

3.9

$$x(nT) \longrightarrow x(-nT)$$

$$X(\omega) = \sum_{n=-\infty}^{+\infty} x(nT) e^{-j\omega nT}$$

$$x(-nT) = x'(nT) \xrightarrow{F} X'(\omega)$$

$$X'(\omega) = \sum_{n=-\infty}^{+\infty} x'(nT) e^{-j\omega nT} = \sum_{n=-\infty}^{+\infty} x(-nT) e^{-j\omega nT}$$

$$\begin{aligned} n \rightarrow -m \quad +\infty & \quad j\omega mT & \quad +\infty & \quad -j(-\omega) mT \\ = \sum_{m=-\infty}^{+\infty} x(mT) e^{j\omega mT} & = \sum_{m=-\infty}^{+\infty} x(mT) e^{-j(-\omega) mT} = X(-\omega) \end{aligned}$$

$$X'(\omega) = X(-\omega)$$

gusu: even part: $Ev[x(n)] = x_e(n) = x_e(-n)$

kisu: odd part: $od[x(n)] = x_o(n) = x_o(-n)$

$$x(n) = x_e(n) + x_o(n) \longrightarrow \frac{1}{2}[x(n) + x(-n)] = x_e(n)$$

$$x(-n) = x_e(-n) + x_o(-n)$$

$$x(-n) = x_e(n) - x_o(n) \longrightarrow \frac{1}{2}[x(n) - x(-n)] = x_o(n)$$

$$X_e(\omega) = \sum_{n=-\infty}^{+\infty} x_e(n) e^{-j\omega n} = \frac{1}{2} \sum_{n=-\infty}^{+\infty} [x(n) + x(-n)] e^{-j\omega n} = \frac{1}{2} [X(\omega) + X(-\omega)]$$

If $x(n)$ real: $\rightarrow X(-\omega) = X^*(\omega)$

$$X_e(\omega) = \frac{1}{2} [X(\omega) + X^*(\omega)] = \text{Re}[X(\omega)]$$

$$\begin{aligned} X_o(\omega) = F[x_o(n)] &= \sum_{n=-\infty}^{+\infty} x_o(n) e^{-j\omega n} = \frac{1}{2} \sum_{n=-\infty}^{+\infty} [x(n) - x(-n)] e^{-j\omega n} = \frac{1}{2} [X(\omega) - X^*(\omega)] \\ &= j \text{Im}[X(\omega)] \end{aligned}$$