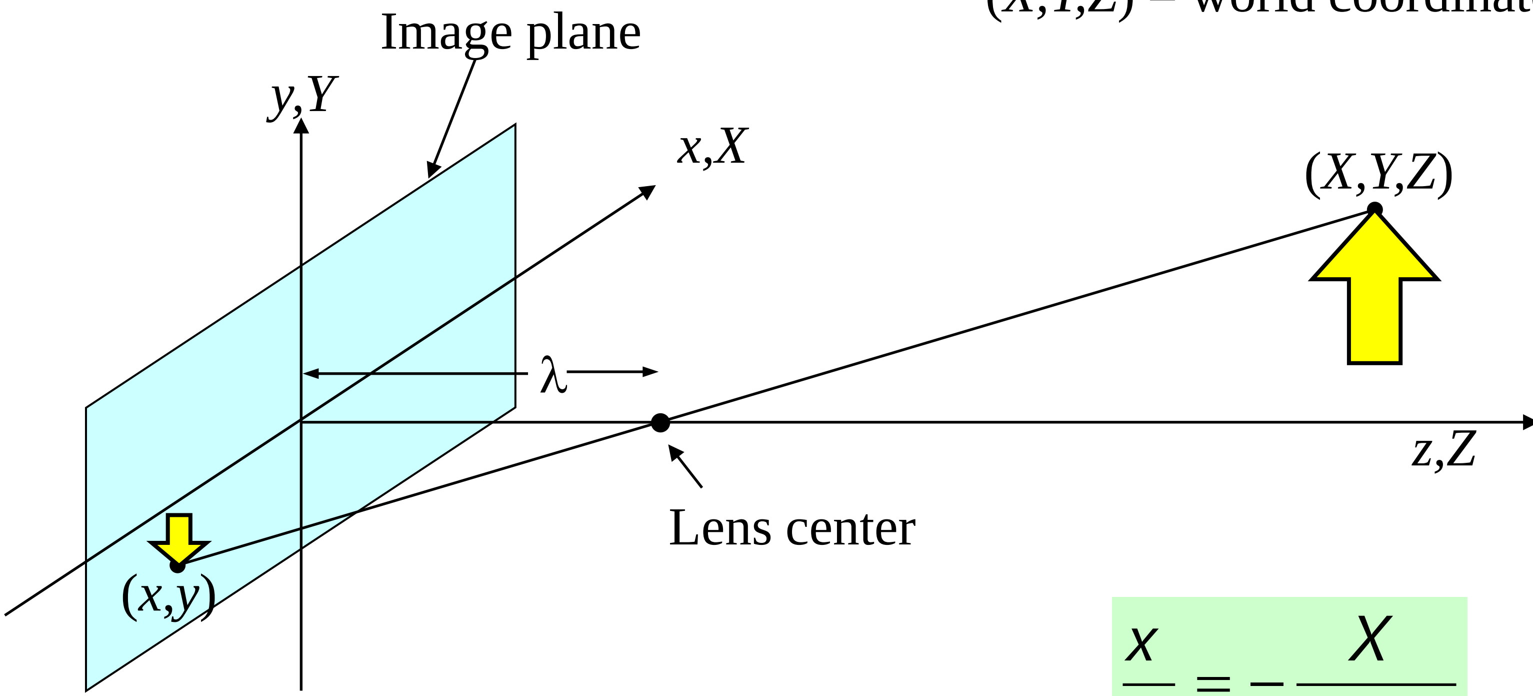
Digital Image Processing:

Imaging Geometry: Perspective Transformation

(X,Y,Z) = world coordinate



(x,y,z) = Camera coordinate system

λ = focal length

$$\frac{x}{\lambda} = -\frac{X}{Z-\lambda}$$

$$\frac{y}{\lambda} = -\frac{Y}{Z-\lambda}$$

Eq. 1.1

Imaging Geometry: Perspective Transformation (cont.)

Relation between camera coordinate (*x*,*y*,*z*) and real world coordinate (*X*,*Y*,*Z*) are given by

$$c = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \dfrac{\lambda X}{\lambda - Z} \\ \dfrac{\lambda Y}{\lambda - Z} \\ \dfrac{\lambda Z}{\lambda - Z} \end{bmatrix} \qquad \text{Eq. 1.2}$$

Since on the image plane *z* is always zero, *z=0*, we consider only (x,y) while z is neglected.

Imaging Geometry: Perspective Transformation (cont.)

Equation 1.2 is not linear because of Z in the dividers so we introduce the ***homogeneous coordinate*** to solve this problem.

Cartesian coordinate $$w = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Homogeneous coordinate $$w_h = \begin{bmatrix} kX \\ kY \\ kZ \\ k \end{bmatrix}$$

k = nonzero constant

To convert from the homogeneous coordinate w_h to the Cartesian coordinate w, we divide the first 3 components of w_h by the fourth component.

Imaging Geometry: Perspective Transformation (cont.)

The perspective transformation matrix for the homogeneous coordinate:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\lambda} & 1 \end{bmatrix}$$

Perspective transformation becomes:

$$c_h = Pw_h = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\lambda} & 1 \end{bmatrix} \begin{bmatrix} kX \\ kY \\ kZ \\ k \end{bmatrix} = \begin{bmatrix} kX \\ kY \\ kZ \\ \frac{-k(Z-\lambda)}{\lambda} \end{bmatrix}$$

Eq. 1.3

Imaging Geometry: Perspective Transformation (cont.)

From homogeneous coordinate

$$c_h = \begin{bmatrix} kX \\ kY \\ kZ \\ \dfrac{-k(Z-\lambda)}{\lambda} \end{bmatrix}$$

We get camera coordinate in the image plane:

$$c = \begin{bmatrix} kX \cdot \dfrac{\lambda}{-k(Z-\lambda)} \\ kY \cdot \dfrac{\lambda}{-k(Z-\lambda)} \\ kZ \cdot \dfrac{\lambda}{-k(Z-\lambda)} \end{bmatrix} = \begin{bmatrix} \dfrac{\lambda X}{\lambda - Z} \\ \dfrac{\lambda Y}{\lambda - Z} \\ \dfrac{\lambda Z}{\lambda - Z} \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Imaging Geometry: Inverse Perspective Transformation

$$w_h = P^{-1} c_h \quad \text{where} \quad P^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\lambda} & 1 \end{bmatrix} \qquad \text{Eq. 1.4}$$

Inverse Perspective Transformation (cont.)

For an image point (x_0,y_0), since on the image plane z=0, we have

$$c_h = \begin{bmatrix} kx_0 \\ ky_0 \\ 0 \\ k \end{bmatrix}$$

We get the real world coordinate :

$$w_h = P^{-1}c_h = \begin{bmatrix} kx_0 \\ ky_0 \\ 0 \\ k \end{bmatrix} \quad \text{or} \quad w = \begin{bmatrix} x_0 \\ y_0 \\ 0 \end{bmatrix} \rightarrow ???$$

Since the perspective transformation maps 3-D coordinates to 2-D Coordinates, we cannot get the inverse transform.

Inverse Perspective Transformation (cont.)

To find the solution, let

$$c_h = \begin{bmatrix} kx_0 \\ ky_0 \\ kz \\ k \end{bmatrix}$$

We get

$$w_h = P^{-1}c_h = \begin{bmatrix} kx_0 \\ ky_0 \\ kz \\ \frac{k(z+\lambda)}{\lambda} \end{bmatrix} \quad \text{or} \quad w = \begin{bmatrix} X = \frac{\lambda x_0}{\lambda + z} \\ Y = \frac{\lambda y_0}{\lambda + z} \\ Z = \frac{\lambda z}{\lambda + z} \end{bmatrix} \qquad \text{Eq. 1.5}$$

Inverse Perspective Transformation (cont.)

From Eq. 1.5,

We get
$$z = \frac{\lambda Z}{\lambda - Z} \qquad \text{Eq. 1.6}$$

Substituting Eq. 1.6 into Eq.1.5, we get

$$X = \frac{x_0}{\lambda}(\lambda - Z)$$
$$Y = \frac{y_0}{\lambda}(\lambda - Z) \qquad \text{Eq. 1.7}$$

Equations 1.7 show that inverse perspective transformation requires information of at least one component of the world coordinate of the point.

Camera Model

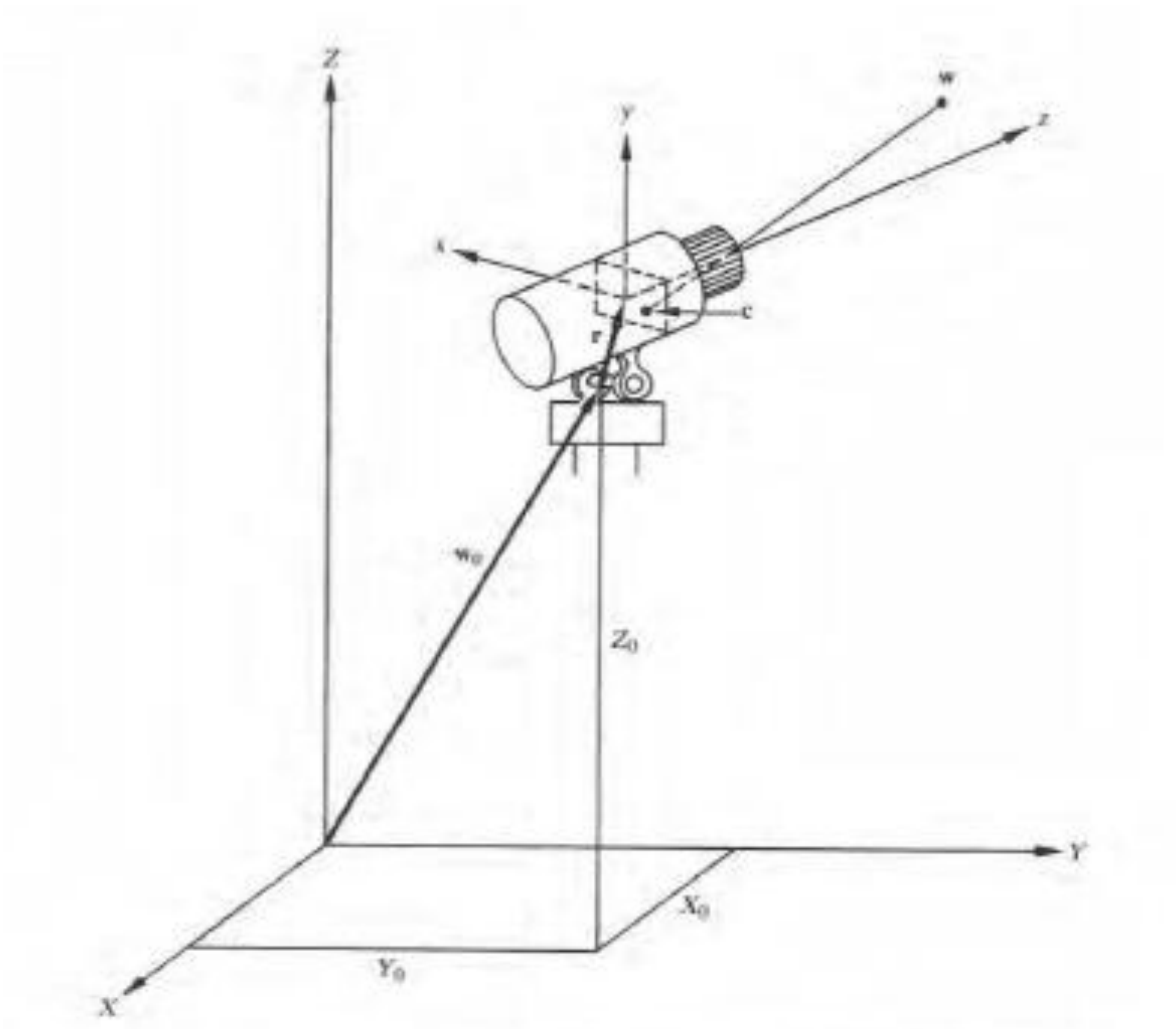


Figure 2.18 *Imaging geometry with two coordinate systems. (From Fu, Gonzalez, and Lee [1987].)*

Camera Model

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & -X_0 \\ 0 & 1 & 0 & -Y_0 \\ 0 & 0 & 1 & -Z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\mathbf{R} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta\cos\alpha & \cos\theta\cos\alpha & \sin\alpha & 0 \\ \sin\theta\sin\alpha & -\cos\theta\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

<u>Transformation Matrix</u>

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & -r_1 \\ 0 & 1 & 0 & -r_2 \\ 0 & 0 & 1 & -r_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Camera Model

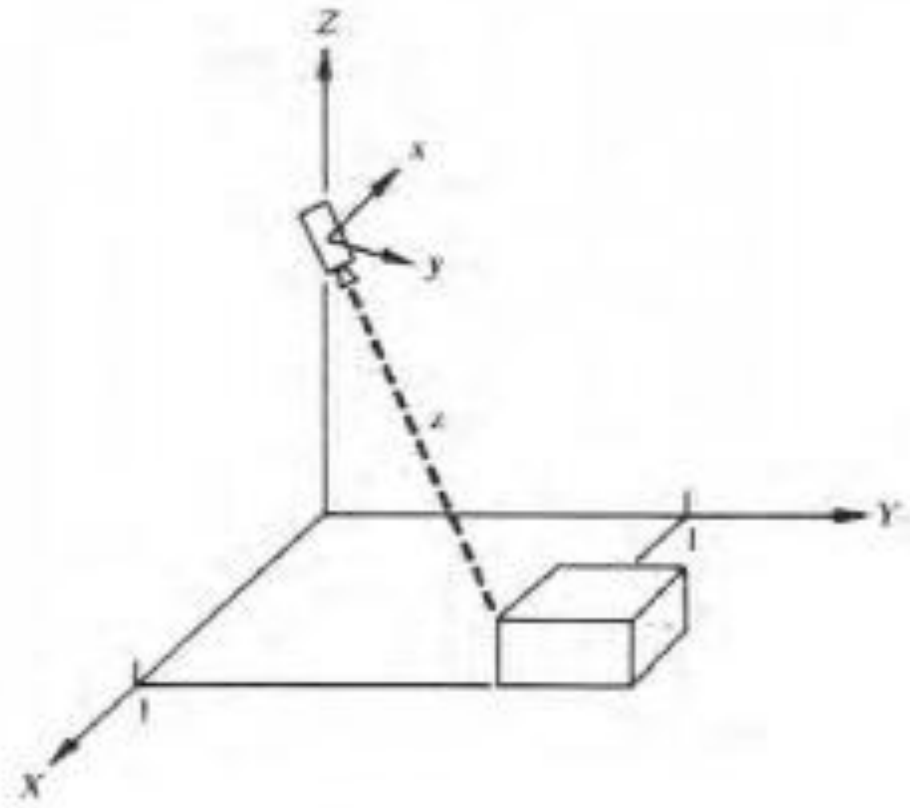


Figure 2.19 *Camera viewing a 3-D scene. (From Fu, Gonzalez, and Lee [1987].)*

Camera Model

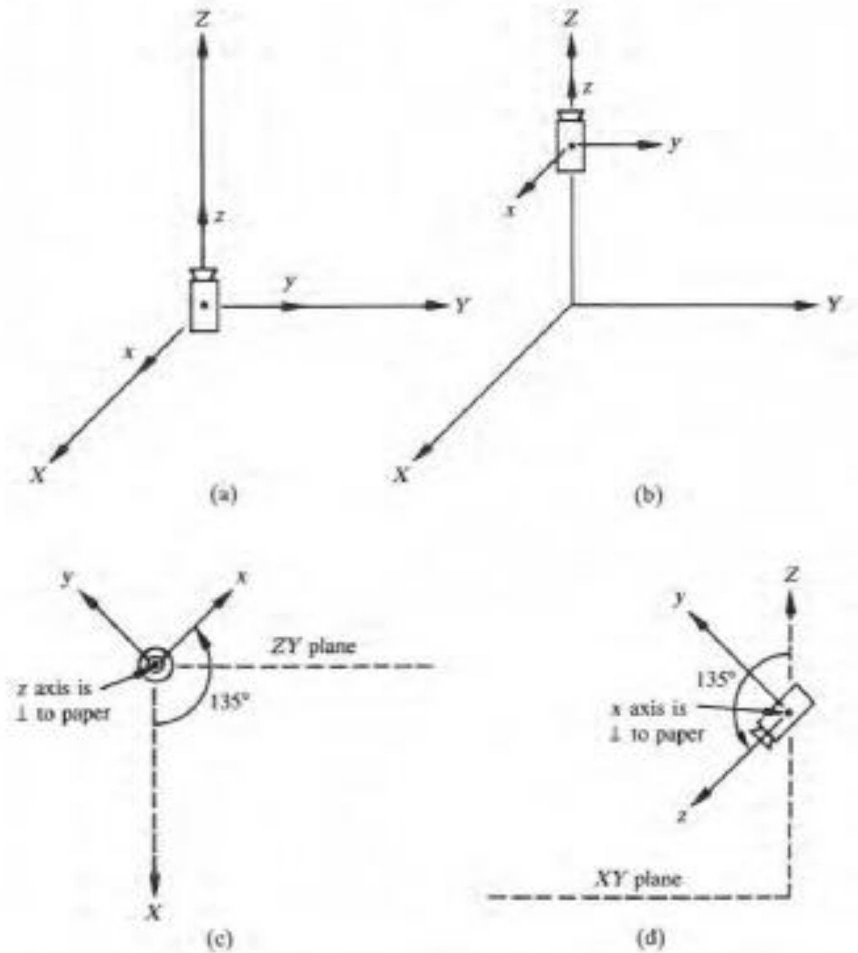


Figure 2.20 *(a) Camera in normal position; (b) gimbal center displaced from origin; (c) observer view of rotation about z axis to determine pan angle; (d) observer view of rotation about x axis for tilt. (From Fu, Gonzalez, and Lee [1987].)*

Camera Calibration

$$\begin{bmatrix} c_{h1} \\ c_{h2} \\ c_{h3} \\ c_{h4} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Cartesian form

$$x = c_{h1}/c_{h4}$$

and

$$y = c_{h2}/c_{h4}.$$

Stereo Imaging

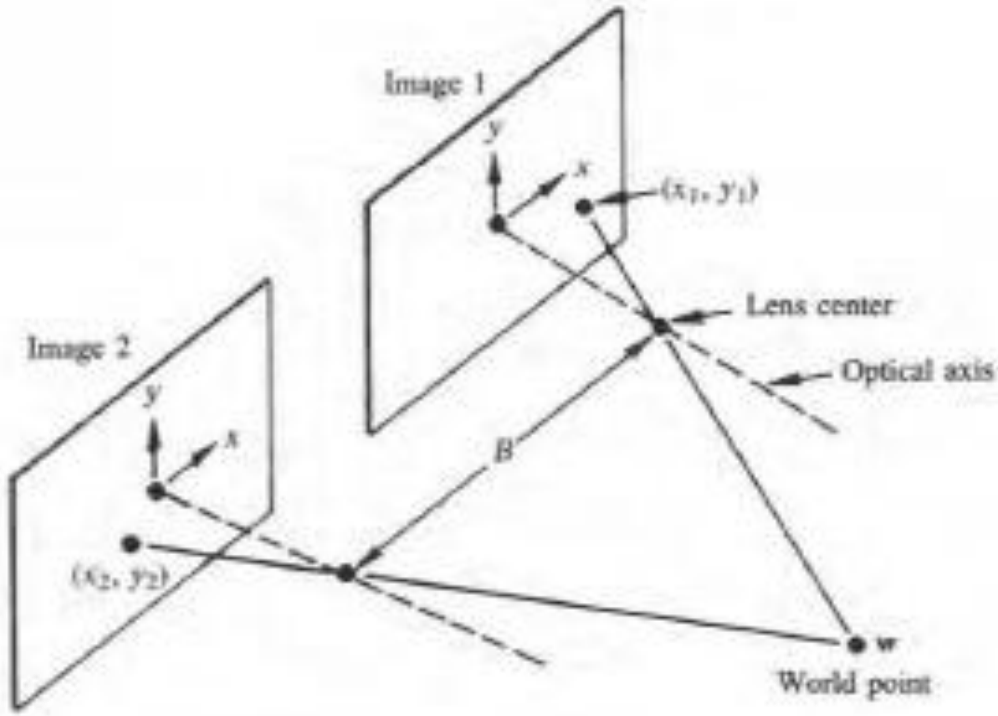


Figure 2.21 *Model of the stereo imaging process. (From Fu, Gonzalez, and Lee [1987].)*

Camera Coordinate system

$$X_1 = \frac{x_1}{\lambda}(\lambda - Z_1)$$

$$X_2 = \frac{x_2}{\lambda}(\lambda - Z_2).$$

$$X_2 = X_1 + B$$

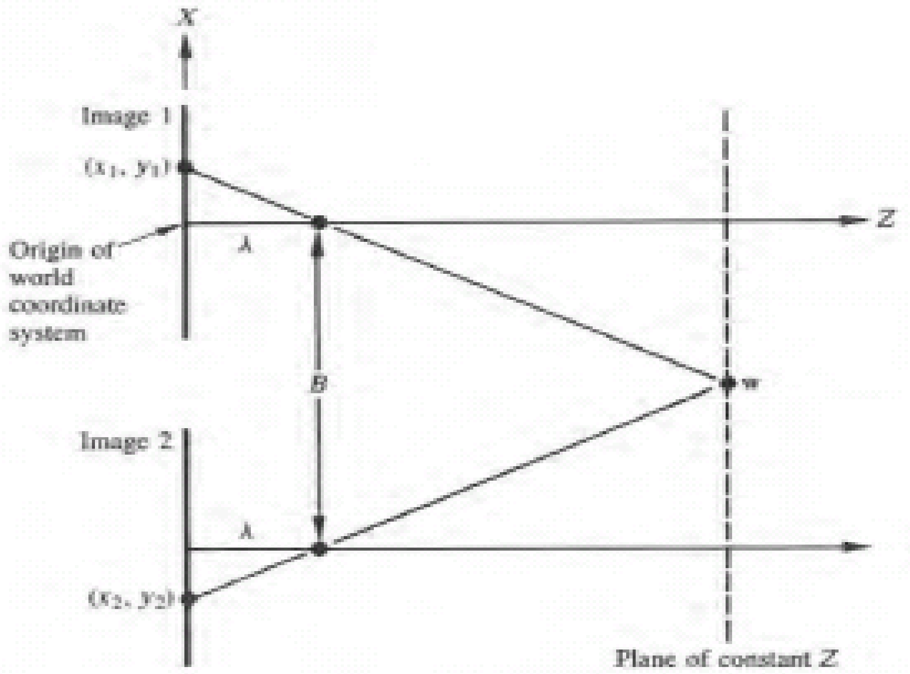


Figure 2.22 *Top view of Fig. 2.21 with the first camera brought into coincidence with the world coordinate system. (From Fu, Gonzalez, and Lee [1987].)*