

デジタル信号処理  
～ レポート 2 ～

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## 1 問題 1.9(a)

$$y(nT) = 2 * x(nT) + x(nT - T) + 0.5 * (nT - 2T) + 0.1 * (nT - 3T)$$

入力信号は  $-2T = 0, -T = 0, 0 = 1, T = 0, 2T = -1, 3T = 0, 4T = 0$  なので

$$y(0) = 2 \cdot y(1) = 1 \cdot y(2) = -1.5 \cdot y(3) = -0.9 \cdot y(4) = -0.5 \cdot y(5) = -0.1$$

$$y(6) \ y(9) = 0$$

## 2 問題 1.9(b)

$$y(nT) = 2 * x(nT) + x(nT - T) + 0.5 * (nT - 2T) + 0.1 * (nT - 3T)$$

入力信号は  $-2T = 0, -T = 0, 0 = 1, T = 0, 2T = -1, 3T = 0, 4T = 0$  なので

$$y(0) = 2 \cdot y(1) = 1 \cdot y(2) = -1.5 \cdot y(3) = -0.9 \cdot y(4) = -0.5 \cdot y(5) = -0.1$$

$$y(6) \ y(9) = 0$$

## 3 問題 1.9(c)

$$y(nT) = a_0 x(nT) + a_1 x(nT - T) + a_2 x(nT - 2T) + b_1 y(nT - T) + b_2 y(nT - 2T)$$

$a_0 = 0, a_1 = 0.3236, a_2 = -0.36, b_1 = 1.2944, b_2 = -0.64$  より

$$y(nT) = 0.3236 * x(nT - T) - 0.36 * x(nT - 2T) + 1.2944 * y(nT - T) - 0.64 * y(nT - 2T)$$

入力信号は  $-2T = 0, -T = 0, 0 = 1, T = 0, 2T = -1, 3T = 0, 4T = 0$  なので

$$y(0) = 0 \cdot y(1) = 0.3236 \cdot y(2) = 0.0588678 \cdot y(3) = -0.4545055 \cdot y(4) = -0.2659873$$

$$y(5) = -0.0534105 \cdot y(6) = 0.1010974 \cdot y(7) = 0.1650431 \cdot y(8) = 0.1489295 \cdot y(9) = 0.0871468$$

### 3.1 問題 1.9(d)

$$x_1(nT) = x(nT) + b_1 x_1(nT - T) + b_2 x_1(nT - 2T)$$

$$y(nT) = a_0 x_1(nT) + a_1 x_1(nT - T) + a_2 x_1(nT - 2T)$$

$a_0 = 0, a_1 = 0.3236, a_2 = -0.36, b_1 = 1.2944, b_2 = -0.64$  より

$$x_1(nT) = x(nT) + 1.2944 * x_1(nT - T) - 0.64 * x_1(nT - 2T)$$

$$y(nT) = 0.3236 * x_1(nT - T) - 0.36 * x_1(nT - 2T)$$

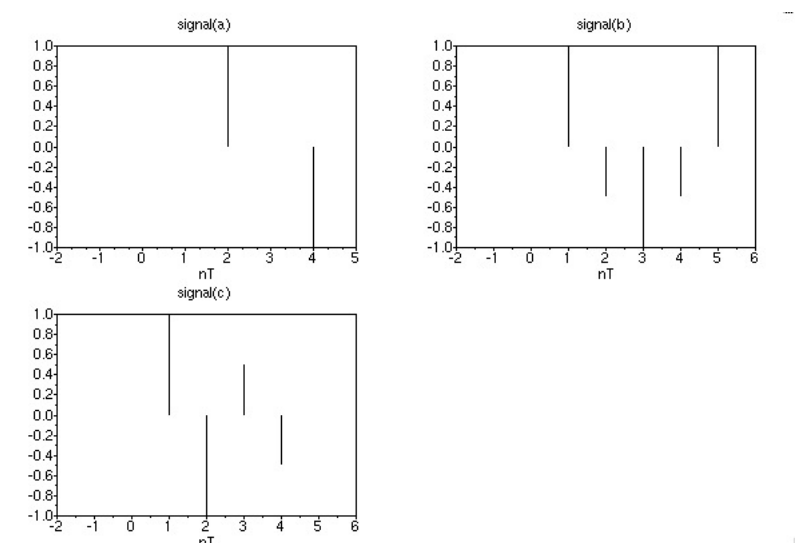
入力信号は  $-2T = 0, -T = 0, 0 = 1, T = 0, 2T = -1, 3T = 0, 4T = 0$  なので

$$y(0) = 0 \cdot y(1) = 0.3236 \cdot y(2) = 0.0588678 \cdot y(3) = -0.4545055 \cdot y(4) = -0.2659873$$

$$y(5) = -0.0534105 \cdot y(6) = 0.1010974 \cdot y(7) = 0.1650431 \cdot y(8) = 0.1489295 \cdot y(9) = 0.0871468$$

## 4 問題 1.11

### 4.1 問題 1.11(c)



## 5 問題 1.12

$$\begin{aligned}
y_{(a)}(nT) &= x(nT) + x(nT - T) + n(nT - 2T) + n(nT - 3T) + n(nT - 4T) \\
y_{(b)}(nT) &= x(nT) - x(nT - 5T) + y(nT - T) \\
&= x(nT) - x(nT - 5T) + \{x(nT - T) - x(nT - 6T) + y(nT - 2T)\} \\
&= x(nT) - x(nT - 5T) + \{x(nT - T) - x(nT - 6T) + \{x(nT - 2T) - x(nT - 7T) + y(nT - 3T)\}\} \\
&\vdots \\
&= x(nT) - x(nT - 5T) + \{x(nT - T) - x(nT - 6T) + \{x(nT - 2T) - x(nT - 7T) + \{x(nT - 3T) \\
&\quad - x(nT - 8T) + \{x(nT - 4T) - x(nT - 9T) + y(nT - 5T)\}\}\} \\
&= x(nT) + x(nT - T) + x(nT - 2T) + x(nT - 3T) + x(nT - 4T) - x(nT - 5T) - x(nT - 6T) \\
&\quad - x(nT - 7T) - x(nT - 8T) - x(nT - 9T) + y(nT - 5T) \\
&= x(nT) + x(nT - T) + x(nT - 2T) + x(nT - 3T) + x(nT - 4T) - \mathbf{x(nT - 5T)} - x(nT - 6T) \\
&\quad - x(nT - 7T) - x(nT - 8T) - x(nT - 9T) + \mathbf{x(nT - 5T)} - x(nT - 10T) + y(nT - 6T)\} \\
&= x(nT) + x(nT - T) + x(nT - 2T) + x(nT - 3T) + x(nT - 4T) - x(nT - 6T) \\
&\quad - x(nT - 7T) - x(nT - 8T) - x(nT - 9T) - x(nT - 10T) + y(nT - 6T)\} \\
&= x(nT) + x(nT - T) + x(nT - 2T) + x(nT - 3T) + x(nT - 4T) \\
&\quad - x(nT - 7T) - x(nT - 8T) - x(nT - 9T) - x(nT - 10T) - x(nT - 11T) + y(nT - 7T)\} \\
&\vdots \\
&= x(nT) + x(nT - T) + x(nT - 2T) + x(nT - 3T) + x(nT - 4T) \\
&\quad - x(\infty) - x(\infty + 1) - x(\infty + 2) - x(\infty + 3) - x(\infty + 4) + y(\infty)\}
\end{aligned}$$

実際は  $x(nT - \infty T)$  なんて存在しないので

$$\begin{aligned}
&= x(nT) + x(nT - T) + x(nT - 2T) + x(nT - 3T) + x(nT - 4T) \\
&\quad - x(\infty) - x(\infty + 1) - x(\infty + 2) - x(\infty + 3) - x(\infty + 4) + y(\infty)\} \\
&\simeq x(nT) + x(nT - T) + x(nT - 2T) + x(nT - 3T) + x(nT - 4T)
\end{aligned}$$

よって (a) と (b) はほとんど等価である

## 6 問題 2.1

### 6.1 問題 2.1(2)

$$y(nT) = x^2(nT) + x(nT + T)$$

### 6.1.1 線形性

A,B は任意の実定数

$$\begin{aligned}R[Ax_1(nT) + Bx_2(nT)] &= \{Ax_1(nT) + Bx_2(nT)\}^2 + \{Ax_1(nT + T) + Bx_2(nT + T)\} \\ &= A^2x_1^2(nT) + 2ABx_1(nT)x_2(nT) + B^2x_2^2(nT) + Ax_1(nT + T) + Bx_2(nT + T) \\ AR[x_1(nT)] + BR[x_2(nT)] &= A[x_1^2(nT) + x_1(nT + T)] + B[x_2^2(nT) + x_2(nT + T)] \\ &= A[x_1^2(nT) + Ax_1(nT + T) + Bx_2^2(nT) + Bx_2(nT + T)]\end{aligned}$$

よって線形性はない

## 6.2 問題 2.1(4)

$$y(nT) = a^n(nT + T)(|a| < 1)$$

### 6.2.1 線形性

A,B は任意の実定数

$$\begin{aligned}R[Ax_1(nT) + Bx_2(nT)] &= R[Aa_1^n x(nT - T) + Ba^n x_2(nT - T)] \\ &= Aa_1^n x(nT - T) + Ba^n x_2(nT - T) \\ BR[x_1(nT)] + BR[x_2(nT)] &= AR[a_1^n x(nT - T)] + BR[a^n x_2(nT - T)] \\ &= Aa_1^n x(nT - T) + Ba^n x_2(nT - T)\end{aligned}$$

よって線形性がある

## 6.3 問題 2.1(6)

$$y(nT) = \{an + x(nT + 2T)\}^2 = a^2n^2 + 2anx(nT + 2T) + x^2(nT + 2T)$$

### 6.3.1 線形性

A,B は任意の実定数

$$\begin{aligned}R[Ax_1(nT) + Bx_2(nT)] &= a^2n^2 + 2an[Ax_1(nT + 2T) + Bx_2(nT + 2T)] + [Ax_1(nT + 2T) + Bx_2(nT + 2T)]^2 \\ BR[x_1(nT)] + BR[x_2(nT)] &= A(a^2n^2 + 2anx_1(nT + 2T)) + B(a^2n^2 + 2anx_2(nT + 2T)) + \dots\end{aligned}$$

よって線形性はない

## 7 問題 2.2

### 7.1 問題 2.2(b)

$$\begin{aligned}x_1(nT) &= x(nT - T) \\x_2(nT) &= x_1(nT - T) \\y(nT) &= \{x(nT) + 2 * x_1(nT)\} + x_2(nT) + \{3 * x(nT) + 4 * x_1(nT)\} = 4 * x(nT) + 6 * x_1(nT) + x_2(nT) \\&= 4 * x(nT) + 6 * x(nT - T) + x(nT - 2T)\end{aligned}$$

よってインパルス応答は

$$y(0) = 4, y(1) = 6, y(2) = 1 \cdot y(3) \quad (9) = 0$$

### 7.2 問題 2.2(f)

$$\begin{aligned}x_1(nT) &= x(nT - T) \\x_2 &= 3 * \{x(nT - T) + 2 * x_1(nT - T)\} \\x_3 &= 4 * \{x(nT - T) + 2 * x_1(nT - T)\} + x_2(nT - T) \\y(nT) &= 5 * \{x(nT) + 2 * x_1(nT)\} + x_3(nT) \\&= 5 * \{x(nT) + 2 * x_1(nT)\} + 4 * \{x(nT - T) + 2 * x_1(nT - T)\} + x_2(nT - T) \\&= 5 * x(nT) + 10 * x_1(nT) + 4 * x(nT - T) + 8 * x_1(nT - T) + 3 * \{x(nT - 2T) + 2 * x_1(nT - 2T)\} \\&= 5 * x(nT) + 10 * x(nT - T) + 4 * x(nT - T) + 8 * x(nT - 2T) + 3 * x(nT - 2T) + 6 * x(nT - 3T) \\&= 5 * x(nT) + 14 * x(nT - T) + 11 * x(nT - 2T) + 6 * x(nT - 3T)\end{aligned}$$

よってインパルス応答は

$$y(0) = 5, y(1) = 14, y(2) = 11 \cdot y(3) = 6 \cdot y(4) \quad y(9) = 0$$

## 参考文献

[1] 例題で学ぶデジタル信号処理

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