Original with Solution
M.R.Asharif 2009/6/18

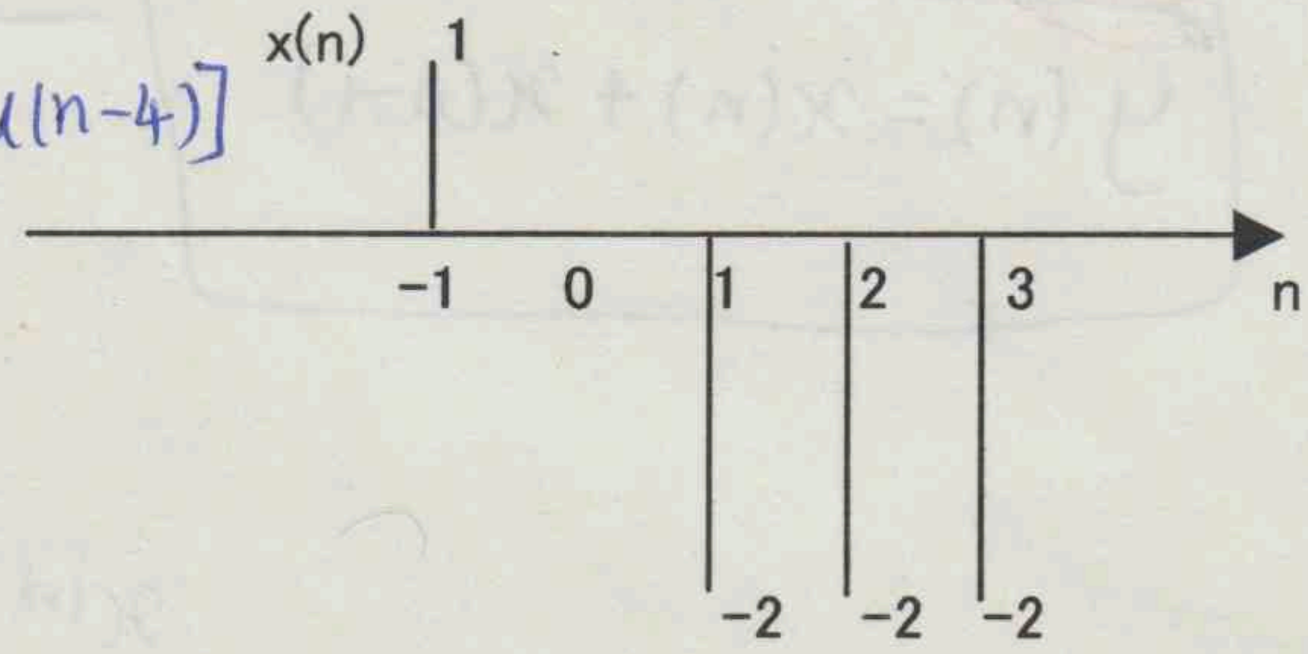
Digital Signal Processing
Undergraduate Course Student's Name:
Mid-Term Examination Sudent's No.
2009.6.19 [write your answer in the blocks, each one 10 score]

University of the Ryukyus
Faculty of Engineering
Dept. of Information Eng.
Prof. M.R. Asharif

~~~~~~

1. 図で信号、x(n)、を unit step (u(n))、とδ(n)、関数を用いて表現せよ。

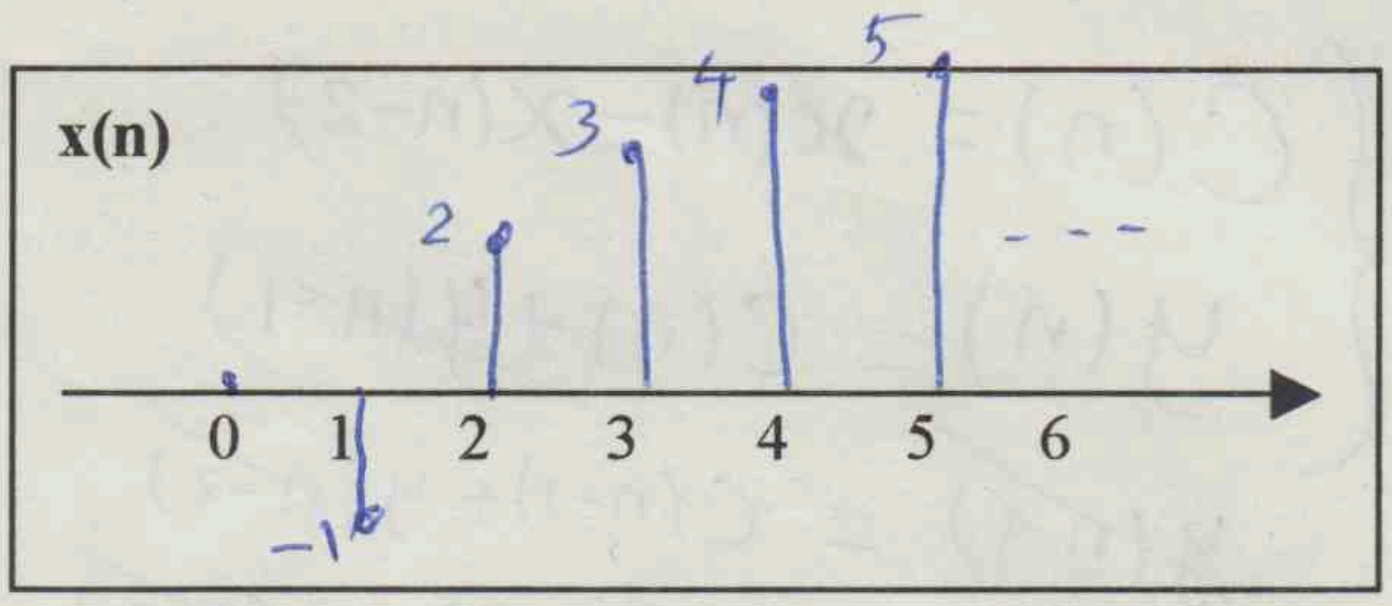
$x(n) = \delta(n+1) - [2u(n-1) - 2u(n-4)]$



x(n)= $\delta(n+1) - 2[u(n-1) - u(n-4)]$

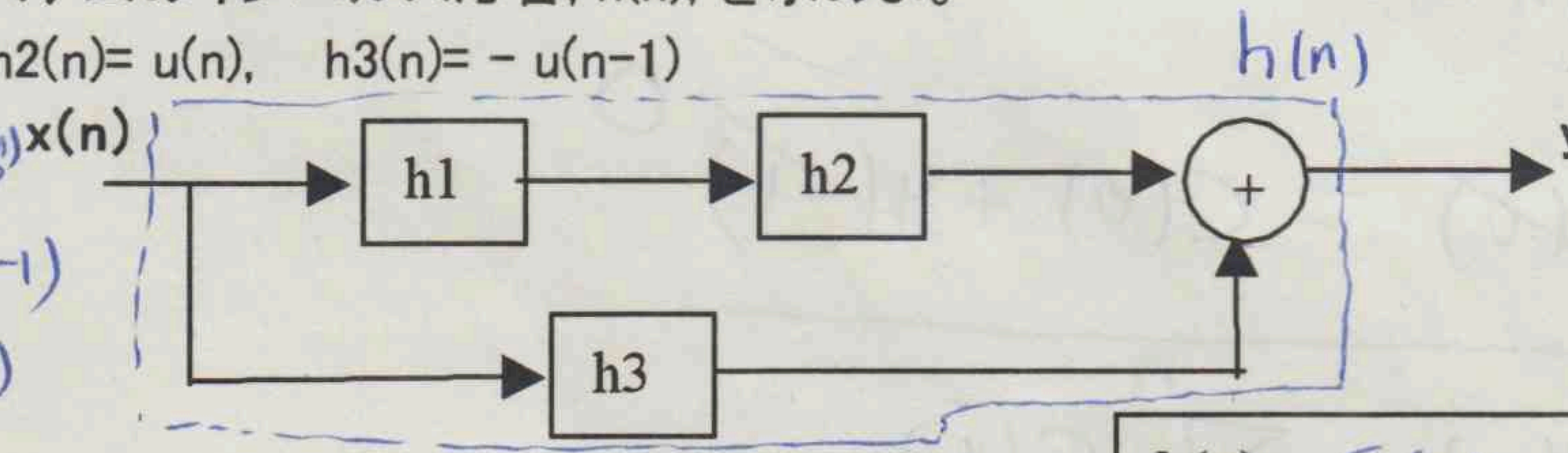
2. 次の信号の5 サンプルまでをポロットせよ。

$$x(n) = -2\delta(n-1) + nu(n)$$



3. 図に示す離散時間システムのインパルス応答, h(n), を求めよ。
ただし: h1(n)=δ(n), h2(n)= u(n), h3(n)= − u(n−1)

$h(n) = h_1(n) * h_2(n) + h_3(n)$
$h(n) = \delta(n) * u(n) - u(n-1)$
$h(n) = u(n) - u(n-1)$
$h(n) = \delta(n)$



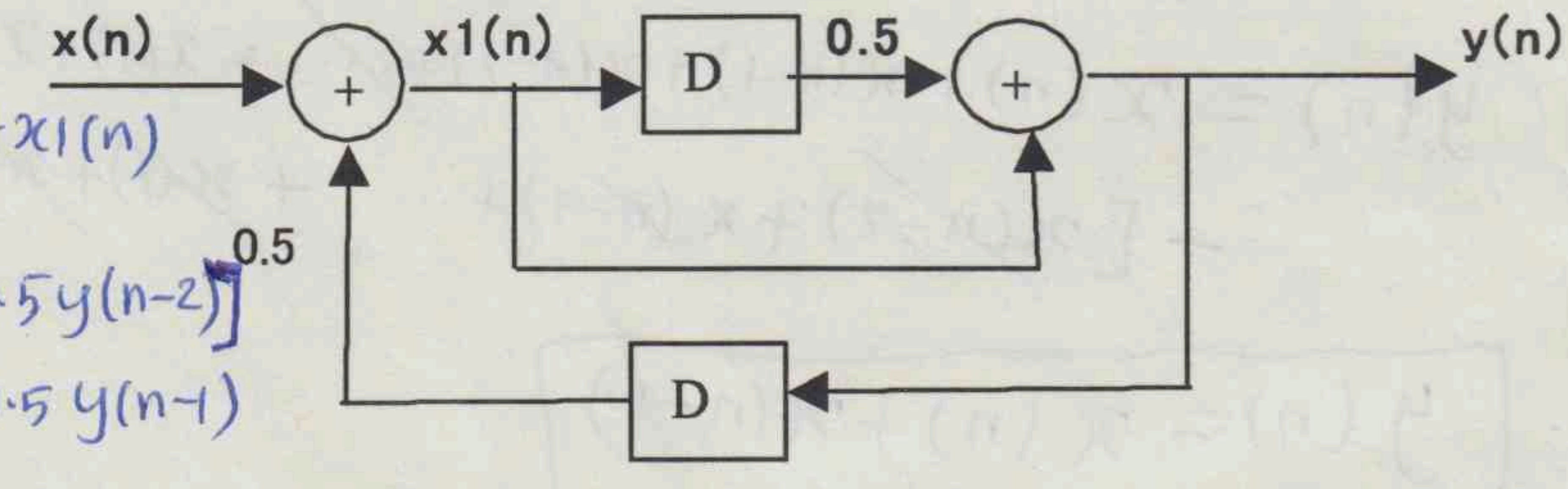
h(n)= $\delta(n)$

4. 図に示す離散時間システム(IIR Digital Filter) の差分方程式を指出せよ。
T=1

$x_1(n) = x(n) + 0.5y(n-1)$

$y(n) = 0.5x_1(n-1) + x_1(n)$

$y(n) = 0.5[x(n-1) + 0.5y(n-2)] + x(n) + 0.5y(n-1)$



y(n)= $x(n) + 0.5x(n-1) + 0.5y(n-1) + 0.25y(n-2)$

5. 以下 2 個の入出力を示すシステムの線形性、時不変性、因果性、を判定し、○か×で示せよ。

1) $y(nT)=e^{n}x(nT+T)$
2) $y(nT)=1 + x(nT)$

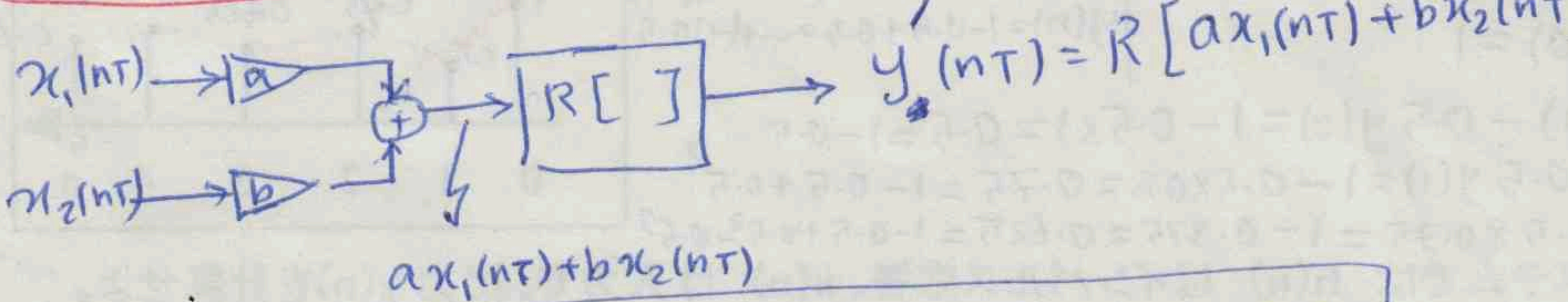
	Linearity,	Shift Invariance,	Causality
1)	○	×	×
2)	×	○	○
~~~~~~

5) –

1- $y(nT) = e^{n}\, x(nT+T) = R[x(nT)]$

For Linearity investigation

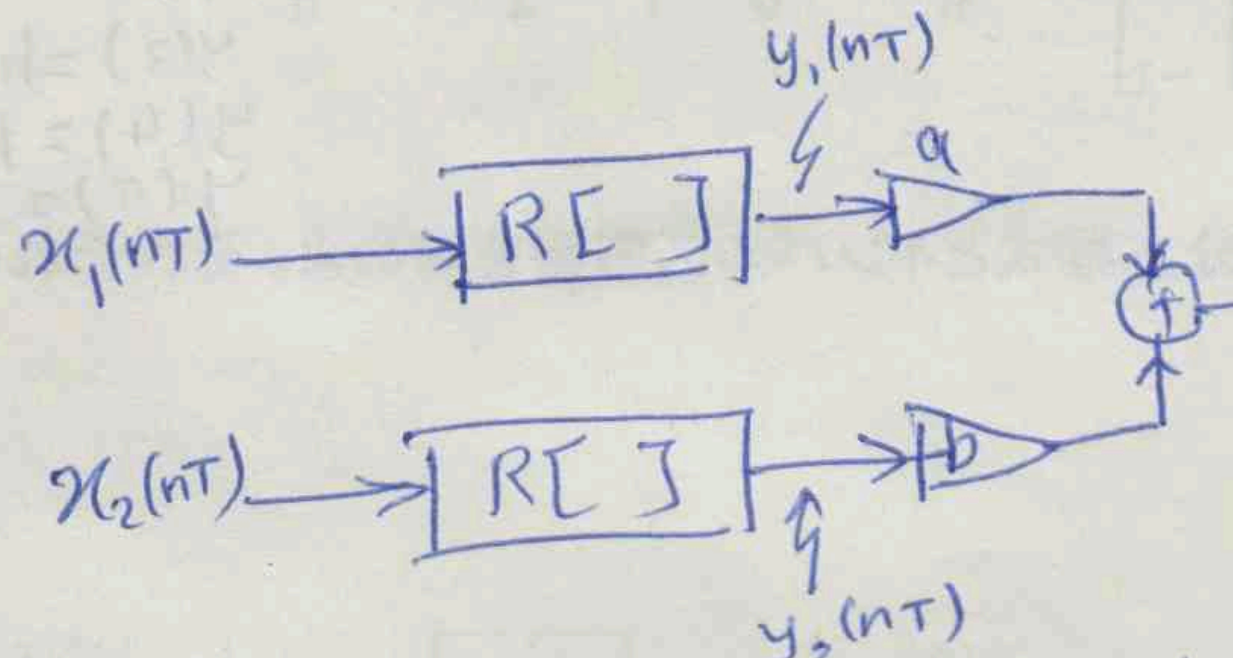
Fig.(1)



$$\boxed{y'(nT) = e^{n}[a x_1(nT+T) + b x_2(nT+T)]} \quad (1)$$

Next, we investrigate the following structure:

Fig.(2)



$y''(nT) = a y_1(nT) + b y_2(nT)$

$$\boxed{y''(nT) = a\, e^{n} x_1(nT+T) + b\, e^{n} x_2(nT+T)}$$

Eq.(1) and Eq.(2) are same, then the system is Linear.

2- $y(nT) = 1 + x(nT)$

Now, let's try Linearity for the above system:

From structure, in Fig.1:

$$y'(nT) = R[a x_1(nT) + b x_2(nT)] = \boxed{1 + a x_1(nT) + b x_2(nT)} \quad (3)$$

From structure, in Fig.2:

$$y''(nT) = a y_1(nT) + b y_2(nT) = a[1 + x_1(nT)] + b[1 + x_2(nT)]$$

$$\boxed{y''(nT) = a + a x_1(nT) + b + b x_2(nT)} \quad (4)$$

Eq.(3) is not equal to Eq.(4), then this system is not Linear.

Let's try SI for system in (1): For SI we should Compare the following two structures:

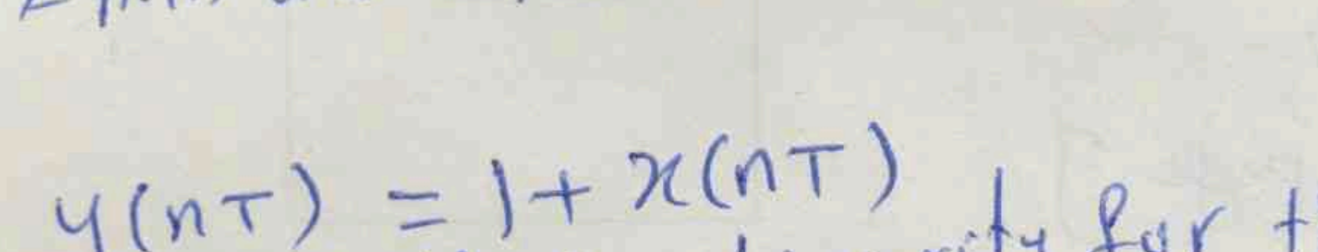
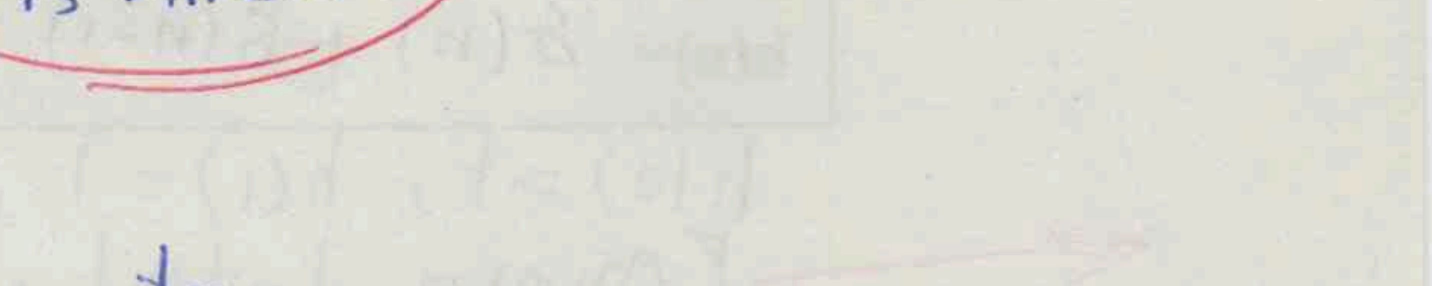


Fig.3



$$y'(nT) = e^{(n-K)}\, x(nT-KT+T)$$

$$y''(nT) = e^{n}\, x(nT-KT+T)$$

Then: $y' \neq y'' \Longrightarrow$ Systems is not S.I.

For system in (2)

$$y'(nT) = 1 + x(nT-KT)$$

$$y''(nT) = 1 + x(nT-KT)$$

Then $y'(nT) = y''(nT) \Rightarrow$ this system is S.I.

For Causality, because both systems are not LSI, just we search whether $y(nT)$ is dependent on persent and past samples of $x(nT)$ or not:

For system (1) dependes on future samples of $x(nT)$, then not Causal, but system (2) is Casual

Original with Solution

M-R-Asharif 2009/6/18

$y(n)_{n\to\infty} = \frac{1}{1-.5} = 0.6666\cdots$

6 ・次の LSI システムでは、$x(n) = u(n)$ は入力で、出力 y(n)の５サンペルを計算せよ。

y(−1)=0

$$y(n) = x(n) - 0.5\,y(n-1)$$

$y(4) = 1-0.5+0.5^2-0.5^3+0.5^4$

$\vdots$

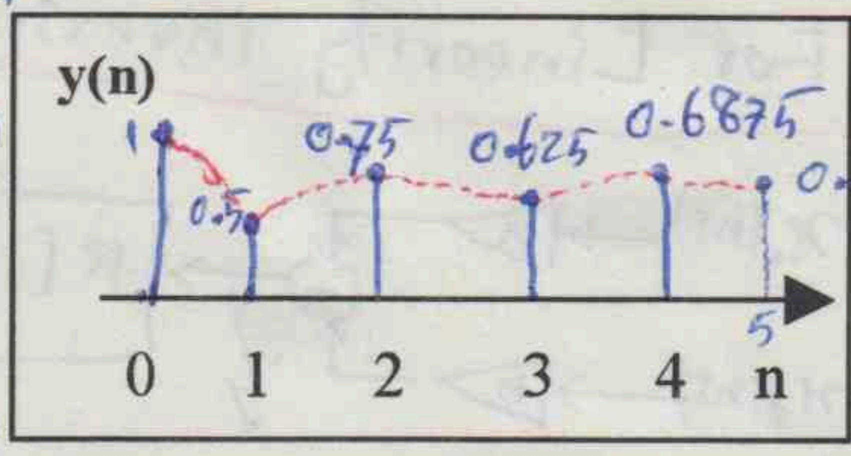
$y(n) = 1-0.5+0.5^2+\cdots+(-1)^n 0.5^n$

$y(0) = u(0) = 1$

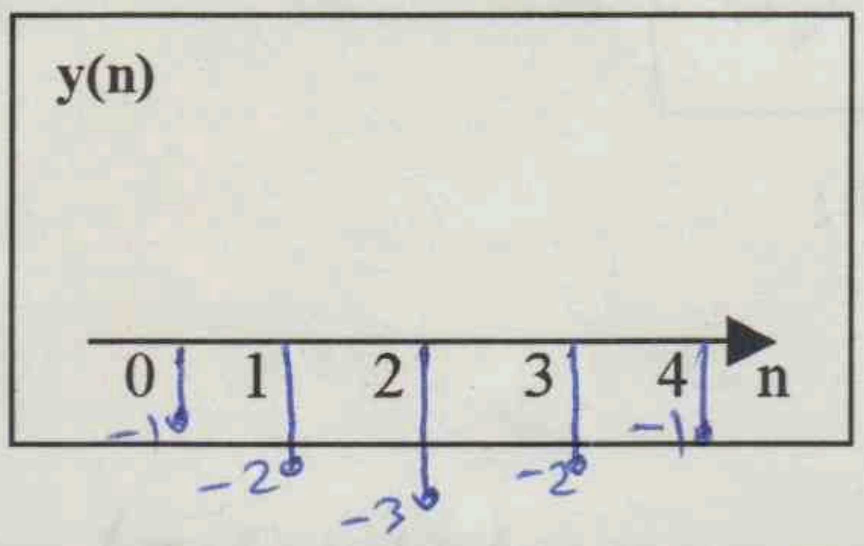
$y(1) = u(1) - 0.5\,y(0) = 1-0.5\times 1 = 0.5 = 1-0.5$

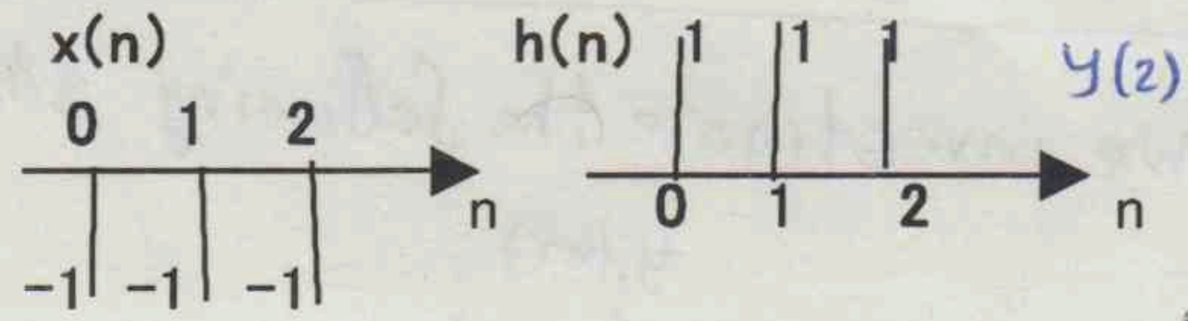
$y(2) = 1-0.5\,y(1) = 1-0.5\times 0.5 = 0.75 = 1-0.5+0.5^2$

$y(3) = 1-0.5\times 0.75 = 1-0.375 = 0.625 = 1-0.5+0.5^2-0.5^3$



7 ・次の LSI システムでは h(n) はインパルス応答、x(n) は入力で、出力 y(n)を計算せよ。





$y(n) = \sum_{n=0}^{4} h(k)x(n-k)$, $y(0) = h(0)x(0) = \boxed{-1}$, $y(1) = h(0)x(1) + h(1)x(0)$

$y(1) = -1-1 = \boxed{-2}$

$y(2) = h(0)x(2) + h(1)x(1) + h(2)x(0)$

$y(2) = -1-1-1 = \boxed{-3}$

$y(3) = h(1)x(2) + h(2)x(1) = \boxed{-2}$

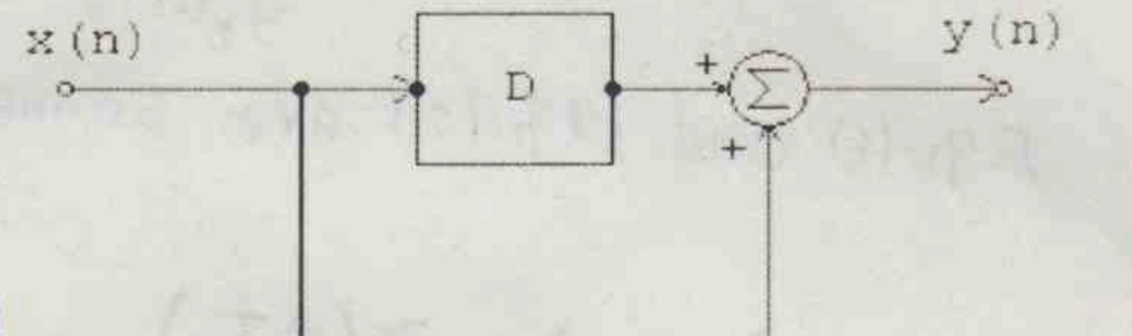
$y(4) = h(2)x(2) = \boxed{-1}$

$y(5) = \boxed{0}$

8ー以下の２個の回路(FIR & IIR Digital Filters)の差分方程式とインパルス応答を求めよ。それぞれのインパルス応答が同じである事を確かめよ。

y(−1)=0, x(−1)=0

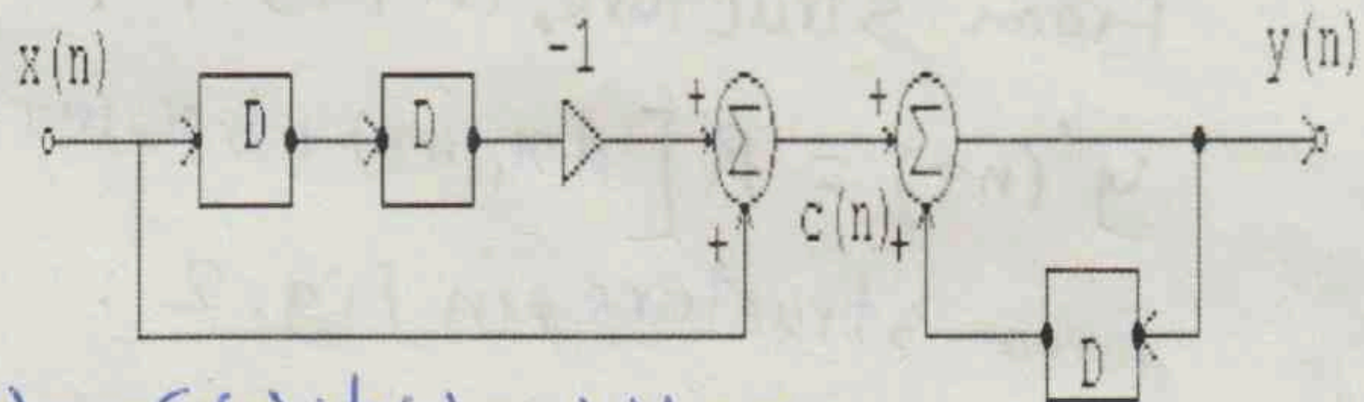
y(n)=	$x(n) + x(n-1)$
h(n)=	$\delta(n) + \delta(n-1)$



$h(0)=1$, $h(1)=1$, $h(2)=0$, ~~~

[more detail in previous page]

y(n)=	$x(n) - x(n-2) + y(n-1)$
h(n)=	$\delta(n) + \delta(n-1)$



$x(n) = \delta(n)$

$h(n) = \delta(n) - \delta(n-2) + h(n-1)$

$h(0) = 1$

$h(1) = h(0) = 1$

$h(2) = -\delta(0) + h(1) = -1+1 = 0$

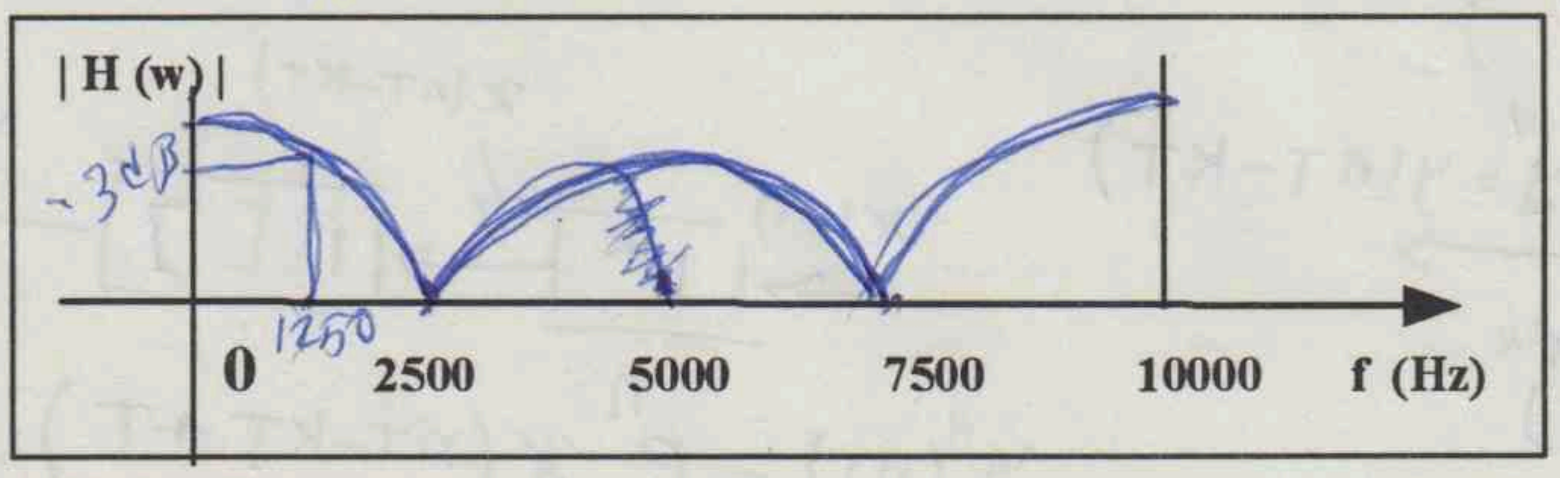
$h(3) = h(2) = 0$

9ー次の離散時間システムのフーリエ変換|H(w)|と Arg[H(w)]を求めよ。

$$h(nT) = 0.5\delta(nT+T) + 0.5\delta(nT-T)$$

T=0.1msの時、｜H(w)｜をプロットせよ。

f=~~2500~~ 1250 Hz で ｜H(w)｜(dB)を求めよ。



| H (w) | = $\lvert \cos\omega T \rvert$	**20log |H(w)| =** −3dB **10 f=~~2500~~** 1250
arg [H (w)] = $0, \pi$	

$H(w) = \sum_{n=-1}^{1} h(nT)e^{-j\omega nT}$

$|H(w)| = 0.5e^{j\omega T} + 0.5e^{-j\omega T} = |\cos\omega T|$

T = 0.1 msec.

$|H(w)| = \cos(2\pi f\times 0.1\times 10^{-3}) = \cos\left(\frac{2\pi f}{10000}\right)$

$H(w)\big|_{f=0} = 1$, $H(w)\big|_{f=5000} = \cos(\pi) = |-1| = 1$

$H(w)\big|_{f=10000} = \cos(2\pi) = 1$

$H(w)\big|_{f=2500,7500} = 0$

$H(w)\big|_{f=1250} = \cos\left(\frac{2\pi\times 1250}{10000}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

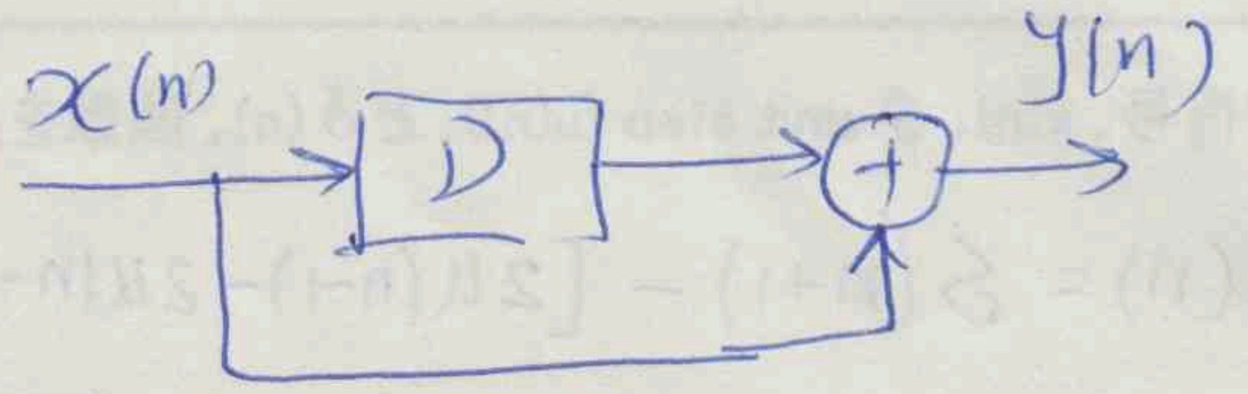
$20\log_{10}\frac{\sqrt{2}}{2} = -10\log_{10}2 = \boxed{-3\text{ dB}}$

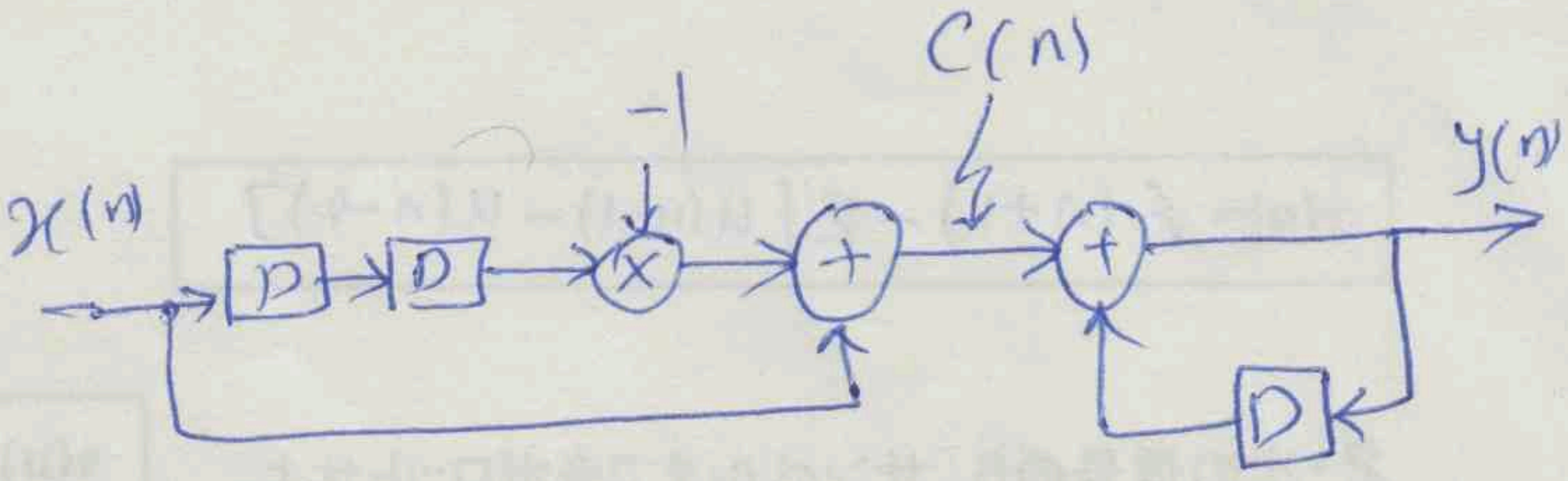
Original with Solution

2004/6/18

8 -

$$\boxed{y(n) = x(n) + x(n-1)}$$





$$\begin{cases} C(n) = x(n) - x(n-2) \\ y(n) = C(n) + \cancel{y(n-1)} \end{cases}$$

$$\cancel{y(n-1)} = C(n-1) + \cancel{y(n-2)}$$

$$\cancel{y(n-2)} = C(n-2) + \cancel{y(n-3)}$$

$$\vdots$$

$$\cancel{y(0)} = C(0) + \cancel{y(-1)}^{\,0}$$

$$y(n) = \sum_{i=0}^{n} C(i)$$

$$y(n) = \sum_{i=0}^{n} x(i) - x(i-2)$$

$$y(n) = x(n) + x(n-1) + \cancel{x(n-2)} + \cancel{\cdots} + \cancel{x(1)} + \cancel{x(0)}$$

$$- [\cancel{x(n-2)} + \cancel{x(n-3)} + \cdots + \cancel{x(1)} + \cancel{x(0)}]$$

$$\boxed{y(n) = x(n) + x(n-1)}$$

Then, two Circuits are equal.