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$$H(e^{j\omega T}) = \frac{1}{3}\sum_{n=0}^{2} e^{-j\omega nT} = \frac{1}{3}\left[1 + e^{-j\omega T} + e^{-j2\omega T}\right]$$

$$= \frac{1}{3}\,\frac{1-e^{-j3\omega T}}{1-e^{-j\omega T}} = \frac{1}{3}\,\frac{e^{-j\frac{3\omega T}{2}}\left(e^{j\frac{3\omega T}{2}} - e^{-j\frac{3\omega T}{2}}\right)}{e^{-j\frac{\omega T}{2}}\left(e^{j\frac{\omega T}{2}} - e^{-j\frac{\omega T}{2}}\right)}$$

$$H(e^{j\omega T}) = \frac{1}{3}e^{-j\omega T}\,\frac{\sin\frac{3\omega T}{2}}{\sin\frac{\omega T}{2}}$$

$$\left|H(e^{j\omega T})\right| = M(\omega) = \frac{1}{3}\,\frac{\sin\frac{3\omega T}{2}}{\sin\frac{\omega T}{2}} = \frac{1}{3}(1+2\cos\omega T)$$

$$\angle H(e^{j\omega T}) = -\cancel{j}\,\omega T$$

[A] Fourier Transform of Discrete-Time Signal:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(n)\,e^{-j\omega n}$$

Inverse Fourier Transform of Discrete-Time Signal:

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\,e^{j\omega n}\,d\omega$$

To see why this is true:

we calculate $\hat{x}(n)$ such as:

$$\hat{x}(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\,e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\sum_{m=-\infty}^{+\infty} x(m)\,e^{-j\omega m}\right)e^{j\omega n}\,d\omega$$

Interchanging the order of integration and summation:

$$\hat{x}(n) = \sum_{m=-\infty}^{+\infty} x(m)\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega(n-m)}\,d\omega\right)$$

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$$M^2(\omega) = \frac{1}{9}\left[(1+\cos\omega T + \cos 2\omega T)^2 + (\sin\omega T + \sin 2\omega T)^2\right] =$$

$$\cancel{= \frac{1}{9}\left[\left(1+\cos^2\omega T + (\cos^2\omega T - \sin^2\omega T)^2\right.\right.} + 2\cos\omega T + 2\cos 2\omega T + 2\cos\omega T\cos 2\omega T) +$$

$$\cancel{\left.(+\sin^2\omega T + 4\sin\omega T\cos^2\omega T + 2\sin\omega T\sin 2\omega T)\right]}$$

$$\cancel{= \frac{1}{9}\left[\right.}$$

$$= \frac{1}{9}\left[(1 + \cancel{\cos^2\omega T} + \cos^2 2\omega T + 2\cos\omega T + 2\cos 2\omega T + 2\cos\omega T\cos 2\omega T) + (\cancel{\sin^2\omega T} + \sin^2 2\omega T + 2\sin\omega T\sin 2\omega T)\right]$$

$$= \frac{1}{9}(1+1+1 + 2\cos\omega T + 2\cos 2\omega T + 2\cos\omega T)$$

$$= \frac{1}{9}(\cancel{3} + 4\cos\omega T + 4\cos^2\omega T - \cancel{2}) =$$

$$= \frac{1}{9}(1 + 4\cos\omega T + 4\cos^2\omega T) = \frac{1}{9}(1+2\cos\omega T)^2$$

putting (5) into (4):

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty}\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} X_c(\Omega)\, e^{j\Omega nT}\, d\Omega\right] e^{-jn\omega} \qquad (7)$$

Interchanging Σ with integral in Eq.(7):

$$X(e^{j\omega}) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X_c(\Omega)\left[\sum_{n=-\infty}^{+\infty} e^{jn(\Omega T-\omega)}\right] d\Omega \qquad (8)$$

but we have:

$$\sum_{n=-\infty}^{+\infty} e^{jn(\Omega T-\omega)} = \frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta\left(\Omega-\frac{\omega}{T}+\frac{2\pi k}{T}\right) \qquad (9)$$

Then (8),(9):

$$X(e^{j\omega}) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X_c(\Omega)\times\left[\frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta\left(\Omega-\frac{\omega}{T}+\frac{2\pi k}{T}\right)\right] d\Omega$$

$$= \frac{1}{T}\sum_{k=-\infty}^{+\infty}\int_{-\infty}^{+\infty} X_c(\Omega)\,\delta\left(\Omega-\frac{\omega}{T}+\frac{2\pi k}{T}\right) d\Omega$$

$$= \frac{1}{T}\sum_{k=-\infty}^{+\infty} X_c\left(\frac{\omega}{T}-\frac{2\pi k}{T}\right)$$

$$\boxed{X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X_c\left(\Omega-\frac{2\pi k}{T}\right)}$$

from 高專攻 ppt Demonstration

Then $X(e^{j\omega})$ is a periodic function of $\frac{\omega}{T}=\Omega$ with period 2π.

(See Continue on back page)

but we had:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega(n-m)}\, d\omega = \frac{\sin[\pi(n-m)]}{\pi(n-m)} = \begin{cases} 1 & \text{for } m=n \\ 0 & \text{for } m\neq n\end{cases}$$

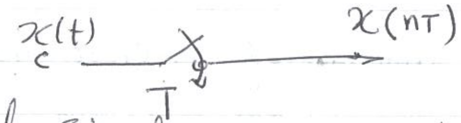
Then: $$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega(n-m)}\, d\omega = \delta(n-m)$$

So that:

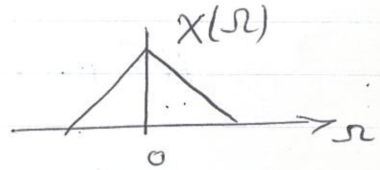
$$\hat{x}(n) = \sum_{m=-\infty}^{+\infty} x(m)\,\delta(n-m) = x(n)$$

and has been proved.

Sampling Theorem: [B]



Fourier Transform for Analog Signal:



$$X_c(\Omega) = \int_{-\infty}^{+\infty} x_c(t)\, e^{-j\Omega t}\, dt \qquad (1)$$

$$x_c(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X_c(\Omega)\, e^{j\Omega t}\, d\Omega \qquad (2)$$

Fourier Transform for Discrete-Time signal: $\omega = \Omega T$ (3)

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-jnT\Omega} = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-jn\omega} \qquad (4)$$

from Eq(2): for $t=nT$:

$$x(nT) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X_c(\Omega)\, e^{j\Omega nT}\, d\Omega \qquad (5)$$

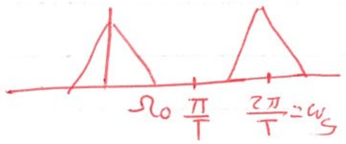
on the other hand:

$$x(nT) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega \qquad (6)$$

Nyquist rate for maximum frequency Ω_0 is:

$$\Omega_0 < \frac{\cancel{2}\pi}{T} = \frac{\omega_s}{2} = \cancel{2}\pi f_s$$

$$-\pi \leq \omega \leq \pi$$



In this case we can recover the analog signal from its sampling $x(nT)$ as follows:

$$\boxed{x_c(t) = \sum_{k=-\infty}^{+\infty} x_c(kT)\,\frac{\sin\left[\frac{\pi}{T}(t-kT)\right]}{\left[\frac{\pi}{T}(t-kT)\right]}}$$

This formula is obtained as follows: (see 82 page my writing)

$$x_c(t) = \frac{1}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} X_c(\Omega)\, e^{j\Omega t}\, d\Omega$$

$$-\frac{\pi}{T} < \Omega < \frac{\pi}{T} \;:\quad X(e^{j\omega}) = X(e^{j\Omega T}) = \frac{1}{T} X_c(\Omega)$$

$$x_c(t) = \frac{1}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} T\cdot X(e^{j\Omega T})\, e^{j\Omega t}\, d\Omega$$

but:

$$X(e^{j\omega}) = X(e^{j\Omega T}) = \sum_{k=-\infty}^{+\infty} x(kT)\, e^{-j\Omega T k}$$

$$x_c(t) = \frac{T}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \left[\sum_{k=-\infty}^{+\infty} x(kT)\, e^{-j\Omega T k}\right] e^{j\Omega t}\, d\Omega$$

$$x_c(t) = \sum_{k=-\infty}^{+\infty} x(kT)\left[\frac{T}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} e^{j\Omega(t-kT)}\, d\Omega\right]$$

$$\boxed{x_c(t) = \sum_{k=-\infty}^{+\infty} x(kT)\,\frac{\sin\left[\left(\frac{\pi}{T}\right)(t-kT)\right]}{\left(\frac{\pi}{T}\right)(t-kT)}}$$

We can proof the periodicity of $X(e^{j\omega})$ by other way as follows:

From: Two Eqs. (5), (6), we have:

$$x(nT) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X_c(\Omega)\, e^{j\Omega nT}\, d\Omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} X_c(\Omega)\, e^{j\Omega nT}\, d\Omega = \frac{1}{2\pi}\sum_{k=-\infty}^{+\infty}\int_{\frac{k-1}{2T}2\pi}^{\frac{k+1}{2T}2\pi} X_c(\Omega)\, e^{j\Omega nT}\, d\Omega = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\int_{-\frac{2\pi}{2T}}^{\frac{2\pi}{2T}} X_c\left(\Omega - \frac{2\pi k}{T}\right) \times e^{j\Omega nT}\, d\Omega$$

$$= \frac{1}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \sum_{k=-\infty}^{+\infty} X_c\left(\Omega - \frac{2\pi k}{T}\right) e^{j\Omega nT}\, d\Omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

Therefore:

$$\boxed{X(e^{j\omega}) = \frac{1}{T}\sum X_c\left(\Omega - \frac{2\pi k}{T}\right)}$$

$$\boxed{X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X_c\left(\Omega - \frac{2\pi k}{T}\right)}$$

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$\hookrightarrow \omega = 0 \quad |H(\omega)|^2 = \frac{1}{1.25-1} = \frac{1}{0.25} = 4$

$10\log_{10}|H(\omega)|^2 = 10\log_{10} 4 = 6\text{ dB}$

at $f = 2500\text{ Hz} \rightarrow \omega = 2\pi f = 2\pi \times 2500$

$|H(\omega)|^2 = \frac{1}{1.25 - \cos(2\pi \times 2500 \times 0.1 \times 10^{-3})} = \frac{1}{1.25 - \cos\frac{\pi}{2}} = 0.8$

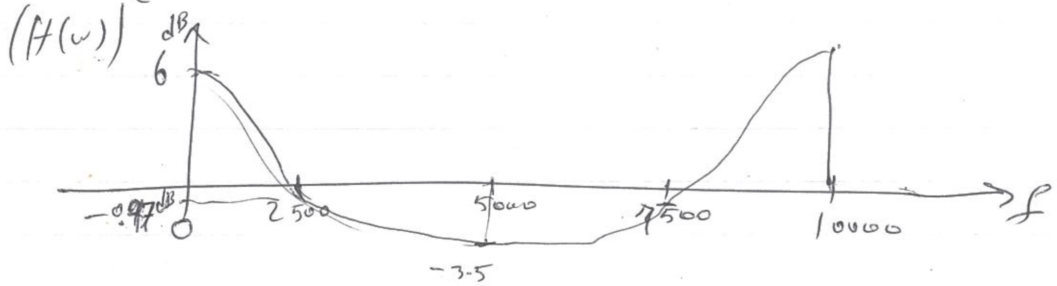
$10\log_{10} 0.8 = \boxed{-0.97\text{ dB}}$

at $f = 5000\text{ Hz}$: $|H(\omega)|^2 = \frac{1}{1.25 - \cos\pi} = \frac{1}{2.25} = 0.44$

$10\log_{10} 0.44 = -3.5\text{ dB}$

at $f = 7500\text{ Hz}$: $|H(\omega)|^2 = \frac{1}{1.25} = 0.8 \rightarrow \boxed{-0.97\text{ dB}}$

at $f = 10000\text{ Hz}$ $|H(\omega)|^2 = 4 \rightarrow 6\text{ dB}$



We can draw for phase too.

[C]

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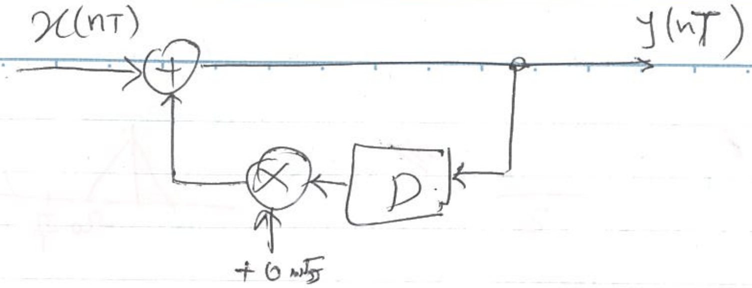

$y(nT) = x(nT) + 0.5\,y(nT-T)$

The impulse response is:

$$h(nT) = 0.5^n\, u(nT)$$

The Frequency Response is:

$$H(e^{j\omega T}) = \sum_{n=-\infty}^{+\infty} h(nT)\, e^{-j\omega nT}$$

$$H(e^{j\omega T}) = \sum_{n=0}^{\infty} 0.5^n e^{-j\omega nT} = \frac{1}{1-0.5e^{-j\omega T}}$$

$$= \frac{1}{1-0.5(\cos\omega T - j\sin\omega T)} = \frac{1}{1-0.5\cos\omega T + 0.5j\sin\omega T}$$

$$|H(e^{j\omega T})|^2 = \frac{1}{(1-0.5\cos\omega T)^2 + 0.25\sin^2\omega T} = \frac{1}{1.25-\cos\omega T}$$

For: $T = 0.1\text{ ms}$

$$\angle H(e^{j\omega T}) = -\text{Arctan}\,\frac{0.5\sin\omega T}{1-0.5\cos\omega T}$$

$\hookleftarrow$