# FFT 2014/6/27

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nK}{N}}$$

$$W = e^{-j\frac{2\pi}{N}}$$

$$X(K) = \sum x(n)\, W_N^{nK}$$

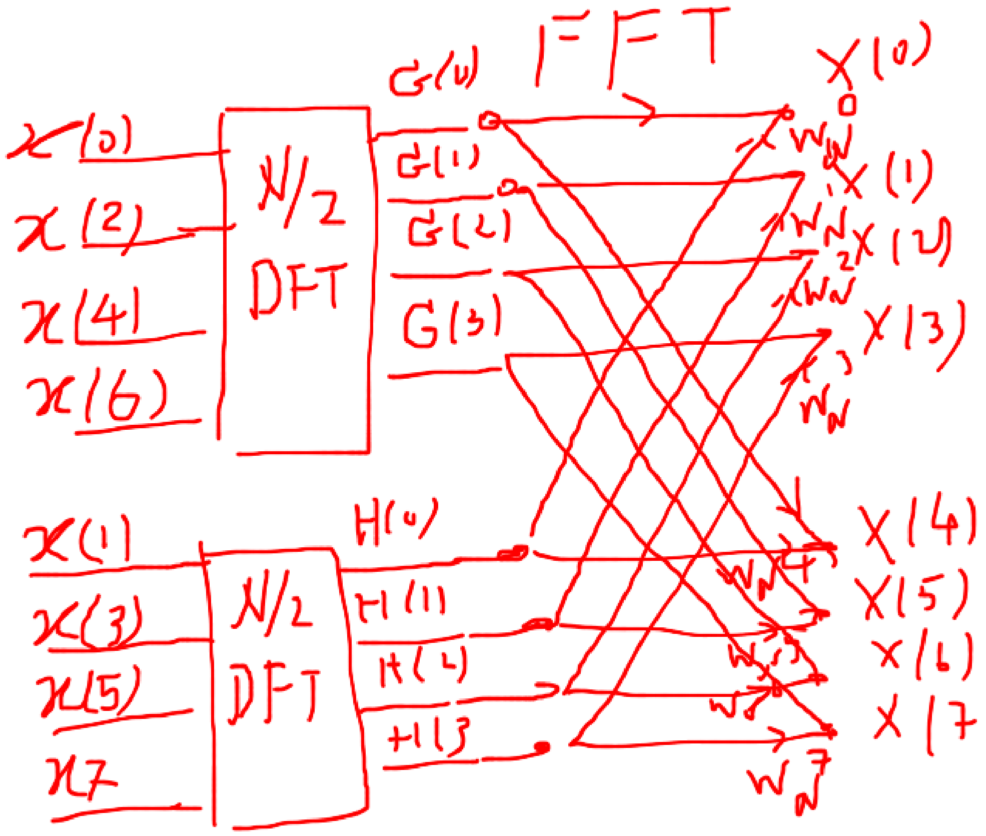
$$N = 2^{\nu}$$

$$X(k) = \sum_{r=0}^{N/2-1} x(2r)\, W_N^{2rk} + \sum_{r=0}^{N/2-1} x(2r+1)\, W_N^{(2r+1)K}$$

$$W_N^{2} = W_{N/2} = e^{-j\frac{2\pi}{N}\times 2} = e^{-j\frac{2\pi}{N/2}}$$

$$X(K) = \sum_{r=0}^{N/2-1} x(2r)\, W_{N/2}^{rK} + W_N^{K} \sum_{r=0}^{N/2-1} x(2r+1)\, W_{N/2}^{rK}$$

$$X(k) = G(K) + W_N^{K} H(K)$$




$$X(0) = G(0) + W_N^0 H(0)$$

$$X(1) = G(1) + W_N^1 H(1)$$

- - $G(4) = G(0)$, $G(5) = G(1)$

$$X(4) = G(0) + W_N^4 H(0)$$

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FFT

DFT $\doteq N^2$ Multiplication

$2\ DFT)_{N/2} \doteq 2\left(\frac{N}{2}\right)^2 + N =$

$$N + \frac{N^2}{2} < N^2$$

$N = 8$

$40 < 64$

$$G(K) = \sum_{r=0}^{N/2-1} g(r)\, W_{N/2}^{rK} = \sum_{\ell=0}^{N/4-1} g(2\ell)\, W_{N/2}^{2\ell K}$$

$$+ \sum_{\ell=0}^{N/4-1} g(2\ell+1)\, W_{N/2}^{(2\ell+1)K}$$

$$G(K) = \sum_{\ell=0}^{N/4-1} g(2\ell)\, W_{N/4}^{\ell K} + W_{N/2}^{K} \sum_{\ell=0}^{N/4-1} g(2\ell+1)\, W_{N/4}^{\ell K}$$

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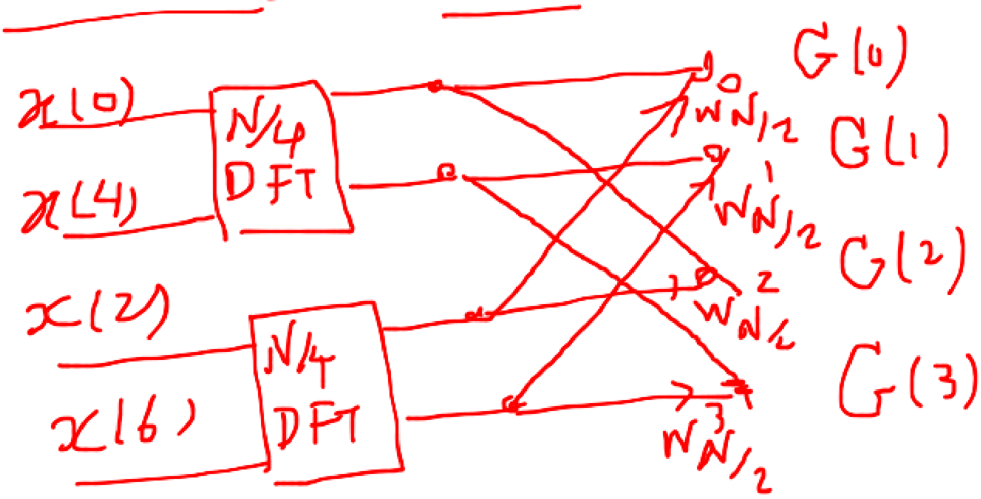
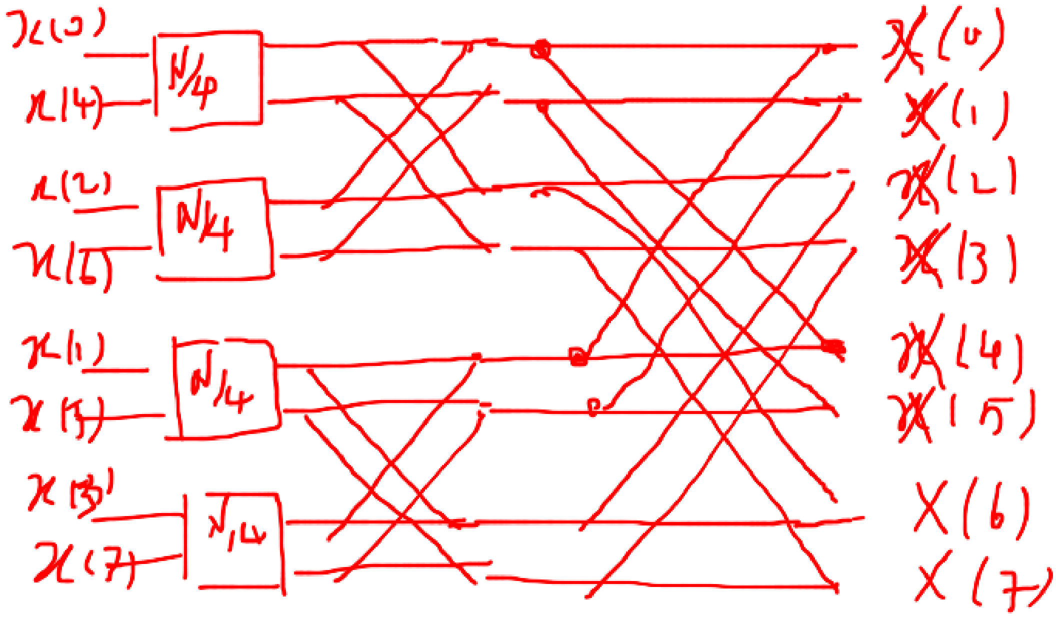
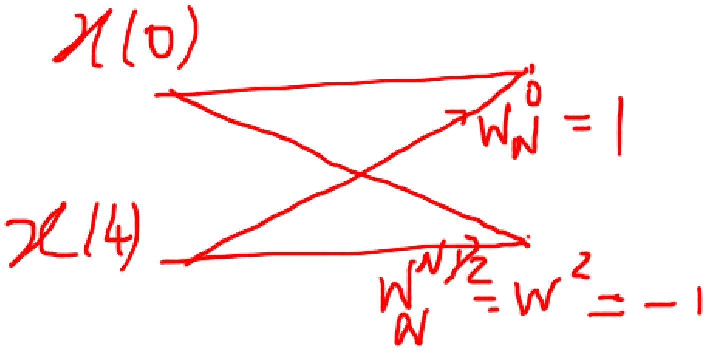
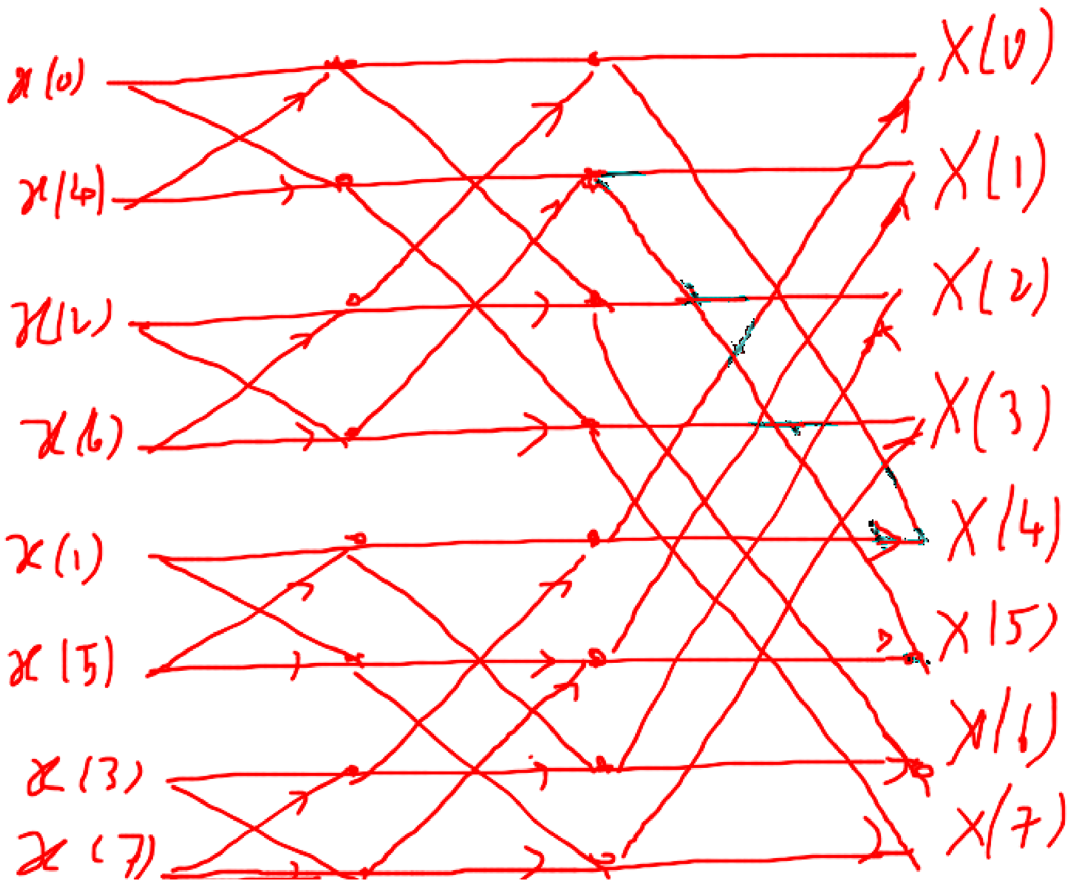



x(0)
x(4)
$W_N^0 = 1$
$W_N^{N/2} = W^2 = -1$


x(0)
x(4)
x(2)
x(6)
x(1)
x(5)
x(3)
x(7)
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)

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$N = 2^{\nu}$, stage: $\nu = \log_2 N$

1) $N + 2\left(\frac{N}{2}\right)^2$

2) $\frac{N}{2} + 2\left(\frac{N}{4}\right)^2$

$\nu$) $N \log_2 N = N \cdot \nu$

$N = 2^{10} = 1024$

DFT

$N^2 = 2^{20} = 1{,}048{,}576$

$\frac{FFT}{DFT} = \frac{1}{100}$

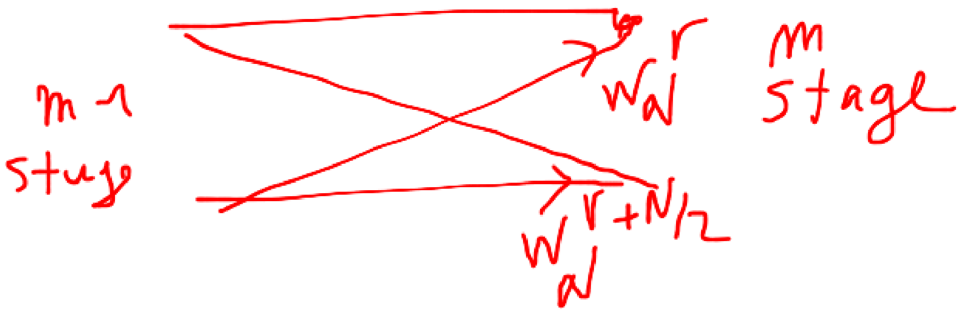
FFT

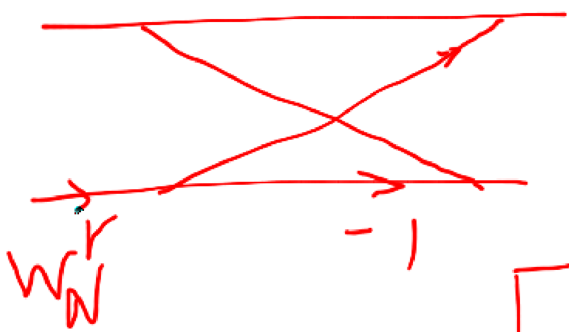
$N \log_2 N =$
10,240

# FFT

$$W_N^{N/2} = e^{-j\frac{2\pi}{N}\times\frac{N}{2}} = e^{-j\pi} = -1$$

$$W_N^{r+N/2} = W_N^{N/2}\cdot W_N^{r} = -W_N^{r}$$




FFT

$$\boxed{\frac{N}{2}\log_2 N}$$

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$X_m(\ell)$ $m$ : stage

$\ell = 0, 1, \cdots N-1$

$m = 1, 2, \cdots \nu = \log_2 N$

$$\begin{cases} X_m(p) = X_{m-1}(p) + W_N^r X_{m-1}(q) \\ X_m(q) = X_{m-1}(p) - W_N^r X_{m-1}(q) \end{cases}$$

| | |
|---|---|
| $X_0(0) = x(0)$ | $X_0(4) = x(1)$ |
| $X_0(1) = x(4)$ | $X_0(5) = x(5)$ |
| $X_0(2) = x(2)$ | $X_0(6) = x(3)$ |
| $X_0(3) = x(6)$ | $X_0(7) = x(7)$ |

in-place - Computation

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# Bit-reversed order

$N = 8 = 2^3$

$X_0(000) = x(000)$

$X_0(001) = x(100)$

$X_0(010) = x(010)$

$X_0(011) = x(110)$

$X_0(100) = x(001)$

$X_0(101) = x(101)$

$X_0(110) = x(011)$

$X_0(111) = x(111)$