Mathematical Morphology

Morphological Filters

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February 18, 2013

[1]The slides has been designed by using LaTeX.

Set Theory and Basic Operations

Mathematical Morphology has build up based on Set Theory. As well, the non-linear morphological operators are basically based on set operations.

How can we relate signal and images to sets?

To answer this question, let's start with binary images;

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Set Theory and Basic Operations

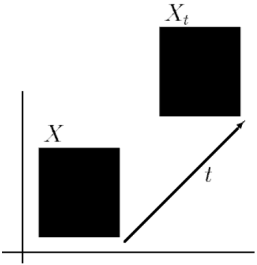
This relation is as follows;
X is a set of vectors in space $\mathbb{R}^2$.
Considering a binary image composed of values 0 and 1. This image can be express as a set in this way:
With respect to an origin, every pixel is a coordination of a vector which point to that pixel from origin. The value of pixel 0 or 1 determines that whether the vector belongs to the set X or its compliment X^c.

Set Theory and Basic Operations

Similar to normal mathematic that is based on addition and subtraction, Mathematical Morphology is based some basic operation in set theory.

1. Set Translation: X_t is the translated set of X by vector t, as follows:

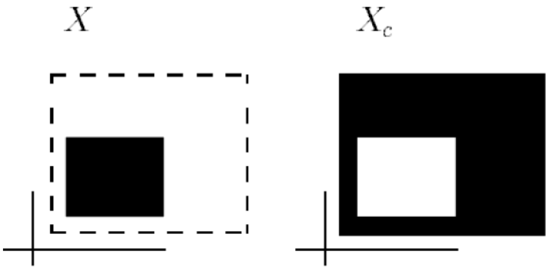
$$X_t = \{x | x - t \in X\} \tag{1}$$



Set Theory and Basic Operations

2. Set Compliment: X^c is the Compliment set of X, as follows:

$$X^c = \{x | x \notin X\} \tag{2}$$



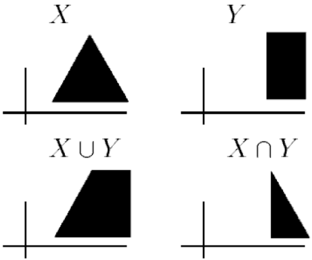
Set Theory and Basic Operations

3. Union of two sets: The union of two sets X and Y denoted as $X \cup Y$ and defined as :

$$X \cup Y = \{x | x \in X \text{ or } x \in Y\} \tag{3}$$

4. Intersection of two sets: The Intersection of two sets X and Y denoted as $X \cap Y$ and defined as :

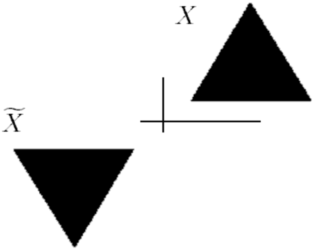
$$X \cap Y = \{x | x \in X \text{ and } x \in Y\} \tag{4}$$



Set Theory and Basic Operations

5. Transpose of a set: The transpose of set X is denoted by $\check{X}$ and defined as :

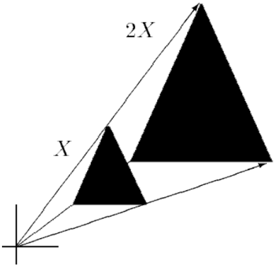
$$\check{X} = \{x \mid -x \in X\} \quad (5)$$



Set Theory and Basic Operations

6. Scaling of a set: The scaled set of X by the scale of α is denoted by αX and defined as :

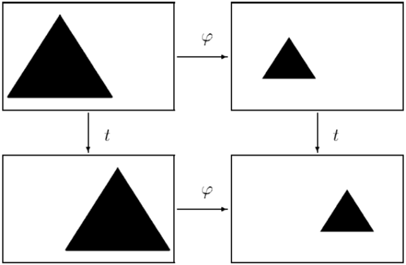
$$\alpha X = \{\alpha x | x \in X\} \tag{6}$$



Set Theory and Basic Operations

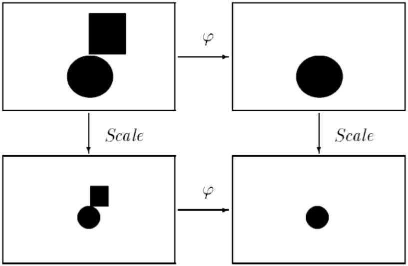
7.Compatible to Translation: In set domain, a transform φ is compatible to translation if

$$\forall t, : \varphi(X_t) = [\varphi(X)]_t \tag{7}$$



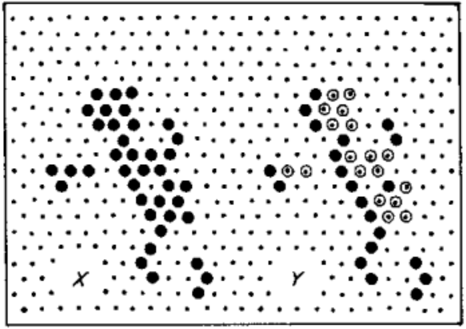
8.Compatible to Scaling: In set domain, a transform φ is compatible to scaling if

$$\varphi_\lambda(X) = \lambda\varphi\left(\frac{X}{\lambda}\right) \tag{8}$$



B1. Step One

A morphological transform



Structuring elements
For X: $B = \{\bullet\}$
For Y: $B' = b_1\, b_2 = \{\circ \overset{\downarrow}{\bullet}\}$

(explanation) ▷ ▷ ▷

Figure II.1. Initial set X, and its transform Y involved in the intercepts measurements.

- ● points of the structuring element which must belong to X
- ○ points of the structuring element which must belong to X^c
- · points of the structuring element with no condition
- ↓ location of the origin associated with the structuring element.

B1. Step One

The transformation is the following:

if $x + b_1$ belongs to the background X, and $x + b_2$ belongs to X, then the transform gives a “one” at $x + b_2$.
When x is allowed to sweep the whole field, a new image is generated.
The number of intercepts of X, is equivalently the number of picture points of Y, and equals 20.

Homework:
Write the above transform just using mathematical notations.

B1. Step One
Another transform: The connectivity number

The connectivity number **(i.e. the number of particles minus their holes)** can be calculated by the following similar approach.

The associated structuring elements are now triangular configurations, involving adjacent points of the grid. *N* equals

the following difference:

$$N = \mathscr{N}\begin{bmatrix} & \check{1} & \\ 0 & & 0 \end{bmatrix} - \mathscr{N}\begin{bmatrix} 1 & & \check{1} \\ & 0 & \end{bmatrix} \qquad (9)$$

where $\mathscr{N}(*)$ = number of configurations of the type $*$.

In other words, ▷ ▷ ▷

B1. Step One
The Connectivity Number (cont.)

if:
Y is the set of points $x + b_2$ such that
$x + b_1, \quad x + b_2 \in X$ and $x + b_3 \in X^c$

Y' is the set of points $x + b'_3$ such that
$x + b'_1, \quad x + b'_2 \in X^c$ and $x + b'_3 \in X$

then, the connectivity number N is the difference of areas between Y' and Y.

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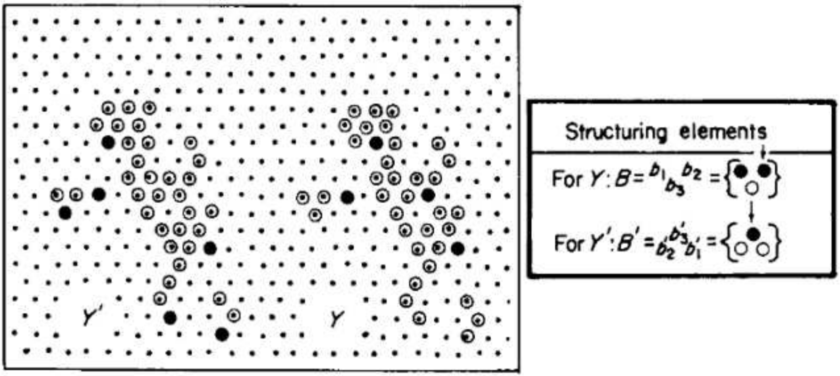
B1. Step One
The Connectivity Number (cont.)



Transforms Y and Y' involved in
the connectivity number computation.

1. Definition of dilation and erosion

We define the eroded set Y of X as the locus of centres x of B_x included in the set X.

This transformation looks like the classical **Minkowski substraction**:

$$X \ominus B = \bigcap_{b \in B} X_b \tag{1}$$

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An Algebraic Study
Erosion :

Indeed, when y sweeps B_o, the point $x + y$ lies in X iff x belongs to the translate X_{-y} of X; i.e.:

$$\begin{aligned} Y &= \{x : B_x \subset X\} \\ &= \bigcap_{y \in B_0} X_{-y} \\ &= \bigcap_{-y \in B_0} X_y \\ &= X \ominus \check{B} \end{aligned} \tag{2}$$

The above definitions of Erosion, allows us to study the algebraic properties of this operation, and also suggests a way to compute it in practice.

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An Algebraic Study
Dilation :

Consider now what happens to the complementary set X^c (i.e. ”the pores”) when eroding set X (the ”grains”).
Obviously, the pores have necessarily been enlarged, since erosion has shrunk the grains. We shall call this operation **Dilation** and denote it by the symbol $\oplus$.

We have:

$$X^c \oplus B = (X \ominus B)^c \tag{3}$$

Dilating the grains is equivalent to eroding the pores, and conversely.

These operations are said to be dual to each other.

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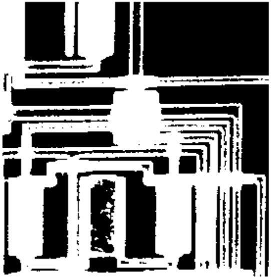
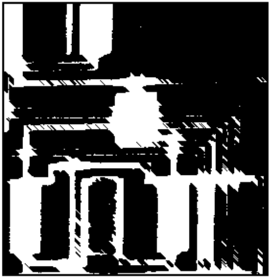
An Algebraic Study
Dilation :



Original Image



Eroded X
Dilated X^c

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An Algebraic Study
Dilation (Local Interpretation):

The dilate $X \oplus B$ is the locus of the centres of the B_x which hit the set X:

$$\begin{aligned} X \oplus \check{B} &= \{x : B_x \cap X \neq \varnothing\} \\ &= \{x : B_x \Uparrow X\} \end{aligned} \tag{4}$$

where the vertical arrow ”$\Uparrow$”, means ”hits”.

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An Algebraic Study
An analytical expression of the dilation:

By applying the (2) and (3) to the pores X^c, we find that dilation of X by B can be expressed as:

$$X \oplus \check{B} = \bigcup_{y \in \check{B}} X_y \tag{5}$$

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An Algebraic Study

An analytical expression of the dilation:

By noting that X is identical to the union of all its points $(X = \bigcup_{y \in X} \{x\})$, we derive from (5) a second more symmetrical expression,

$$\begin{aligned} X \oplus \check{B} &= \bigcup_{\substack{x \in X \\ y \in \check{B}}} \{x + y\} \\ &= \bigcup_{x \in X} \check{B}_x \end{aligned} \tag{6}$$

The first equality is Minkowski addition.

We see also why it was necessary to use the transposed set $\check{B}$ in the definition. Due to this transposition, the roles of X and B in (6) are made symmetrical.

An Algebraic Study
Transposed B in Dilation

A brief example will clearly show the difference between $X \oplus B$, and $X \oplus \check{B}$.
Take for X as well as for B the same equilateral triangle of side a. Let's obtain the following morphological operations:

$\oplus$

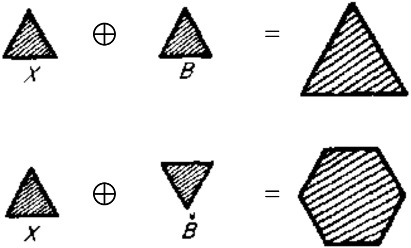
$= ?$

$\oplus$

$= ?$

An Algebraic Study
Transposed B in Dilation

Then $X \oplus B$ is the equilateral triangle of side $2a$, although $X + \check{B}$ turns out to be a regular hexagon of side a.



Homework: Do Dilation and Erosion on the paper at home for different possible shapes.

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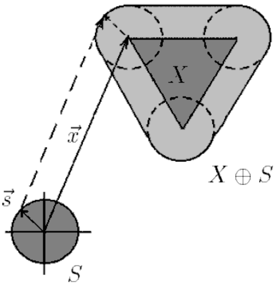
An Algebraic Study
Transposed B in Dilation

For a symmetric B, no matter to be transposed or not.
You can see such case here, as well a better intuition to dilation as

$$X \oplus \check{S} = \{x + s | x \in X \text{ and } s \in S\}$$



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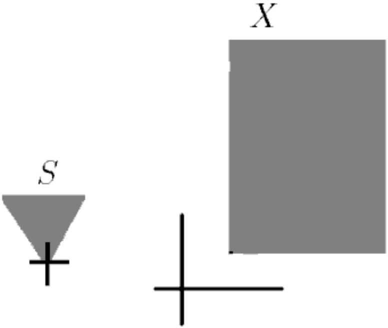
An Algebraic Study

Homework:

Considering the following sets X and S, please draw the the resultant sets of dilation and eriosn of X by S.



An Algebraic Study

Homework:

By going back to equation (4) of this lecture, we see a notation $\Uparrow$ which means "hits". Using a similar analogy, give a definition for erosion of X by B.

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One of the advantages of Erosion and Dilation is these operations are increasing.

Increasing Transform :
In set domains, φ is increasing if

$$X \subset Y \;\Rightarrow\; \varphi(X) \subset \varphi(Y) \tag{7}$$

Both dilation and erosion are compatible to translation;

$$X_t \oplus B = (X \oplus B)_t \tag{8}$$

$$X_t \ominus B = (X \ominus B)_t \tag{9}$$

An Algebraic Study
Extensivity

Dilation has the property of **extensivity** that makes the set more extended (dilated).

Extensivity: If the center of Structuring Element B is located its inside, ($o \in B$), then;

$$X \subset X \oplus B$$

Assuming the condition $o \in B$, dilation adds to the pixels of the set and it does not remove any pixel of it.

We can consider extensivity property in a way that includes the cases ($o \notin B$).

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An Algebraic Study
Extensivity (generalization)

Extensivity: For any arbitrary structuring element B, we have the following property for dilation of X by B;

$$\exists t, \ \forall X, \ X \subset (X \oplus B)_t \tag{10}$$

An Algebraic Study
Anti-Extensivity (generalization)

Erosion has the property of **anti-extensivity** that makes the set smaller (eroded).

Anti-extensivity: If the center of Structuring Element B is located inside it, ($o \in B$), then;

$$X \ominus B \subset X$$

and for the case $o \notin B$,

$$\exists t,\ \forall X,\ (X \ominus B)_t \subset X$$

An Algebraic Study
Commutativity

Dilation is commutative:

$$X \oplus B = B \oplus X \tag{11}$$

Erosion does not have this property.

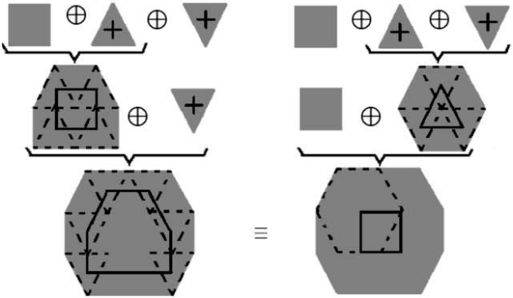
Homework: Using the definition and interpretation of dilation as you learned in this lecture, check the correctness of the following equality.

An Algebraic Study
Commutativity

Dilation is Associative:

$$(X \oplus Y) \oplus T = X \oplus (Y \oplus T)$$

The figure shows this property for dilation.



Homework:

Please check the associative property for erosion ($\ominus$).

The origin $\{o\}$ is the unit element of the semi-group[1]:

$$X \oplus \{0\} = X \ominus \{0\} = X \tag{12}$$

and

$$X \oplus \{x\} = X \ominus \{x\} = X_x \tag{13}$$

[1] A semigroup is a set S together with a binary operation (that is, a function $S \times S \mapsto S$) that satisfies the associative property: For all $a, b, c \in S$, the equation $(a.b).c = a.(b.c)$ holds.

An Algebraic Study
Distributivity

Dilation distributes the union.
We can write:

$$\begin{aligned} X \oplus (B \cup B') &= \bigcup_{x \in X} (B_x \cup B'_x) \\ &= \left(\bigcup_{x \in X} B_x \right) \bigcup \left(\bigcup_{x \in X} B'_x \right) \end{aligned} \tag{14}$$

Which means

$$X \oplus (B \cup B') = (X \oplus B) \cup (X \oplus B') \tag{15}$$

Using duality of operations we have for erosion ▷ ▷ ▷

An Algebraic Study
Distributivity

$$X \ominus (B \cup B') = (X \ominus B) \cap (X \ominus B') \tag{16}$$
$$(X \cap Z) \ominus B = (X \ominus B) \cap (Z \ominus B) \tag{17}$$

The technological implications of the relations (15,16,17) are enormous.
From the first two, we can dilate, or erode X by taking B piece by piece, and then combine the intermediary results by union or intersection respectively.
The relation (17) is treated on **local knowledge**.

Homework : Using duality property and equation (15) obtain the above equations!

An Algebraic Study
Distributivity

We saw that:
Dilation is distributive to $\cup$.
Erosion is distributive to $\cap$.

As well we have the following set inequalities:

$$\begin{aligned} X \oplus (B \cap B') &\subset (X \oplus B) \cap (X \oplus B') && (18) \\ X \ominus (B \cap B') &\supset (X \ominus B) \cup (X \ominus B') && (19) \\ (X \cup Z) \ominus B &\supset (X \ominus B) \cap (Z \ominus B) && (20) \end{aligned}$$

Iterativity

From definition of Dilation and Erosion we can drive that:

$$
\begin{aligned}
(X \oplus Y) \ominus Z &= \bigcap_{z \in Z} \bigcup_{y \in Y} X_{y+z} \\
(X \ominus Z) \oplus Y &= \bigcup_{y \in Y} \bigcap_{z \in Z} X_{y+z}
\end{aligned} \tag{21}
$$

and thus we obtain the (generally strict) set inequality:

$$(X \ominus Z) \oplus Y \subset (X \oplus Y) \ominus Z \tag{22}$$

Simply because, $\cup\cap \subset \cap\cup$!
It is more severe to erode before dilating than to do the reverse.

Iterativity
Iteration of Erosions

$$
\begin{aligned}
(X \oplus B) \ominus B' &= \bigcap_{b' \in B'} (X \ominus B)_{b'} \\
&= \bigcap_{b' \in B'} \bigcap_{b \in B} X_{b+b'} \\
&= \bigcap_{z \in\, B \oplus B'} X_z
\end{aligned}
$$

Then, from the above equations, we can conclude:

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Iterativity
Iteration of Erosions

$$(X \ominus B) \ominus B' = X \ominus (B \oplus B') \quad (23)$$

$$(X \oplus B) \oplus B' = X \oplus (B \oplus B') \quad (24)$$

The second equation is obtained from the first by taking complements.

The above two equations are very useful for technological implications. Here we are provided with a powerful morphological tool, which enables us to decompose a given B (structuring element) into the Minkowski sum of several much simpler B_is.

Iterativity
Decomposing the Structuring Element and Iterative Operation

For example, a pair of points, added to another pair of points, aligned perpendicularly to the first pair, becomes the four vertices of a rectangle.

Such decompositions are particularly useful, theoretically and practically speaking, when dealing with convex structuring elements.

$$\left\{\begin{matrix}\times & \times\\ \times & \times\end{matrix}\right\} = \{\times \;\; \times\} \oplus \left\{\begin{matrix}\times\\ \times\end{matrix}\right\}$$

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ITERATIVITY

Decomposing the Structuring Element and Iterative Operation

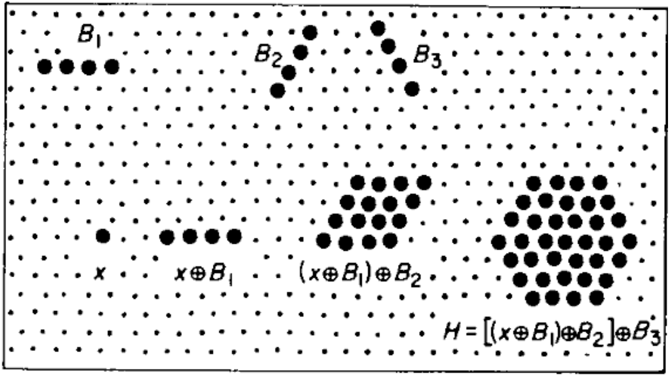
The erosion by the regular hexagon with side length a is the result of three successive erosions by the three segments of length a, which are parallel to the edges of the hexagon.

In other words, bi-dimensional erosions can become iterations of uni-dimensional erosions (this will occur when structuring elements are for example the square, the hexagon (6sides), the dodecagon (12sides), the octagon (8sides), but not the triangle or the pentagon!).

See the next slide! ▷ ▷ ▷

ITERATIVITY
Decomposing the Structuring Element and Iterative Operation

The hexagon considered as the dilation of the segments.



Opening and Closing

Let's start with a question!

After having eroded X by B, is it possible, in general, to recover the initial set by dilating the eroded set $X \ominus B$ by the same B? The answer is No.

Homework: Practically try the to find the reason.

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Opening and Closing

A similar question:

After having dilated X by B, is it possible, in general, to recover the initial set by eroding the dilated set $X \oplus B$ by the same B? The answer is No.

Homework: Practically try the to find the reason.

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Opening

In the answer of both questions, dilation after erosion or erosion after dilation reconstitutes only a part of X, which is simpler and has less details, but may be considered as that part which is the most essential.

We call the opening of X with respect to B the following set, denoted X_B or $X \circ B$:

$$X \circ B = (X \ominus \check{B}) \oplus B \tag{1}$$

$\triangleright \triangleright \triangleright$

Closing

Similarly, we call the closing of X with respect to B (denoted by X^B or $X \bullet B$) the following set:

$$X \bullet B = (X \oplus \check{B}) \ominus B \tag{2}$$

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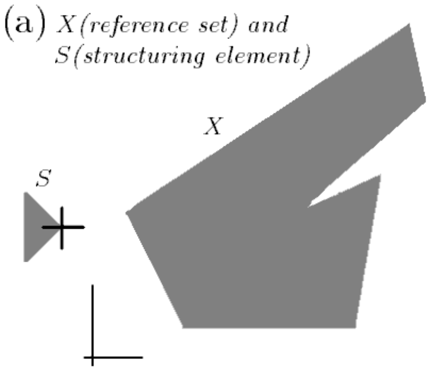
Opening at two steps

Here we graphically show the opening of X by S at two steps of erosion and dilation;



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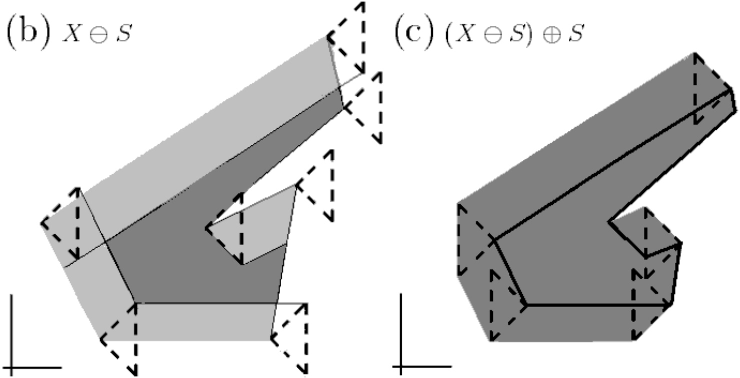
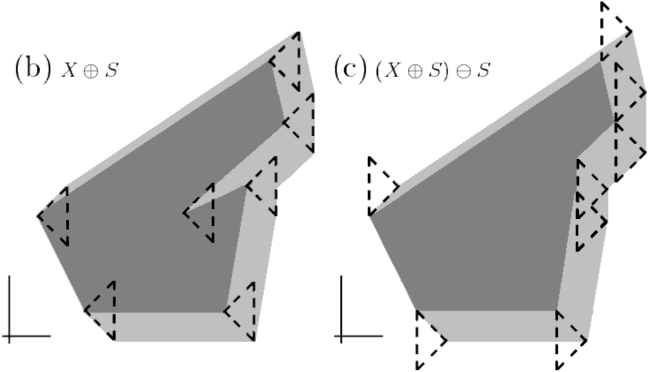
Opening at two steps



Closing at two steps



Opening and Closing
Equivalent Definitions

As well, opening can be equivalently defined as follows:

The opening of X by S is equivalent to the set of union of all translations of S_y where completely covered by X :

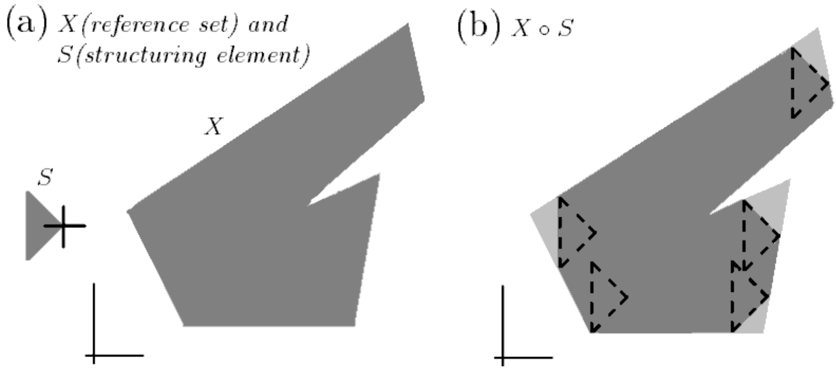
$$X \circ X = \bigcup_{S_y \subset X} S \tag{3}$$

Using this definition of opening, we can obtain opening directly in one step.

See the next slide! ▷ ▷ ▷

Opening and Closing
Equivalent Definitions

It is exactly equivalent to sliding SE interior the border of X, and the exterior border of its trace to be considered as the border of opening set.



Opening and Closing
Equivalent Definitions

Opening and closing are dual with respect to taking complements; opening of the complement X^c of X is precisely the complement of the closing of X, and conversely:

$$(X \circ B)^c = X^c \bullet B \tag{4}$$

$$(x \bullet B)^c = X^C \circ B \tag{5}$$

Considering the duality of opening and closing, ...

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Opening and Closing
Equivalent Definitions

Considering the duality of opening and closing, we can define closing as follows:

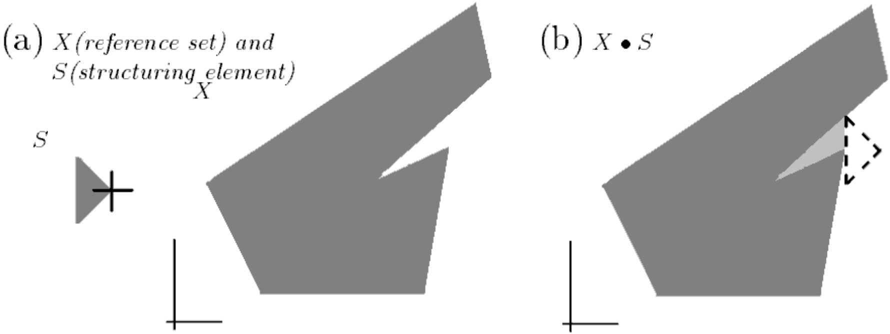
$$
\begin{aligned}
X \bullet S &= \left\{ \bigcup_{S_y \subset X^c} S_y \right\}^c \\
&= \left\{ \bigcup_{(S_y \cap X) = \varnothing} S_y \right\}^c
\end{aligned}
\tag{6}
$$

Using this definition of closing, we can obtain closing directly in one step.

See the next slide! ▷ ▷ ▷

Opening and Closing
Equivalent Definitions

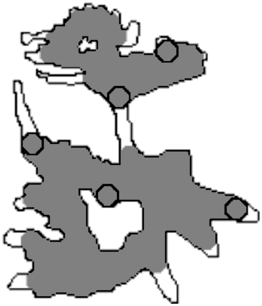
It is exactly equivalent to sliding SE exterior the border of X, and the interior border of its trace to be considered as the border of opening set.

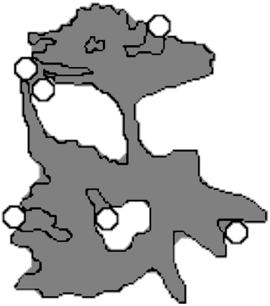


Geometrical Effect of Opening and Closing

Consider the following image. The binary image has been composed of foreground and background (grain and pore). We can geometrically consider it as land and sea with lakes and gulfs inside the land and islands and capes inside the sea. Now, we want to observe the geometrical effects of opening and closing.

Geometrical Effect of Opening and Closing





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Geometrical Effect of opening and closing

Opening operation smooths the contours of X, cuts the narrow isthmuses, suppresses the small islands and the sharp capes of X.

The closing blocks up the narrow channels, the small lakes and the long, thin gulfs of X.

Algebraic properties

In algebra, an application is said to be an **algebraic opening**, if it follow the three following properties

$\forall X$

- Anti-Extensivity: $\varphi(X) \subset X$
- Increasing: $X \subset X' \Longrightarrow \varphi(X) \subset \varphi(X')$
- Idempotent[1] : $\varphi(\varphi(X)) = \varphi(X)$

The morphological opening is an algebraic opening.

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[1] This property is especially determining that whether the operation is filter or not!

Algebraic properties

In algebra, an application is said to be an **algebraic closing**, if it follow the three following properties
$\forall\ X$

- Extensivity: $\varphi(X) \supset X$
- Increasing: $X \subset X' \Longrightarrow \varphi(X) \subset \varphi(X')$
- Idempotent : $\varphi(\varphi(X)) = \varphi(X)$

The morphological opening is an algebraic opening.

▷ ▷ ▷

Idempotence

Among the three properties just described, the most novel is the one of idempotence. Just as for closing, dilation also was an increasing and extensive operation. The difference between the two lies in the fact that when we iterate the same dilation once more, we do not stay with the first dilation but get a larger dilated set. In a certain sense, closing and opening may be considered as more stable than the intermediary steps of dilation and erosion.

Homework: Practice the successive dilations and successive closings over a set and check the idempotence property.

Opening and closing just follows the morphology of the set.

Opening and closing are not sensitive to translation of structuring element.

$$\forall$$

$$\begin{aligned} X \circ S_t &= X \circ S \\ X \bullet S_t &= X \bullet S \end{aligned}$$

This property brings the advantage for opening and closing that make them independent of centre of structuring element. Opening and closing just depend on size and shape of structuring element.

To be open/close for a set

The set X is open for B if

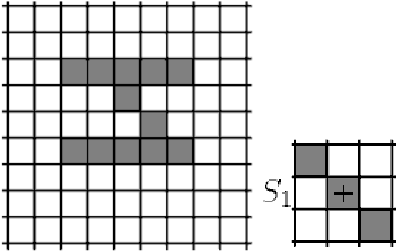
$$X \circ B = X \tag{7}$$

The set X is close for B if

$$X \bullet B = X \tag{8}$$

A Class Practice 

Here, we have a binary set with few points. We try different morphological operations; dilation, erosion, opening and closing using different structuring elements.



(a)input set: X

$X \oplus S_1 = ?$

$X \oplus S_1 \triangleright \triangleright \triangleright$

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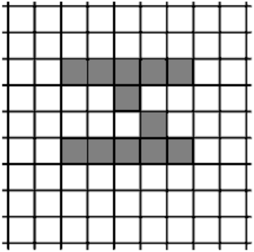

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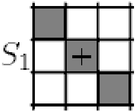

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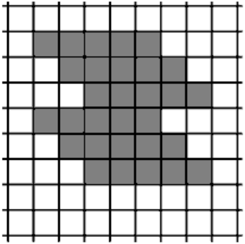

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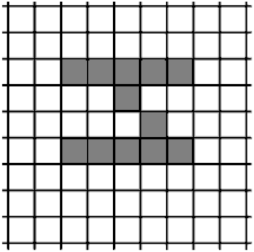

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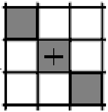
S_1



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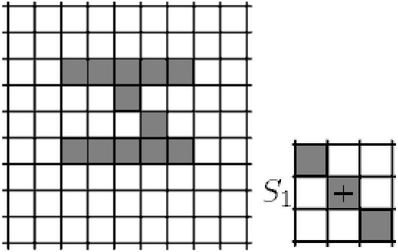

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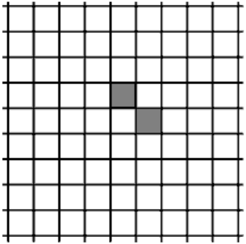

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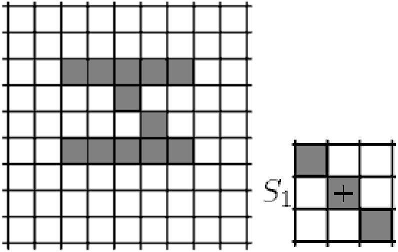

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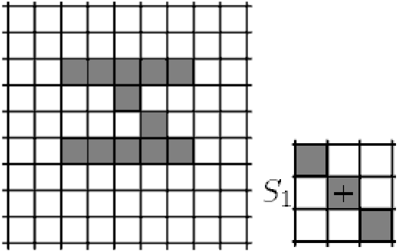

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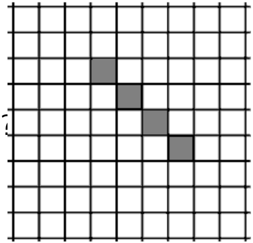

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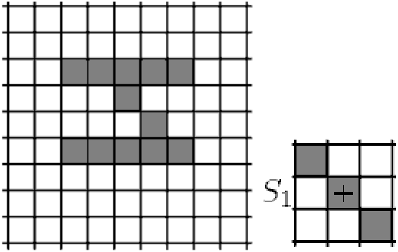

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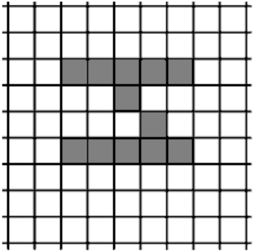
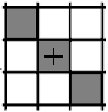


S_1



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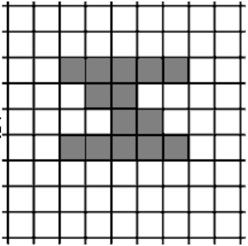

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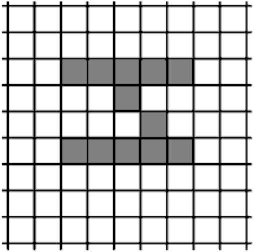
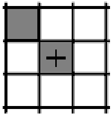


(a)input set: X

S_2



$X \oplus S_2 = ?$

$X \oplus S_2 \triangleright \triangleright \triangleright$

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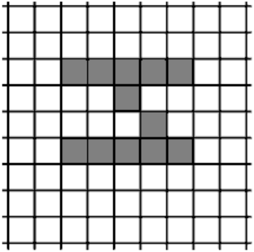
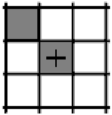
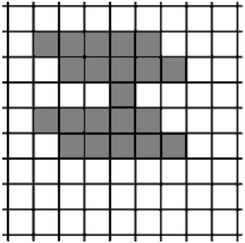


(a)input set: X

S_2 

$X \oplus S_2 =$ 

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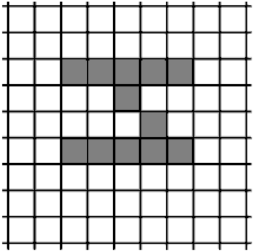

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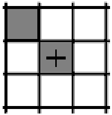
S_2 

$X \ominus S_2 = ?$

$X \ominus S_1$ ▷ ▷ ▷

A Class Practice 

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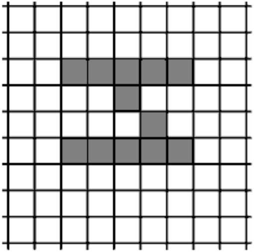

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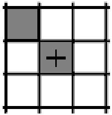
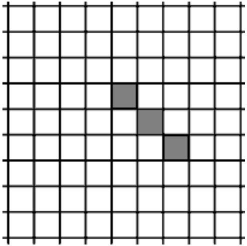
S_2 

$X \ominus S_2 =$ 

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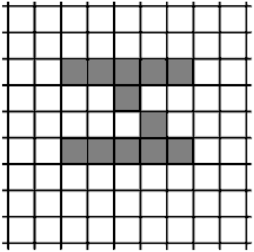

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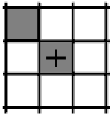
S_2



$$X \circ S_2 = ?$$

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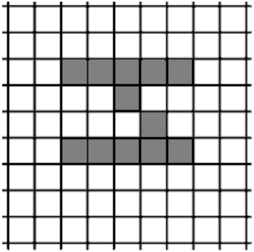

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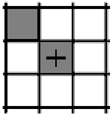
S_2 

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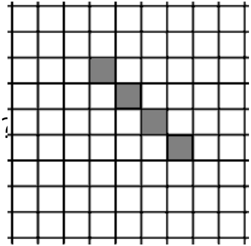

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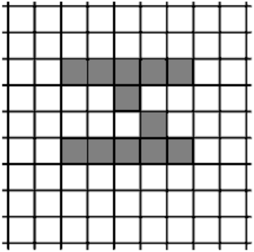

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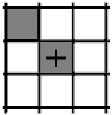

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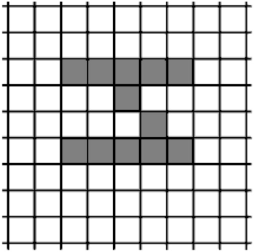

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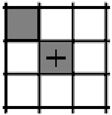
S_2 

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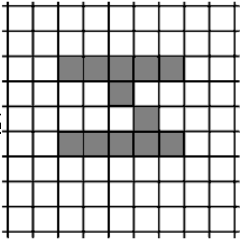
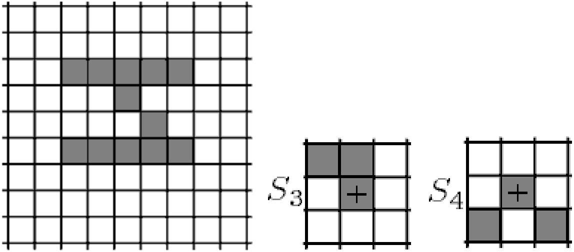


Homework

Having the following X, S_3, S_4, please obtain
$X \oplus S_3, X \ominus S_3, X \circ S_3, X \bullet S_3, X \oplus S_4, X \ominus S_4, X \circ S_4, X \bullet S_4$.



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