My Solution
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2013/6/16

Digital Signal Processing
Undergraduate Course **Student's Name:**
Mid-Term Examination **Sudent's No.**
2013.6.7 [write your answer in the blocks, each one 10-score]

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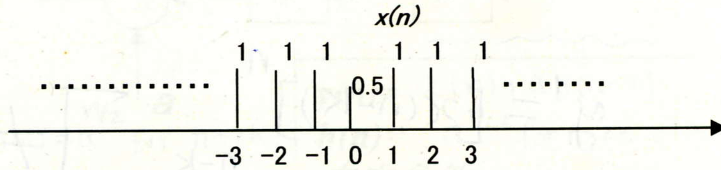
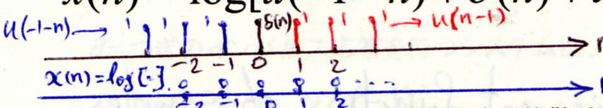
1. 図で信号、x(n)、を、$\delta(n)$ 関数だけを用いて表現せよ。

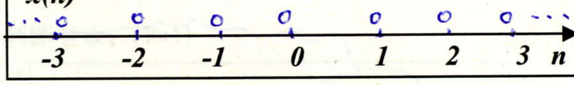
$x(n) = (0.5)^{\delta(n)}$



2・ 次の信号をプロットせよ。 ただし; *n=・・・ −3, −2, −1, 0, 1, 2, 3, ・・・*

$$x(n) = \log[u(-1-n) + \delta(n) + u(n-1)]$$





*3・*以下の二つ差分方程式を満足する離散時間システム(x(n):入力、y(n):出力)を構成と関数求めよ。(T＝1)。

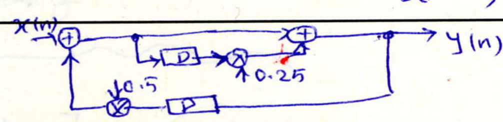
$x1(n) = x(n) + 0.5\,y(n-1)$

$y(n) = x1(n) + 0.25\,x1(n-1)$

$y(n) = x(n) + 0.25x(n-1) + 0.5y(n-1) + 0.125\,y(n-2)$

$y(n) = x(n) + 0.5y(n-1) + 0.25[x(n-1) + 0.5y(n-2)]$

$y(n) = x(n) + 0.25x(n-1) + 0.5y(n-1) + 0.125y(n-2)$



4-図に示す離散時間システムにおいて、*y(n) = x(n)* となるように *a, b* を決定せよ(T=1)。

$a = 1$, $b = -0.5$

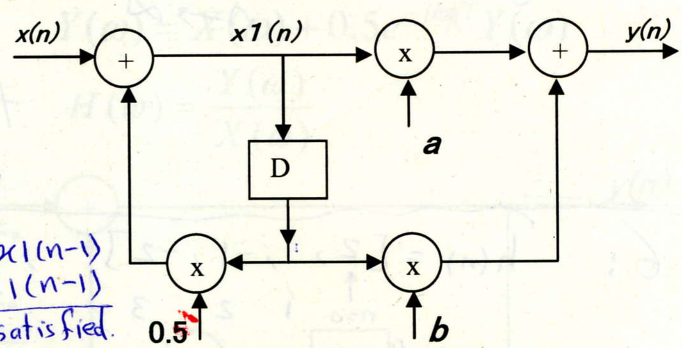
$x1(n) = x(n) + 0.5\,x1(n-1)$

$y(n) = a\,x1(n) + b\,x1(n-1)$

Condition: $y(n) = x(n)$

$y(n) = x(n) = a\,x(n) + 0.5a\,x1(n-1) + b\,x1(n-1)$

if $a = 1$, $b = -0.5$
Then Condition is satisfied.



*5・*以下の入出力を示すシステムの線形性、時不変性、因果性、安定性を判定し、○か×で示せよ。

$y(n) = [x(n)]^n$

[See back for proof.] →

Linearity,	Shift Invariance,	Causality,	Stability
×	×	○	×

*6・*次のシステムでは *h(n)* はインパルス応答、*x(n)* は入力で、出力 *y(n)*を計算せよ。

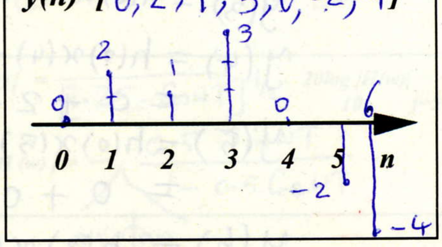
$h(n) = [2, 1, -1, -2]$, $n=0$ ↑

$x(n) = [0, 1, 0, 2]$, $n=0$ ↑

$y(n) = [0, 2, 1, 3, 0, -2, -4]$

$y(n) = x(n) * h(n)$

[See Next Page for Detail Solution.] →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5: $y(n) = [x(n)]^n$

$y' = [a_1 x_1(n) + a_2 x_2(n)]^n$

$y'' = a_1[x_1(n)]^n + a_2[x_2(n)]^n$

$\neq$ Not Linear ✗

$y' = [x(n-K)]^n$

$y'' = [x(n-K)]^{n-K}$

$\neq$ Not S.I ✗

$y(n_0) = [x(n_0)]^{n_0}$ is not function of samples after n_0, then system is Causal O

$\underset{n \to \infty}{y(n) = [x(n)]^n \to \infty}$ for $1 < x(n) \leq M$ $(M > 1)$

$\underset{n \to \infty}{y(n) = [M]^n \to \infty}$

then: Not Stable ✗

6: $h(n) = [\underset{\underset{n=0}{\uparrow}}{2}, \underset{1}{1}, \underset{2}{-1}, \underset{3}{-2}]$, $x(n) = [\underset{\underset{n=0}{\uparrow}}{0}, \underset{1}{1}, \underset{2}{0}, \underset{3}{2}]$

$y(n) = \sum_{i=0}^{4} h(i)\, x(n-i)$

$\boxed{y(n) = [0, 2, 1, 3, 0, -2, -4]}$

$y(0) = h(0) \cdot x(0) = \boxed{0}$

$y(1) = h(0)x(1) + h(1)x(0) = 2 + 0 = \boxed{2}$

$y(2) = h(0)x(2) + h(1)x(1) + h(2)x(0) = 0 + 1 + 0 = \boxed{1}$

$y(3) = h(0)x(3) + h(1)x(2) + h(2)x(1) + h(3)x(0) = 4 + 0 - 1 + 0 = \boxed{3}$

$y(4) = h(0)x(4) + h(1)x(3) + h(2)x(2) + h(3)x(1) + h(4)x(0) =$

$= 0 + 2 + 0 - 2 + 0 = \boxed{0}$

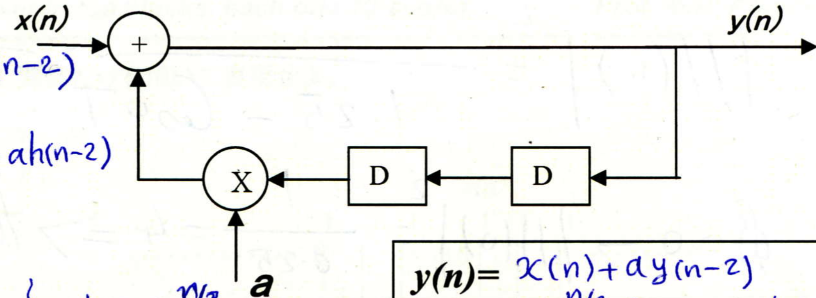
$y(5) = h(0)x(5) + h(1)x(4) + h(2)x(3) + h(3)x(2) + h(4)x(1) + h(5)x(0) =$

$= 0 + 0 - 2 + 0 + 0 + 0 = \boxed{-2}$

$y(6) = h(3)x(3) = \boxed{-4}$

7. 次の回路の差分方程式とインパルス応答を求めて、そしてシステムの安定のため a の条件を求よ。

y(−1)=0



$y(n) = x(n) + a y(n-2)$

$x(n) = \delta(n)$

$y(n) = h(n) = \delta(n) + a h(n-2)$

$h(0) = 1$

$h(1) = 0$

$h(2) = a h(0) = a$

$h(3) = a h(1) = 0$

$h(4) = a h(2) = a^2$

$h(5) = a h(3) = 0$

$h(n) = a^{n/2}$ for $n = 2k$

$h(n) = 0$ for $n = 2k+1$

$\sum_{n=0}^{\infty} |h(n)| = 1 + a + a^2 + a^3 + \cdots = \frac{1}{1-a}$ for: $|a| < 1$

$y(n)=$ $x(n) + a y(n-2)$

$h(n)=$ $a^{n/2}$ for $n=2k$ else zero

安定条件: $|a| < 1$

8− つぎのインパルス応答を持つシステムは、安定かどうか判断せよ。(T=1)

$$h(n) = [\frac{(-1)^n}{1+n}]u(n)$$

安定　　不安定

$h(n) = [1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \cdots \rightarrow 0.693$ without absolute taking

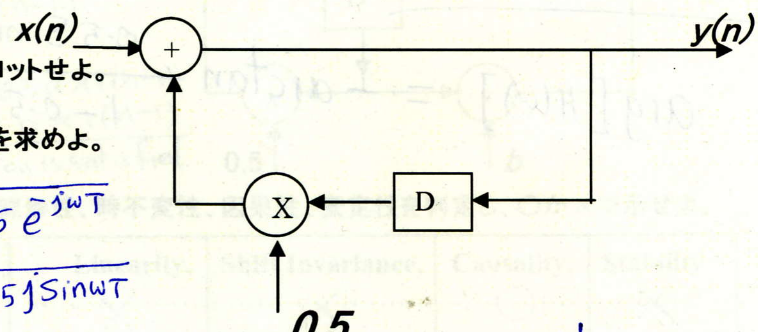
$\sum_{n=0}^{\infty} |h(n)| = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \rightarrow \infty$

9−次の離散時間システムのフーリエ変換 H(w)を求めよ。

ただしシステムの差分方定式: $y(nT) = x(nT) + 0.5 y(nT - T)$

周波数でその表現: $Y(\omega) = X(\omega) + 0.5 e^{-j\omega n T} Y(\omega)$ to be corrected

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$



1− T=0.1msの時、| H(w) | をプロットせよ。

2− Arg[H(w)] を求めよ。

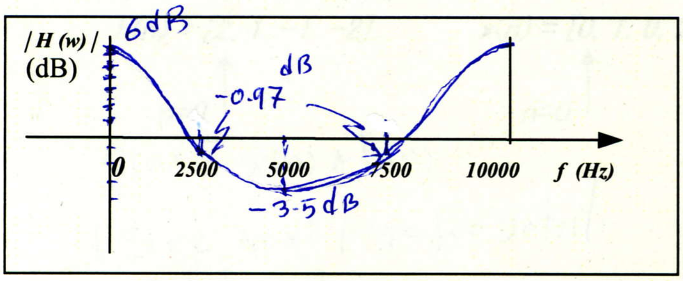
3− f＝5000 Hz で | H(w) | (dB)を求めよ。

$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{1 - 0.5 e^{-j\omega T}}$

$H(\omega) = \frac{1}{1 - 0.5 \cos\omega T + 0.5 j \sin\omega T}$

$|H(\omega)|^2 = \frac{1}{(1 - 0.5\cos\omega T)^2 + 0.25 \sin^2\omega T} \rightarrow |H(\omega)|^2 = \frac{1}{1.25 - \cos\omega T}$

[See next page]



$|H(w)| = \frac{1}{[1.25 - \cos\omega T]^{1/2}}$　　20log |H(w)| = −3.5 dB (10, f=5000)

arg [H (w)] = $-\arctan\frac{0.5 \sin\omega T}{1 - 0.5\cos\omega T}$

9: Continue $\quad T = 0.1^{\,msec.} = 0.1\times10^{-3}\ sec.$

$$|H(\omega)|^2 = \frac{1}{1.25 - \cos\omega T} = \frac{1}{1.25 \cos(2\pi f T)}$$

$$f = 0 \rightarrow |H(0)|^2 = \frac{1}{0.25} = 4 \Rightarrow 10\log_{10}|H(0)|^2 = 10\log_{10} 4 = 6^{\ dB}$$

$$f = 2500^{Hz} \rightarrow |H(\omega)|^2_{f=2500} = \frac{1}{1.25 - \cos(2\pi\times2500\times0.1\times10^{-3})} = \frac{1}{1.25 - \cos\frac{\pi}{2}} = 0.8$$

$$\Rightarrow 10\log|H(\omega)|^2_{f=2500} = 10\log_{10} 0.8 = -0.97^{\ dB}$$

$$f = 5000^{Hz} \rightarrow |H(\omega)|^2_{f=5000} = \frac{1}{1.25 - \cos\pi} = \frac{1}{2.25} = 0.44 \rightarrow 10\log_{10} 0.44 = -3.5^{\ dB}$$

$$f = 7500^{Hz} \rightarrow |H(\omega)|^2_{f=7500} = \frac{1}{1.25 - \cos\frac{3\pi}{2}} = 0.8 \rightarrow 10\log_{10} 0.8 = -0.97^{\ dB}$$

$$f = 10000^{Hz} \rightarrow |H(\omega)|^2_{f=10000} = \frac{1}{1.25 - \cos 2\pi} = 4 \rightarrow 10\log_{10} 4 = 6^{\ dB}$$

$$\arg[H(\omega)] = -\arctan\frac{0.5\sin\omega T}{1 - 0.5\cos\omega T}$$