(n) = x(n) + \cancel{0.5\,x1(n-1)} - \cancel{0.5\,x1(n-1)}$

$y(n) = x(n)$

$h(n) = \delta(n)$

y(n)=	$x(n)$
h(n)=	$\delta(n)$

8. −つぎのインパルス応答を持つシステムは、安定かどうか判断せよ。(T=1)

h(n) = 3⁻ⁿ u(n)

安定	不安定

$S_{(n)} = \sum_{n=0}^{\infty} |h(n)| = \sum_{n=0}^{\infty} 3^{-n} = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

$S(n)_{n\to\infty} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2} < \infty$

9−次の離散時間システムのフーリエ変換 H(w)を求めよ。

1− つぎのインパルス応答を確認せよ。 $y(-1)=0$

$$h(nT) = (-1)^n (0.5)^n u(nT)$$

2− T=0.1msの時、| H(w) | をプロットせよ。

3− Arg[H(w)] をもとめよ。

4− f=2500 Hz で | H(w) | (dB)を求めよ。

$y(n) = x(n) - 0.5\,y(n-1)$

$h(0) = \delta(0) = 1,\ h(1) = -0.5\,h(0) = -0.5$

$h(2) = (-0.5)h(1) = 0.5^2,\ h(3) = -(0.5)^3$

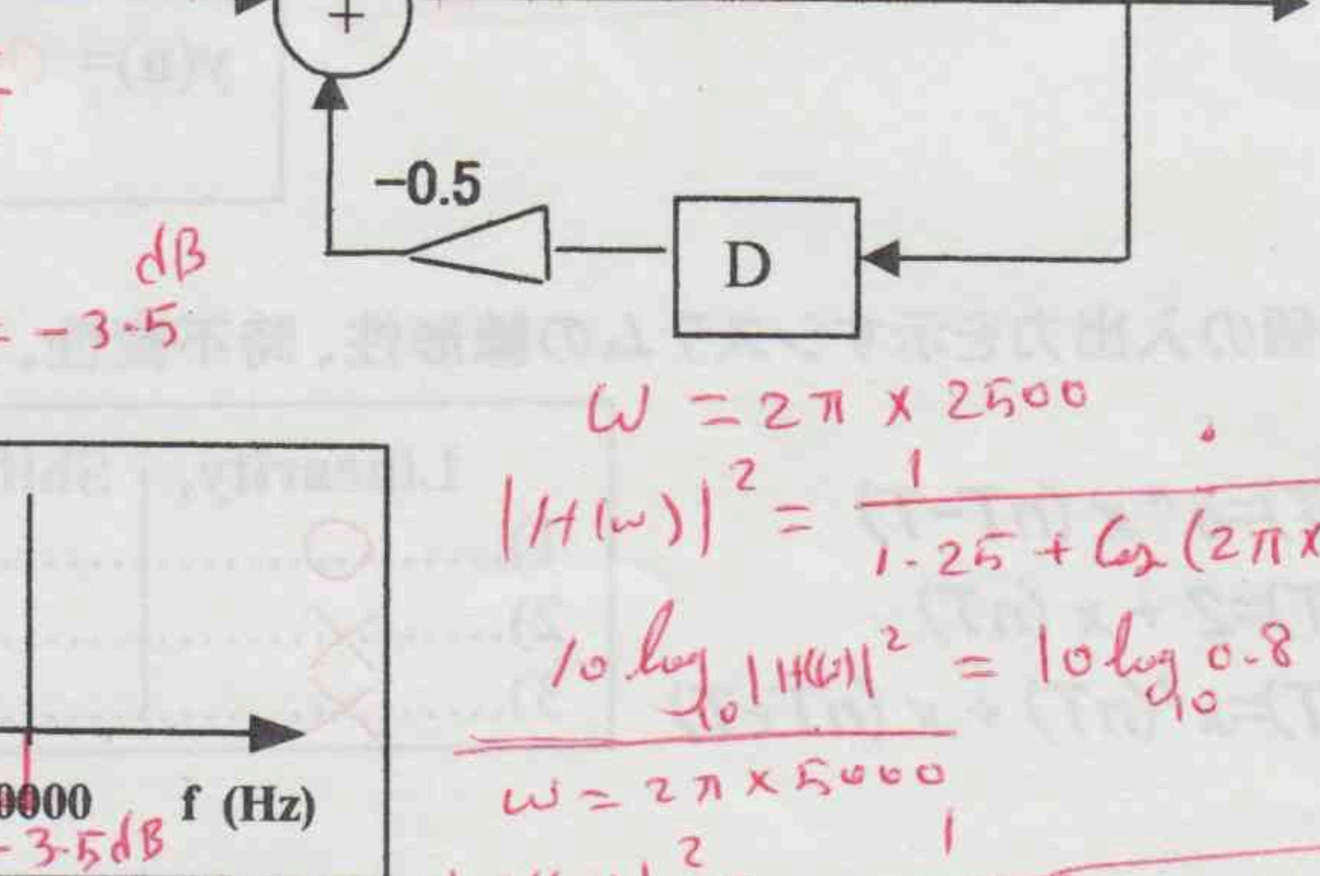
$\ldots \boxed{h(n) = (-1)^n 0.5^n u(n)}$ かくにんしました。

$H(w) = \sum_{n=0}^{\infty} h(nT)\,e^{-jwnT} = \sum_{n=0}^{\infty} (-1)^n 0.5^n e^{-jwnT} = \frac{1}{1+0.5e^{-jwT}}$

$H(w) = \frac{1}{1+0.5\cos wT - j0.5\sin wT}$

$|H(w)|^2 = \frac{1}{(1+0.5\cos wT)^2 + 0.25\sin^2 wT} = \frac{1}{1.25+\cos wT}$

$\angle H(w) = \mathrm{Arctan}\,\frac{0.5\sin wT}{1+0.5\cos wT}$



$w=0 \quad |H(w)|^2 = \frac{1}{2.25} = 0.44 \rightarrow 10\log_{10}|H(w)|^2 = -3.5$ dB

$w = 2\pi\times 2500$

$|H(w)|^2 = \frac{1}{1.25 + \cos(2\pi\times 2500\times 0.1\times 10^{-3})} = \frac{1}{1.25} = 0.8$

$10\log_{10}|H(w)|^2 = 10\log_{10} 0.8 \simeq -1$ dB

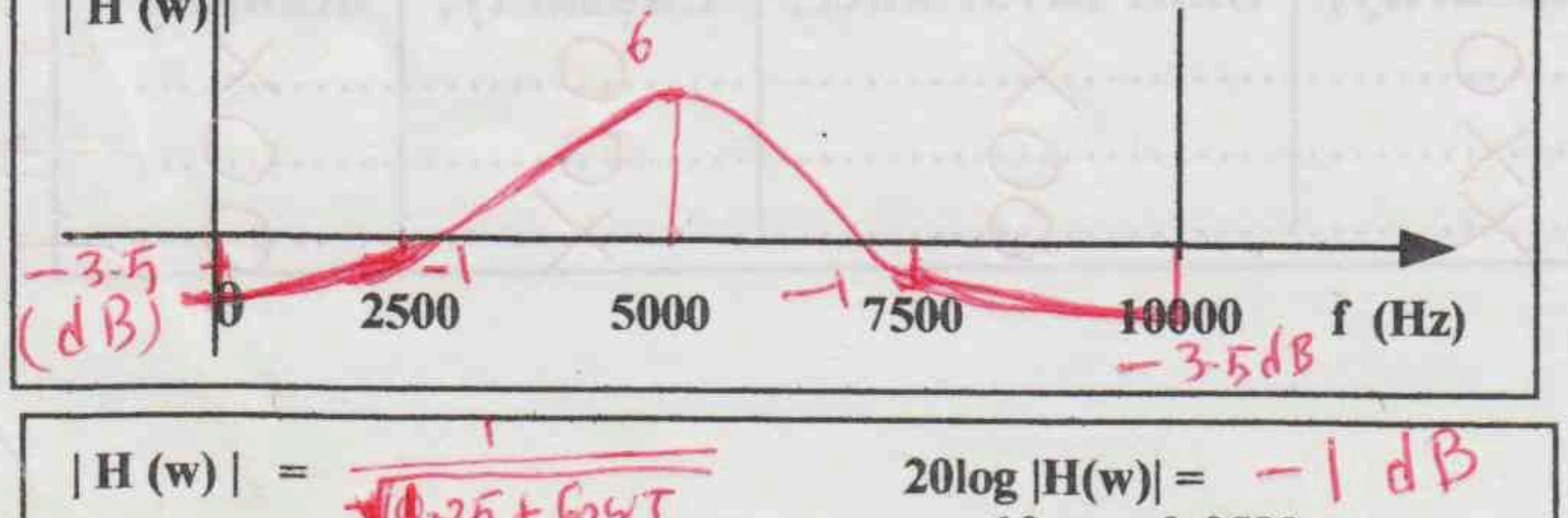
$w = 2\pi\times 5000$

$|H(w)|^2 = \frac{1}{1.25 + \cos 2\pi\times 5000\times 0.1\times 10^{-3}} = \frac{1}{0.25} = 4$

$10\log_{10}|H(w)|^2 = 6$ dB

$w = 2\pi\times 7500 \rightarrow |H(w)|^2 = \frac{1}{1.25} = 0.8 \rightarrow \simeq -1$ dB

$w = 2\pi\times 10000 \rightarrow |H(w)|^2 = \frac{1}{2.25} = 0.44 \rightarrow -3.5$ dB



| H (w) | = $\frac{1}{\sqrt{1.25+\cos wT}}$ 20log |H(w)| = -1 dB
10 f=2500

arg [H (w)] = $\mathrm{Arctan}\,\frac{0.5\sin wT}{1+0.5\cos wT}$