Digital Image Processing:

Histogram Examples (cont…)

A selection of images and their histograms

Notice the relationships between the images and their histograms

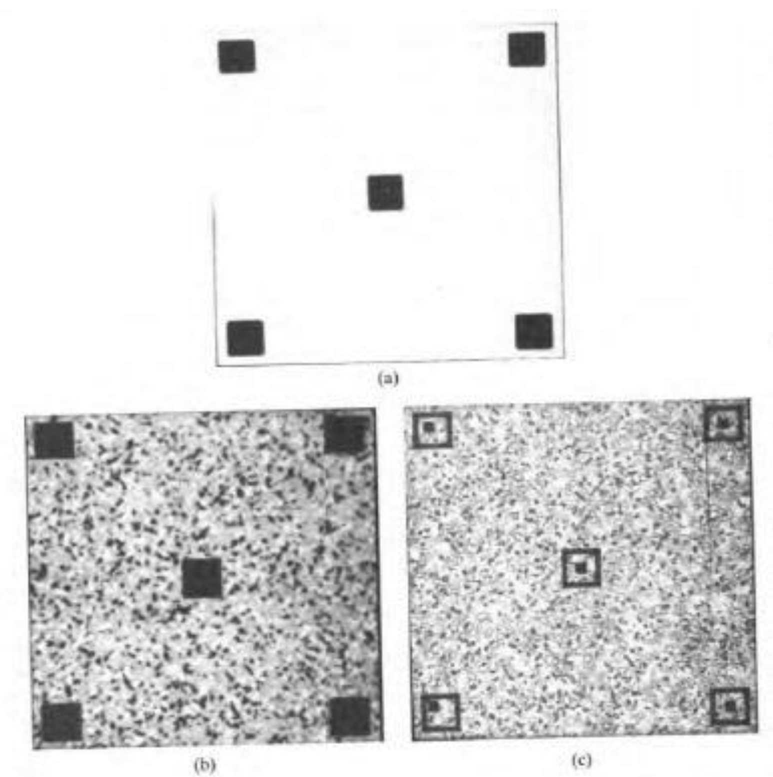
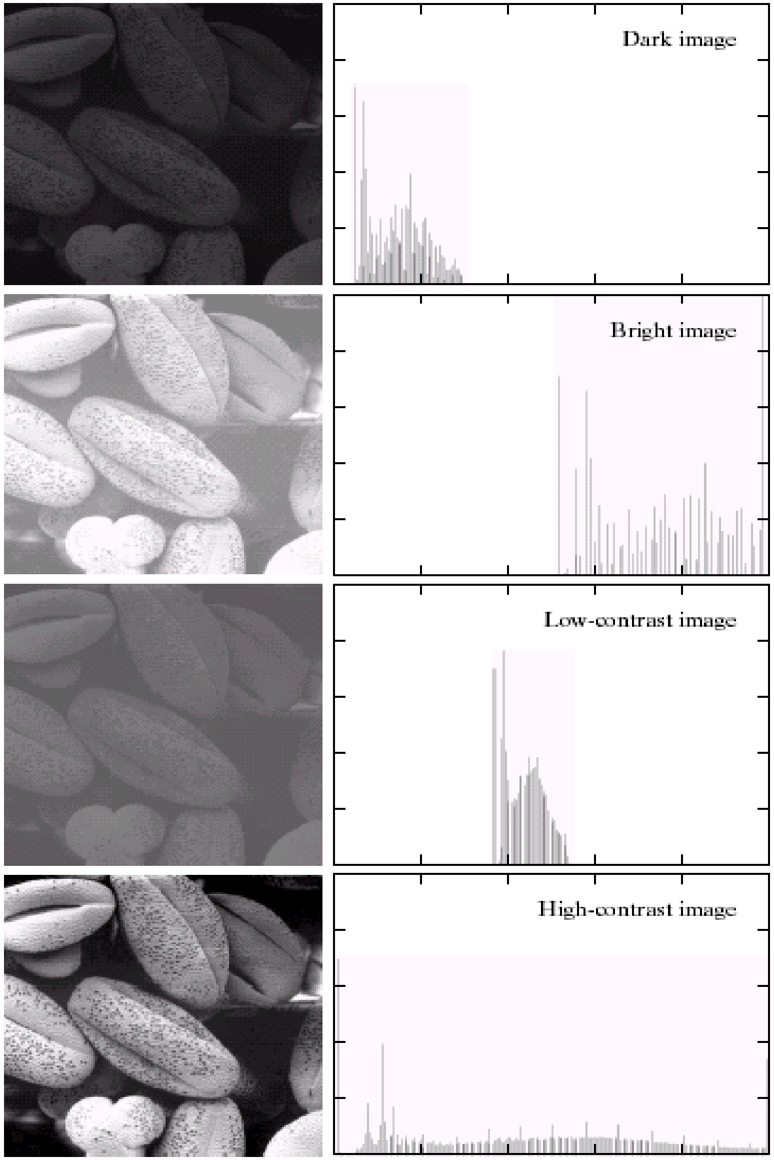


Figure 4.15 (a) Original image; (b) result of global histogram equalization; (c) result of local histogram equalization using a 7 × 7 neighborhood about each pixel. (From Fu, Gonzalez, and Lee [1987].)



Histogram Equalization

$$s = T(r) \qquad (4.2\text{-}2)$$

which produce a level s for every pixel value r in the original image. It is assumed that the transformation function given in Eq. (4.2-2) satisfies the conditions:

(a) $T(r)$ is single-valued and monotonically increasing in the interval $0 \leq r \leq 1$; and

(b) $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$.

Condition (a) preserves the order from black to white in the gray scale, whereas condition (b) guarantees a mapping that is consistent with the allowed range of pixel values. Figure 4.11 illustrates a transformation function satisfying these conditions.

The inverse transformation from s back to r is denoted

$$r = T^{-1}(s) \qquad 0 \leq s \leq 1 \qquad (4.2\text{-}3)$$

where the assumption is that $T^{-1}(s)$ also satisfies conditions (a) and (b) with respect to the variable s.

Histogram Equalization

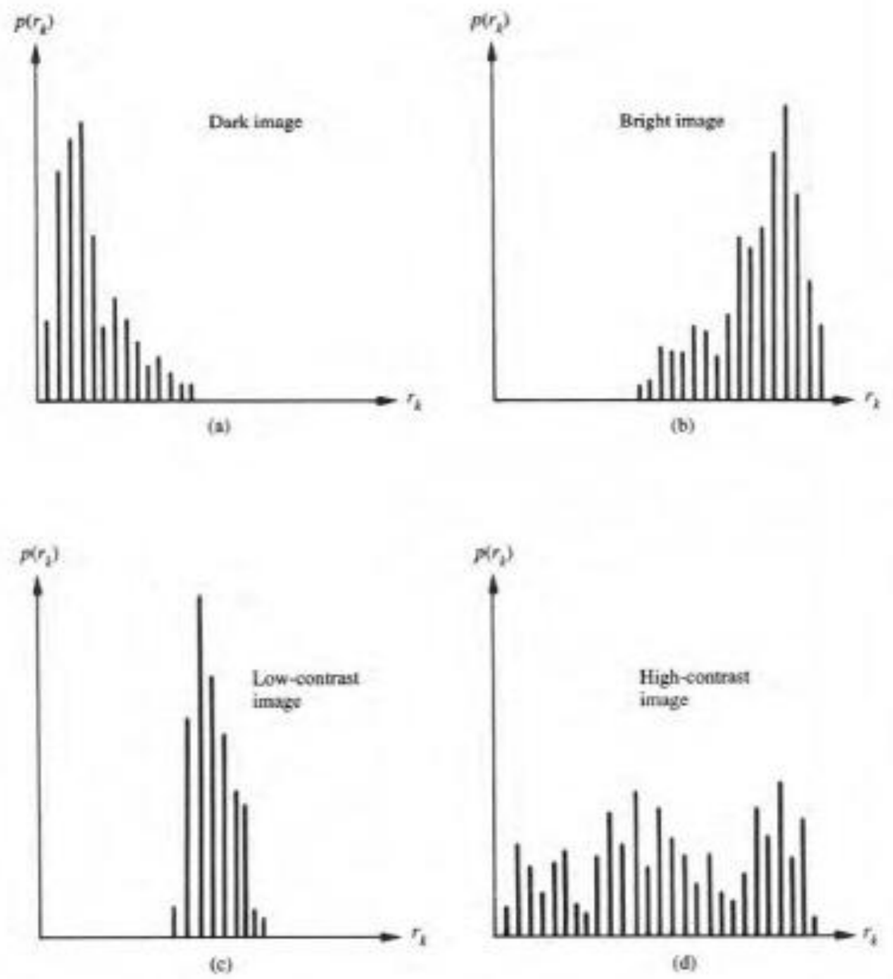


Figure 4.10 *Histograms corresponding to four basic image types.*

Gray-level transformation function

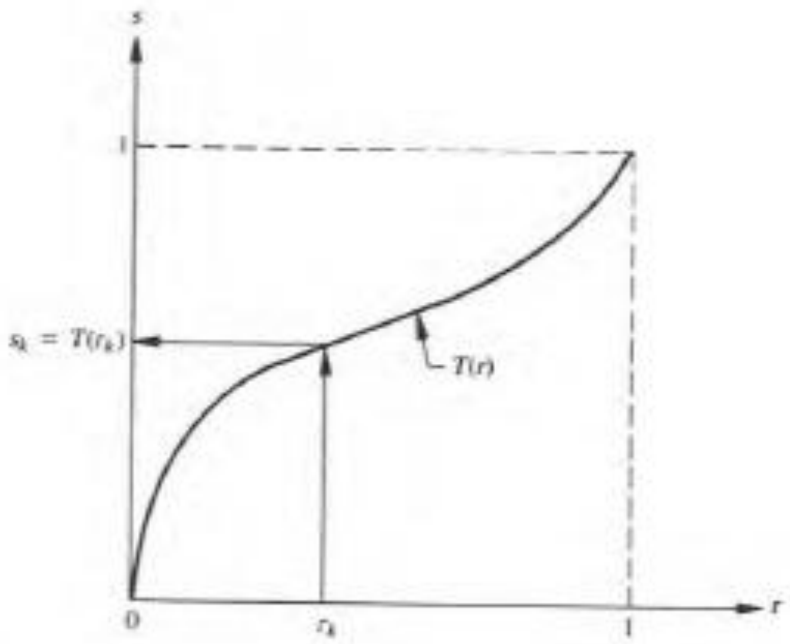


Figure 4.11 A gray-level transformation function.

Probability density function

$$p_s(s) = \left[p_r(r) \frac{dr}{ds} \right]_{r=T^{-1}(s)}$$

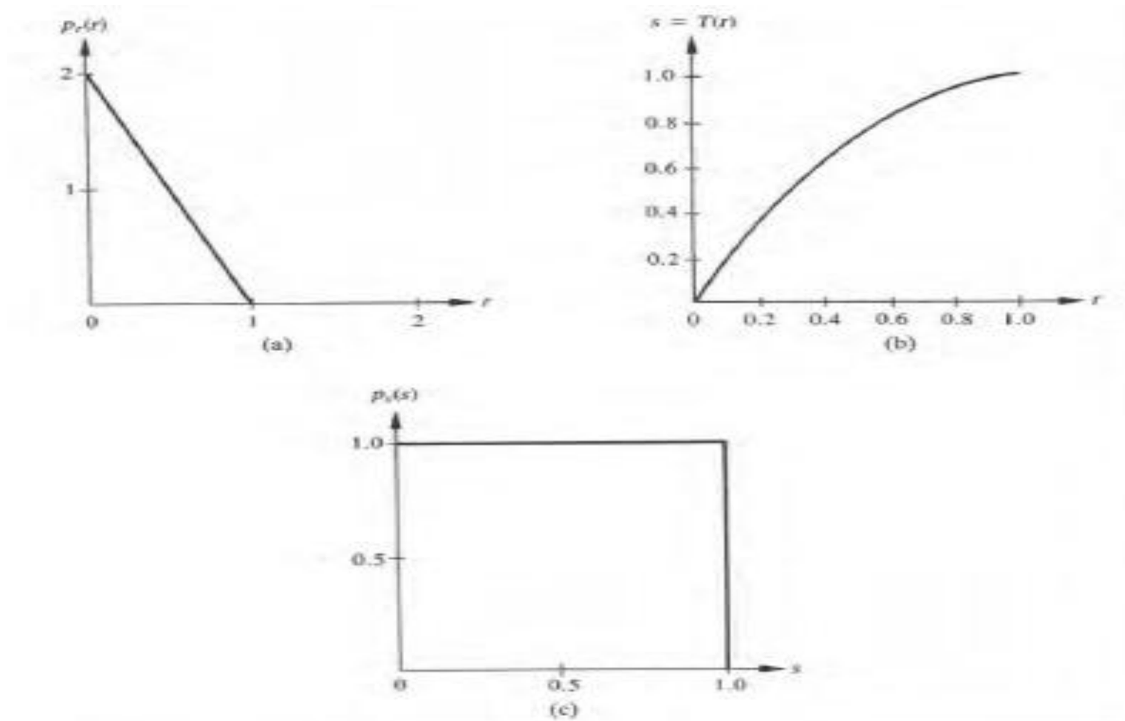


Figure 4.12 *Illustration of the uniform density transformation method: (a) original probability density function; (b) transformation function; (c) resulting uniform density.*

Transformation Function

$$s = T(r) = \int_0^r (-2w + 2)\,dw$$
$$= -r^2 + 2r.$$

Inverse transformation function

$$r = T^{-1}(s) = 1 \pm \sqrt{1 - s}.$$

$$p_s(s) = \left[p_r(r) \frac{dr}{ds} \right]_{r = T^{-1}(s)}$$
$$= \left[(-2r + 2) \frac{dr}{ds} \right]_{r = 1 - \sqrt{1 - s}}$$
$$= \left[(2\sqrt{1 - s}) \frac{d}{ds} (1 - \sqrt{1 - s}) \right]$$
$$= 1 \qquad 0 \le s \le 1$$

Discrete form

$$s_k = T(r_k) = \sum_{j=0}^{k} \frac{n_j}{n}$$

$$= \sum_{j=0}^{k} p_r(r_j) \qquad 0 \le r_k \le 1 \quad \text{and} \quad k = 0, 1, \ldots, L-1.$$

The inverse transformation is denoted

$$r_k = T^{-1}(s_k) \qquad 0 \le s_k \le 1$$

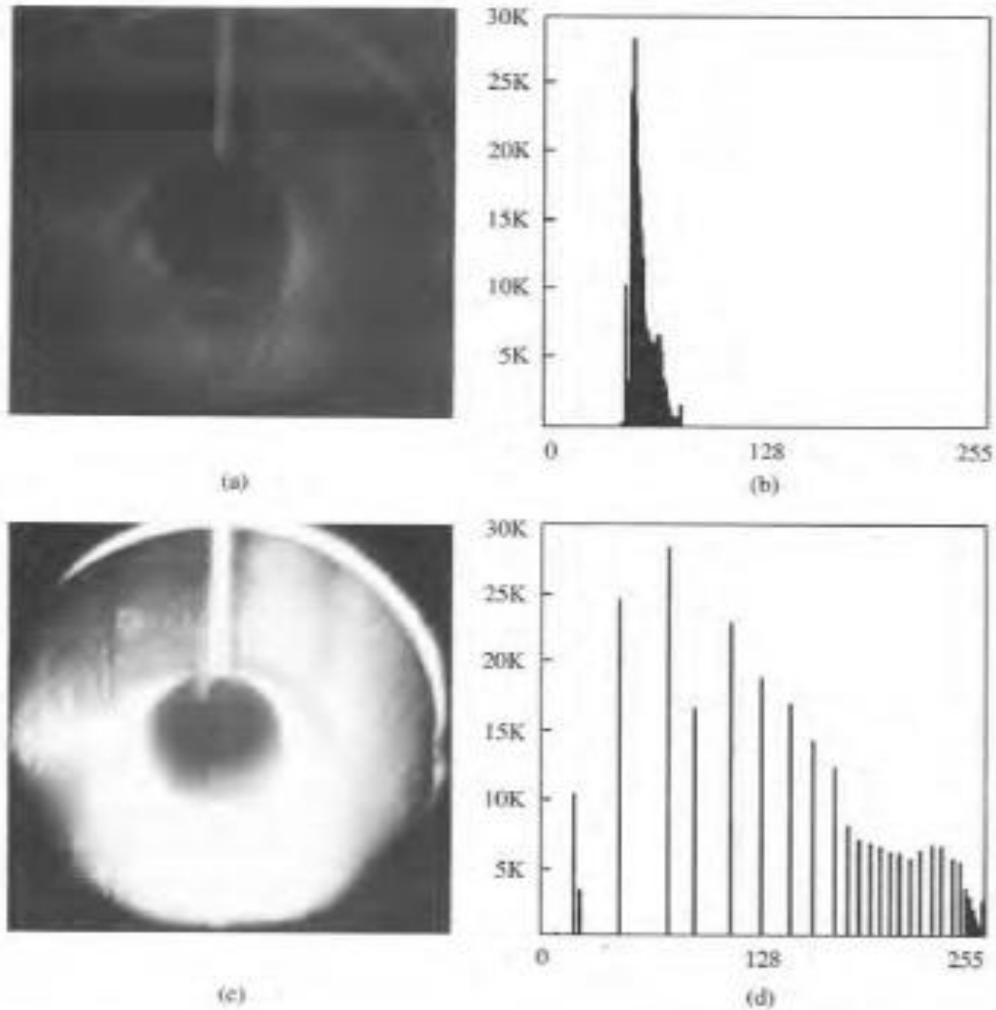


Figure 4.13 *(a) Original image and (b) its histogram; (c) image subjected to histogram equalization and (d) its histogram.*

Histogram specification

$$s = T(r) = \int_0^r p_r(w)\,dw.$$

If the desired image were available, its levels could also be equalized by using the transformation function

$$v = G(z) = \int_0^z p_z(w)\,dw.$$

$$z = G^{-1}(s).$$

$$z = G^{-1}[T(r)]$$

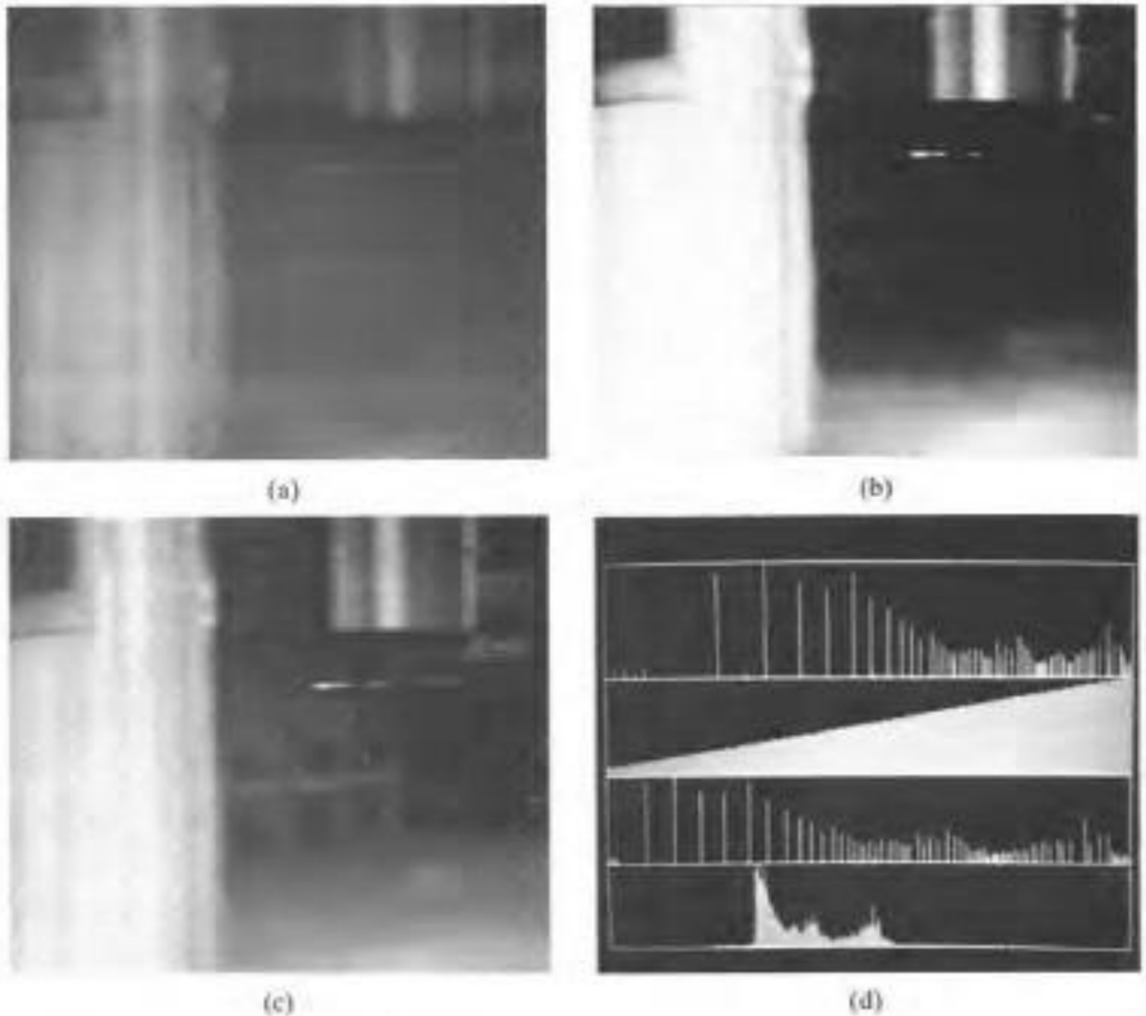
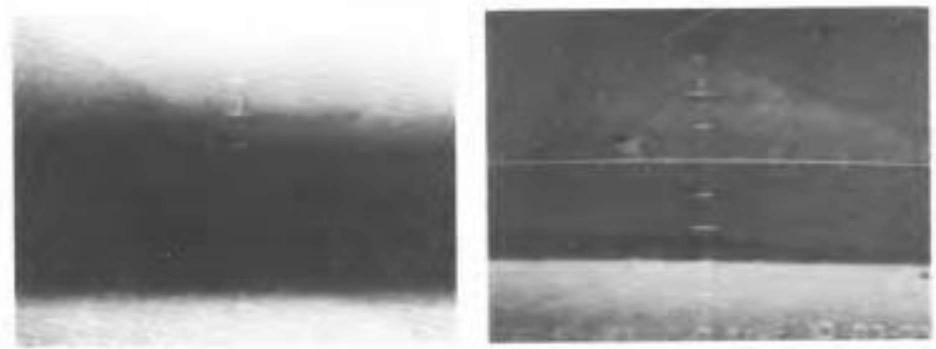


Figure 4.14 *Illustration of the histogram specification method: (a) original image; (b) image after histogram equalization; (c) image enhanced by histogram specification; (d) histograms.*

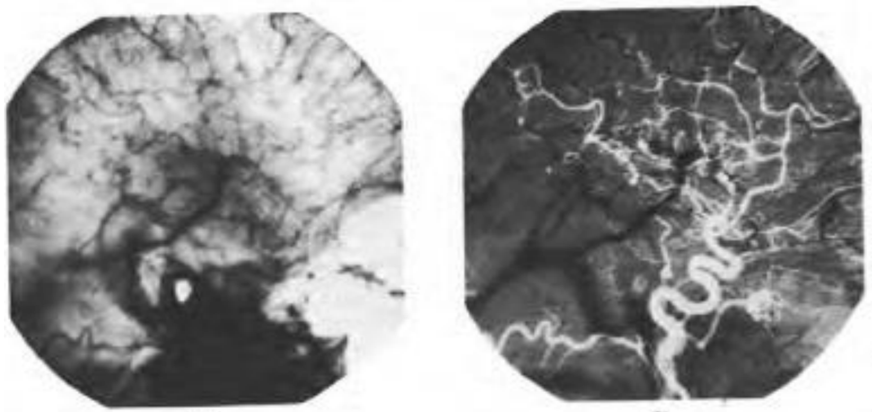
Image Subtraction

- Difference
  - $g(x,y) = f(x,y) - h(x,y)$
- Image
  - Higher-order bit: visually relevant detail
  - Lower-order bit: fine (imperceptible) detail
- Image subtraction + contrast stretching
  - Enhance the difference



(a) (b)

Figure 4.16 Images before and after local enhancement. (From Narendra and Fitch [1981].)



(a) (b)

Figure 4.17 Enhancement by image subtraction: (a) mask image; (b) image (after injection of dye into the bloodstream) with mask subtracted out.

Image Averaging

- Original image f(x,y), noisy image g(x,y)

$$g(x,y) = f(x,y) + \eta(x,y)$$

- Image averaging

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

- Expected value of the average

$$E\{\bar{g}(x,y)\} = f(x,y)$$

- Standard deviation at any point in the average

$$\sigma_{\bar{g}(x,y)} = \frac{1}{\sqrt{K}}\sigma_{\eta(x,y)}$$

Image Averaging Cont'd

- As K increases, variability (noise) of each pixel value decreases
- As K increases, the average approaches the original
- Images need to be registered (aligned)

Image Averaging

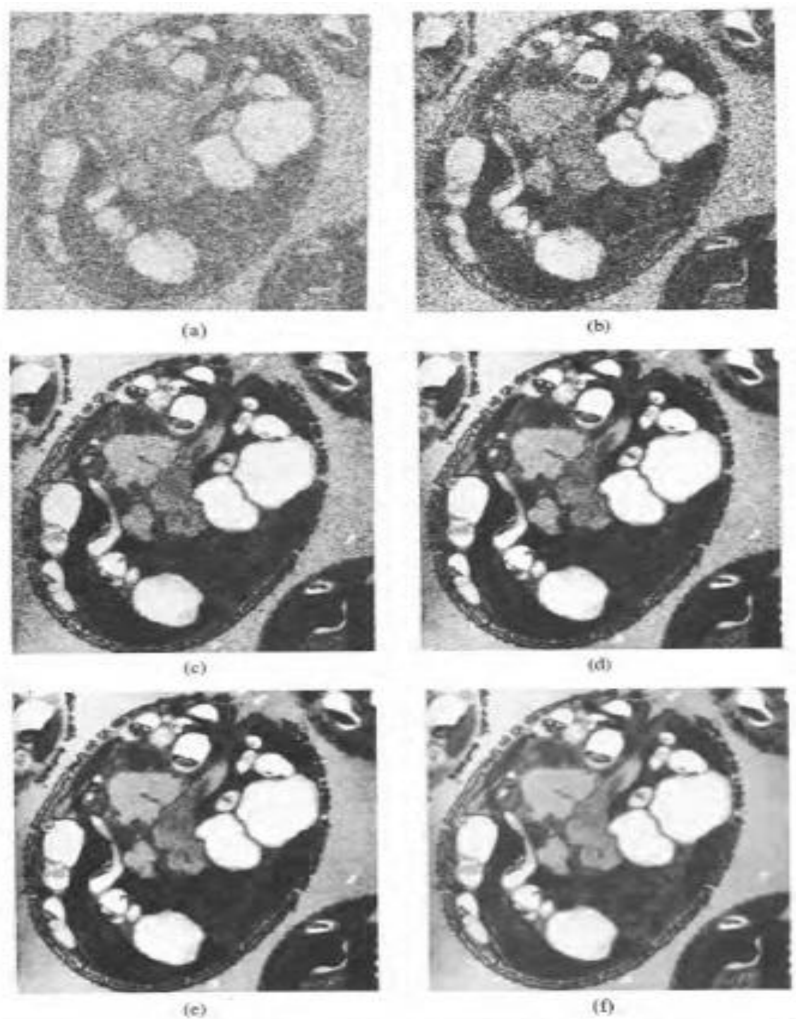
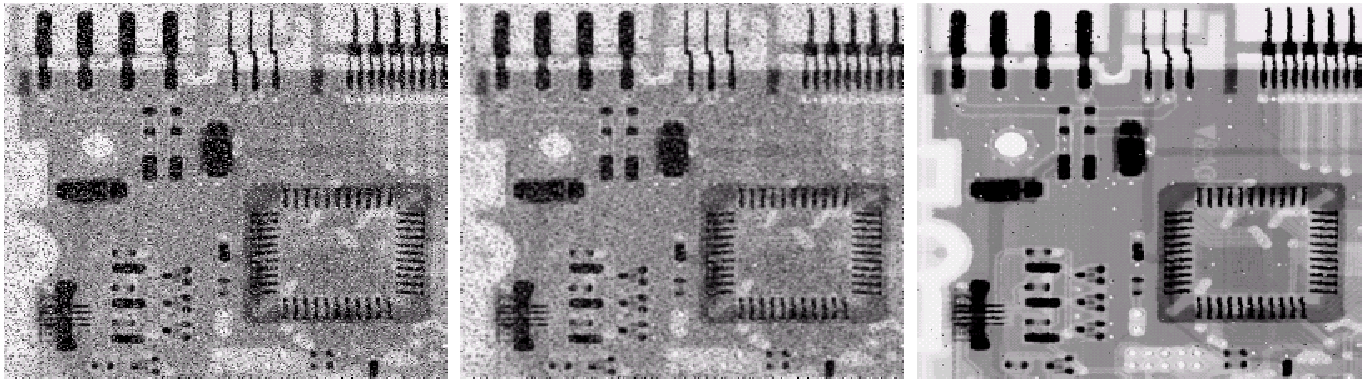


Figure 4.18 Example of noise reduction by averaging: (a) a typical noisy image; (b)–(f) results of averaging 2, 8, 16, 32, and 128 noisy images.

Local Statistic Filters

- Calculate a local statistics and then replace the center pixel value with the calculated statistics.
- Median filter
  - Useful in removing impulsive noise (salt-and-pepper noise) without smoothing the rest of the image.



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

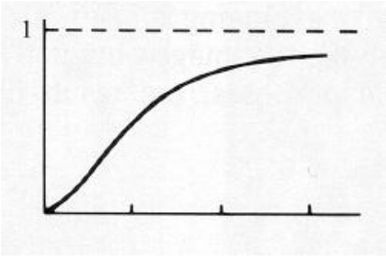
Sharpening Filters

- To highlight fine detail or to enhance blurred detail.
  - smoothing ~ integration
  - sharpening ~ differentiation

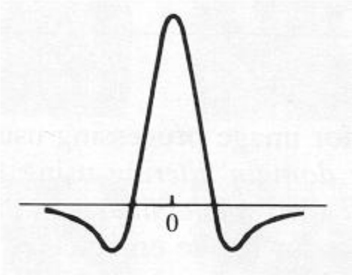
- Categories of sharpening filters:
  - Derivative operators
  - Basic highpass spatial filtering
  - High-boost filtering

Basic Highpass Spatial Filtering

- Cross section of frequency domain filter:



- Cross section of spatial domain filter:



Basic Highpass Spatial Filtering

- The filter should have positive coefficients near the center and negative in the outer periphery:

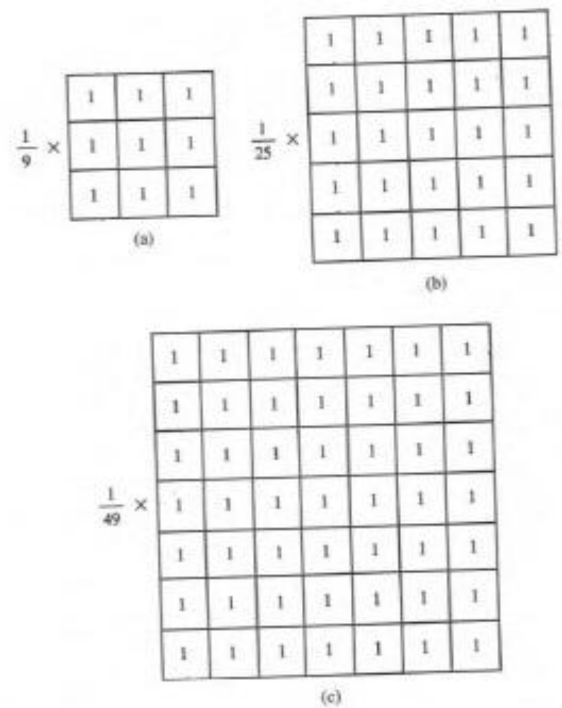


Figure 4.21 Spatial lowpass filters of various sizes.

Spatial Filtering

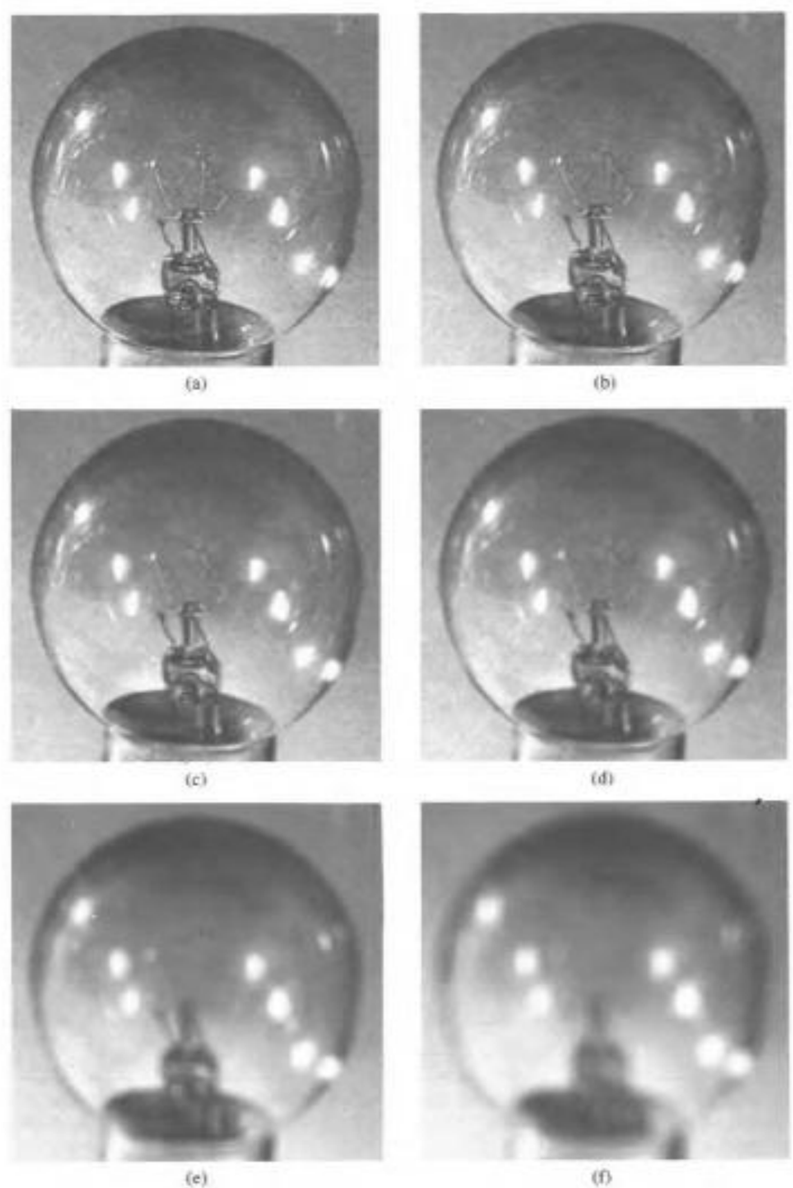


Figure 4.22 *(a) Original image; (b)–(f) results of spatial lowpass filtering with a mask o size $n \times n$, $n = 3, 5, 7, 15, 25$.*

Median filtering

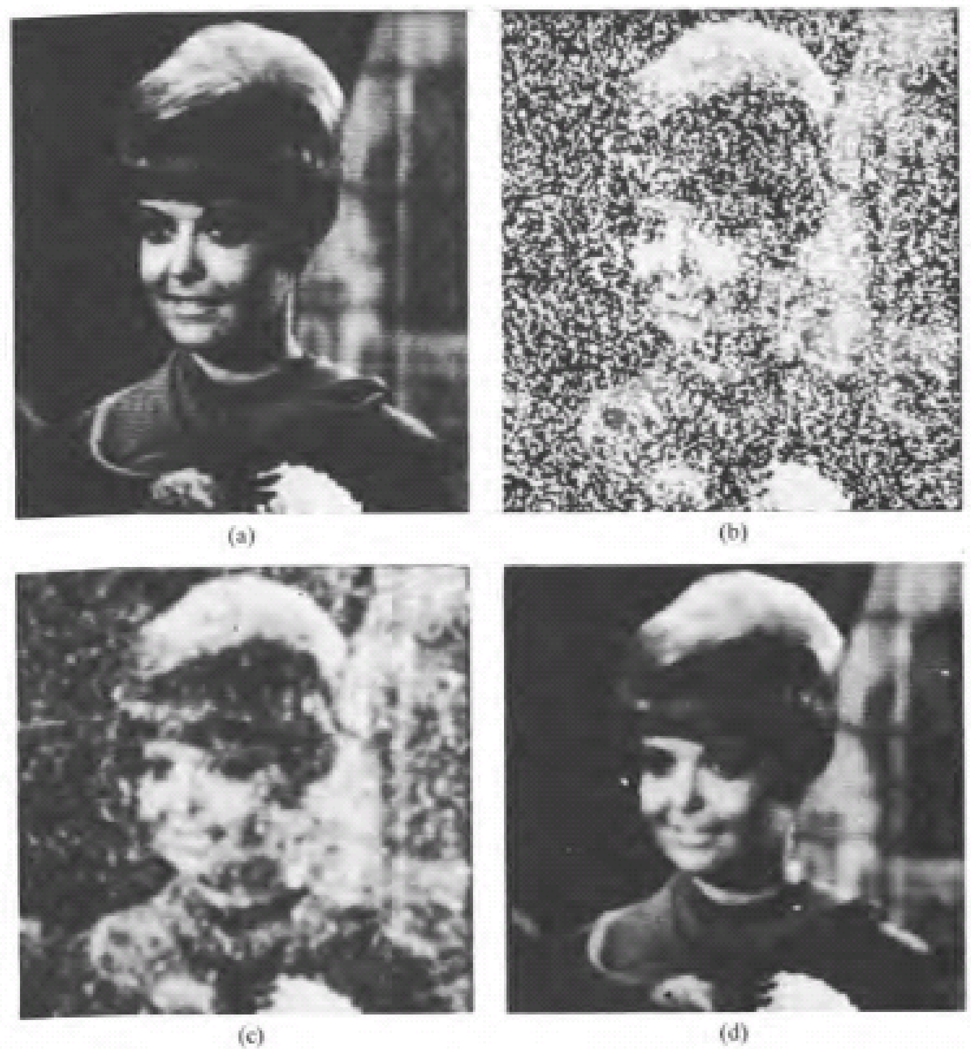


Figure 4.23 (a) Original image; (b) image corrupted by impulse noise; (c) result of 5 × 5 neighborhood averaging; (d) result of 5 × 5 median filtering. (Courtesy of Martin Connor, Texas Instruments, Inc., Lewisville, Tex.)

Basic Highpass Spatial Filtering

- The sum of the coefficients is 0, indicating that when the filter is passing over regions of almost stable gray levels, the output of the mask is 0 or very small.

- Some scaling and/or clipping is involved (to compensate for possible negative gray levels after filtering).

High-boost filter

- High-boost or high-frequency-emphasis filter
  - Sharpens the image but does not remove the low-frequency components unlike high-pass filtering

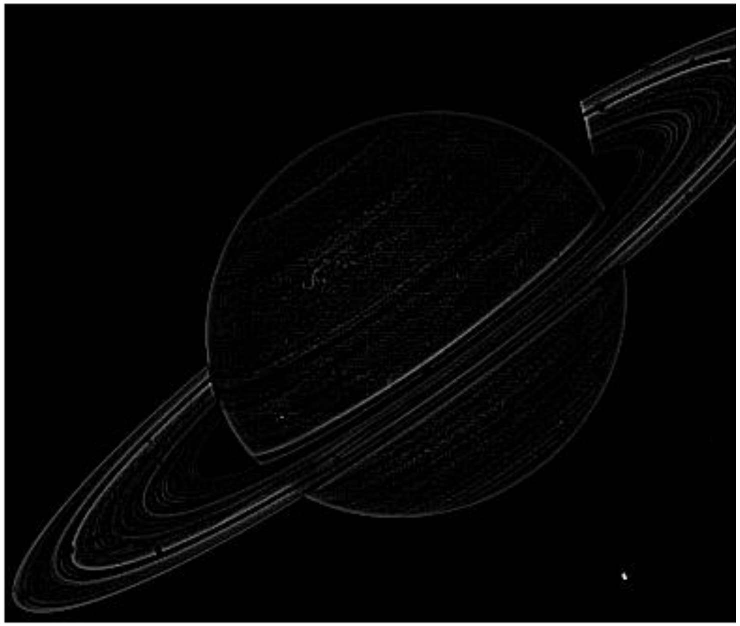
High-boost filter

- High-boost or high-frequency-emphasis filter
  - High pass = Original – Low pass
  - High boost = (A)(Original) - Low pass

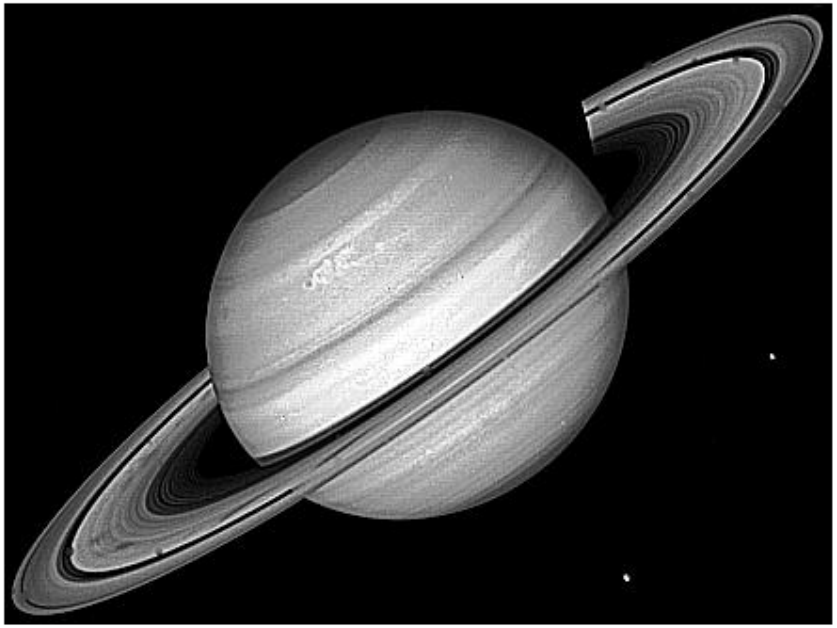
    = (A-1) (Original) +Original – Lowpass

    = (A-1) (Original) + Highpass

High-boost filter



A high-pass filter



A high-boost filter

Derivative Filters

- First order derivative filters

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$|\nabla f| = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

$$|\nabla f| \approx \left| \frac{\partial f}{\partial x} \right| + \left| \frac{\partial f}{\partial y} \right|$$

Derivative Filters

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

FIGURE 3.44 A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum to zero, as expected of a derivative operator.

Derivative Filters

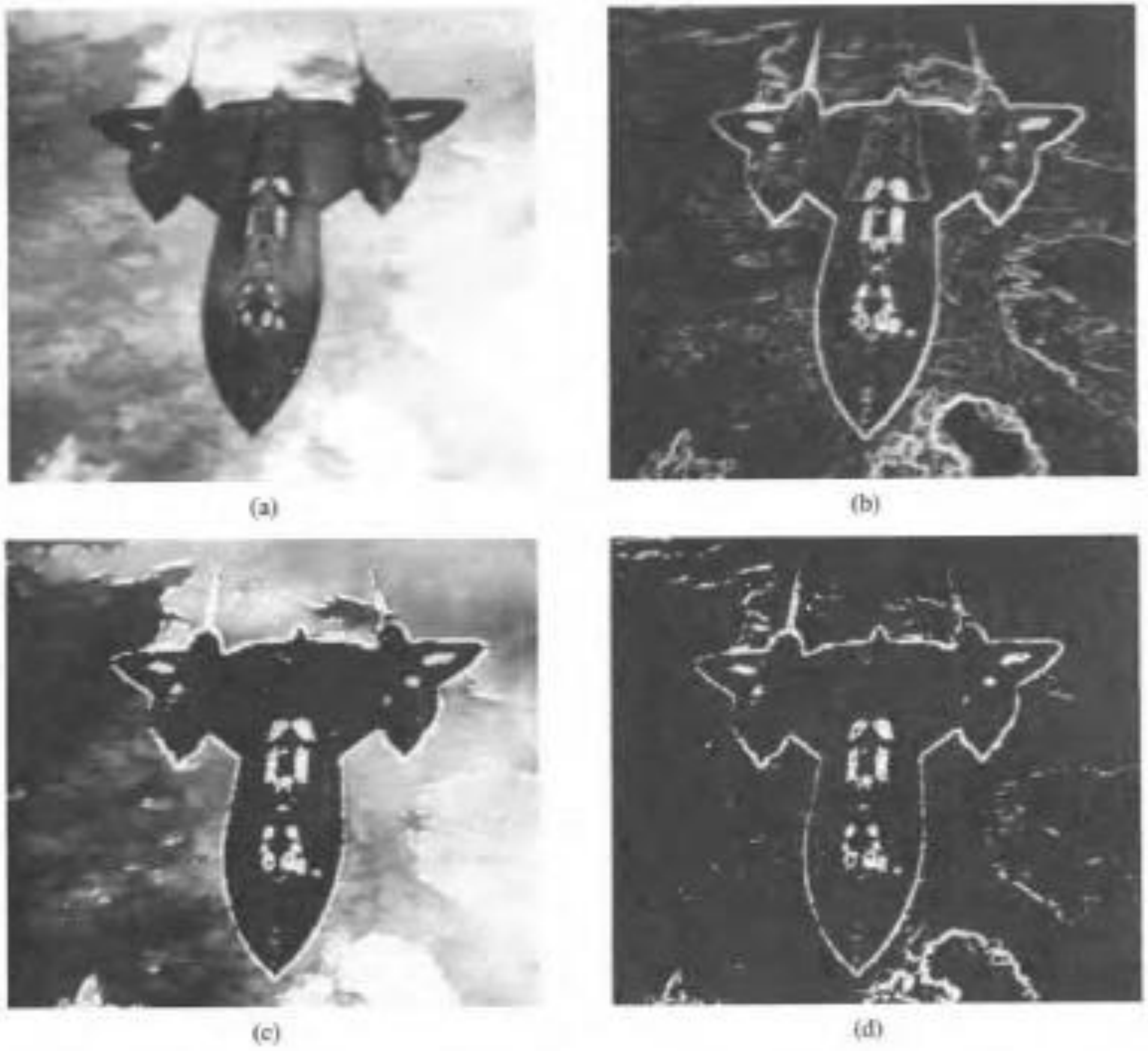


Figure 4.29 Edge enhancement by gradient techniques (see text).