PNS Algorithm and FPGA Design of Wireless OFDM System

Chisato Iramina*, Tomohisa Wada**, Heung-Gyoon Ryu***

* Graduate school of Engineering and Science, University of the Ryukyus, Senbaru 1, Nishihara, Okinawa, 903-0213, Japan

** Department of Information Engineering, University of the Ryukyus

***Department of Electronic Engineering, Chungbuk National University, Cheongju, Chungbuk, 361-763, Korea

chisato@lsi.ie.u-ryukyu.ac.jp, wada@ie.u-ryukyu.ac.jp, ecomm@cbu.ac.kr

***Abstract*— Phase noise seriously affects performance in the OFDM-based wireless communication system. In order to reduce phase noise efficiency, phase noise suppression (PNS) algorithm was proposed. In this paper, OFDM system including PNS algorithm design is simulated by using *Simulink*. As simulation results, phase noise effect can be significantly compensated by PNS algorithm. Compared with the ordinary OFDM system, included PNS algorithm system showed much better BER performance.**

***Keywords*— Phase noise, OFDM, CPE, ICI, PNS, Simulink**

I. INTRODUCTION

Recent years, orthogonal frequency division multiplexing (OFDM) has been adopted in many kinds of wireless communication system such as standards of digital terrestrial television system, DVB-T (digital video broadcasting for terrestrial distribution) and ISDB-T (integrated services digital broadcasting-terrestrial), wireless local area networks (LANs)/metropolitan area networks(MANs) and so on. OFDM is suitable for the high-speed and high data rate communication. However, OFDM is very sensitive to the frequency offset, phase noise and PAPR (peak-to-average power ratio) in the wireless mobile communication system. In this paper, we focus on the effect of phase noise.

Phase noise effect on OFDM signal reception consists of two components. One is common phase error (CPE). And the other is inter-carrier interference (ICI). In OFDM system, it is important to reduce ICI and CPE by phase noise. Phase noise which is generated in the transceiver oscillators may break down carrier orthogonality. It causes seriously system performance degradation. Because of this reason, many researches had studied about phase noise compensation. In 2001, A. G. Armada presented "Understanding the Effects of Phase Noise in OFDM" [1]. Here, she analysed the CPE and ICI effect caused by phase noise. In 2003, Gholami, M.R. and others studied "A new method of phase noise compensation in OFDM" [2]. They compensated the effect of phase noise in a certain extent by equalizer using least square (LS) method. But, there was no significant performance improvement. In 2003, Enescu, A.A. etc addressed "A comparison study over the influence of phase noise on multi-carrier communications" [3]. Here, they introduced a simple CPE compensation method using pilot signals. Similar to these above studies, there were many researches on the compensation for the phase noise effect by digital signal processing (DSP) technique in receiver side [6, 8].

In this paper, we simulate system performance based on OFDM wireless communication, including phase noise suppression (PNS) algorithm by using Simulink in order to convert FPGA level. We calculate bit error rate (BER) considering with phase noise. PNS algorithm can simultaneously compensate CPE and ICI caused by phase noise. This paper is organized as follows. In Section II, an OFDM system model with phase noise is presented. And we analyze about phase noise. In Section III, we show PNS algorithm and Simulink model. Simulation and experimental results are given in Section IV. Section V concludes this paper.

II. PHASE NOISE IN OFDM SYSTEM

Figure 1 shows OFDM transceiver block diagram. Phase noise is generated in transmitter and receiver oscillators that are used for up conversion and down conversion. When phase noise is generated, it is mixed with original OFDM signal. It causes performance degradation. Signal which includes phase noise effect is sent from transmitter antenna to receiver. While sending signal, it passes through channel. In the channel, AWGN (additive white Gaussian noise), and multi-path delay spread are added to the signal.

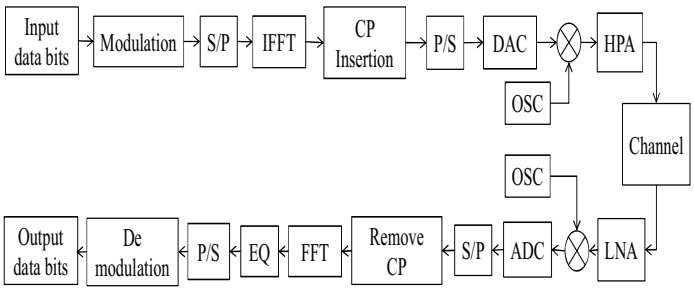


Figure 1. Block diagram of OFDM system

For simplifying, we assume that the channel is AWGN, and multi-path is not exists.

In general, the complex base band OFDM symbol can be written as:

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\cdot e^{j\frac{2\pi}{N}kn} \quad , \tag{1}$$

where $j = \sqrt{-1}$, k is the subcarrier index, N is the total subcarrier number. $X(k)$ means a data symbol for kth subcarrier. Actually, an OFDM symbol is extended by cyclic prefix (CP) in order to deal with multi-path delay spread. In this equation, we don't consider because CP is eliminated in the receiver. And we assume that frequency and time synchronization is perfectly achieved and there is no multipath in channel. So we only consider with phase noise effect in the transceiver. When phase noise is inserted from frequency synthesizer of transceiver, received OFDM symbol can be expressed as follows:

$$y(n) = \{x(n) * h(n) + \nu(n)\}\cdot e^{j\varphi(n)} \quad , \tag{2}$$

where $h(n)$, $\varphi(n)$, $y(n)$ and $\nu(n)$ are channel impulse response, phase noise, received signal and AWGN. And "$*$" means convolution. For simplifying we assume that channel impulse response is $h(n) = 1$. After removed CP and executed FFT, received signal can be expressed as:

$$\begin{aligned} Y(k) &= \sum_{n=0}^{N-1} y(n)\cdot e^{-j\frac{2\pi}{N}nk} \\ &= \sum_{n=0}^{N-1} \{x(n) * h(n) + \nu(n)\}\cdot e^{j\varphi(n)}\cdot e^{-j\frac{2\pi}{N}nk} \\ &= X(k)H(k)Q(0) + \sum_{\substack{l=0 \\ l\neq k}}^{N-1} X(l)H(l)Q(l-k) + V(k) \end{aligned} \quad , \tag{3}$$

where, $Y(k)$, $X(k)$, $H(k)$ are frequency domain expressions of $y(n)$, $x(n)$ and $h(n)$, $V(k)$ is a sampled FFT version of the complex AWGN $\upsilon(n)$ that is multiplied by phase noise, and $Q(l)$ represents phase noise effect on the OFDM signal reception.

$$Q(l) = \frac{1}{N}\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}nl} e^{j\varphi(n)} \quad . \tag{4}$$

As shown (3) and (4), the phase noise effect is composed of two parts. One is the CPE $e^{j\varphi}$ that rotate all the sub carriers equally, and the other is ICI component that has random property. Those two components cause to break subcarrier orthogonality.

Since null subcarriers that are used as guard band, AWGN can be coloured after passing the band pass filter. However, phase noise occurs in the receiver oscillator when it is used for RF (radio frequency) down conversion, so ICI component that is generated by phase noise is still remains. By this reason, phase noise degrades system performance dramatically.

III. DESCRIPTION OF PNS ALGORITHM

PNS algorithm can be divided into two parts of compensation methods, CPE compensation part and ICI cancellation part. According to previous section, CPE component is identical in the whole OFDM symbol duration. It works phase rotation to original signal. So it also appears the same phase rotation in pilot subcarriers. Because of this CPE property, the CPE can be estimated by using comb type pilots.

When the channel transfer function is $H(k) = 1$, the received signal with phase noise can be expressed as following equation:

$$Y(k) = X(k)Q(0) + \sum_{l=-0, l\neq k}^{N-1} X(l)Q(l-k) + V(k) \quad . \tag{5}$$

To estimate CPE, we use the pilot symbol. CPE component can be estimated as follows:

$$CPE(k) = \frac{Y(k)}{X(k)} = Q(0) + \frac{ICI + V(k)}{X(k)} = Q(0) + W(k) \quad . \tag{6}$$

where $ICI = 1/N \sum_{l=0, l\neq k}^{N-1} X(l)Q(l-k)$ and $W(k)$ is the total interference component caused by ICI and AWGN. In (6), $Y(k)$ and $X(k)$ indicate received and transmitted pilot subcarrier.

Finally, the average CPE component r_{cpe} can be calculated by averaging all of pilot carriers:

$$r_{cpe} = \frac{1}{N_P}\sum_{k\in S_P} CPE(k) = Q(0) + \frac{1}{N_P}\sum_{k\in S_P} W(k) \quad , \tag{7}$$

where N_P is the number of pilot, and S_p means pilot symbol. Since CPE component is identical in symbol duration, the more number of pilots per symbol is increased, the more accurate estimation can be expected.

After CPE compensation, received signal can be expressed as:

$$\begin{aligned} \tilde{Y}(k) &= X(k)Q(0)/r_{cpe} + \sum_{l=0, l\neq k}^{N-1} X(l)Q(l-k) \Big/ r_{cpe} + V(k)/r_{cpe} \\ &= \sum_{l=0, l\neq k}^{N-1} X(l)\tilde{Q}(l-k) + W_{ICI+AWGN} \end{aligned} \quad . \tag{8}$$

Even though we can compensate CPE component, there is still existing ICI component and AWGN in (8). In order to compensate phase noise influence of ICI, we use MMSE equalizer. So, transmitted signal is recovered as:

$$\hat{X}(k) = \tilde{Y}(k)\cdot C'(k) \quad , \tag{9}$$

$C'(k)$ is a MMSE coefficient of OFDM receiver including phase noise. It can be expressed as follows[4]:

$$C'(k) = \frac{\tilde{Q}(l-k)H^*(k)}{\left|\tilde{Q}(l-k)H(k)\right|^2 + \frac{\tilde{\sigma}^2}{E_x}} \quad . \tag{10}$$

where $(\cdot)^*$ means conjugate process, where $\tilde{\sigma}^2$ means the variance of $W_{ICI+AWGN}$, and E_x is useful signal power.

Since we assumed that channel estimation is perfectly achieved in this paper, channel frequency response can be

assumed $H(k)=1$. So, in case of considering with only phase noise effect, MMSE coefficient can be expressed as follows:

$$C'(k) = \frac{\tilde{Q}^*(l-k)}{\left|\tilde{Q}(l-k)\right|^2 + \frac{\tilde{\sigma}^2}{E_x}} \quad . \tag{11}$$

In (10) and (11), equation $\tilde{Q}(l-k)$ and $\tilde{\sigma}^2$ are needed.

$\tilde{\sigma}^2$ of data subcarriers can be approximated by estimating in pilot subcarriers that is compensated CPE component. The power $\tilde{\sigma}^2$ of $W_{ICI+AWGN}$ can be obtained as follows:

$$\tilde{\sigma}^2 = \frac{1}{N_P}\sum_{k\in S_P}\left|\tilde{Y}(k)\right|^2 \quad . \tag{12}$$

$\tilde{Q}(l-k)$ can be calculated to minimize cost function by using pilot samples (15).

$$\min_{\tilde{Q}(l-k)} \sum_{k\in S_P}\left|\tilde{Y}(k) - \tilde{Q}(l-k)X(k)\right|^2 \quad . \tag{13}$$

Therefore, $\tilde{Q}(l-k)$ can be also estimated as follows:

$$\tilde{Q}(l-k) = \frac{\sum_{k\in S_P}\tilde{Y}(k)X^*(k)}{\sum_{k\in S_P}\left|X(k)\right|^2} \quad . \tag{14}$$

After estimating $\tilde{Q}(l-k)$ by using (15), more practical factor can be achieved using decision feedback process as follows:

$$\hat{Q}(l-k) = \gamma\tilde{Q}(l-k) + (1-\gamma)\tilde{Q}'(l-k) \quad , \tag{15}$$

where γ is a forgetting factor that is able to express by (16):

$$\gamma = \frac{1}{N_P}\sum_{k\in S_P}\left|X(k) - \tilde{Y}(k)\right|^2 \quad . \tag{16}$$

$\tilde{Q}'(l-k)$ can be achieved as the form of (14) except that detected data sample S_D is used for estimation. After estimating $\hat{Q}(l-k)$, we can update coefficient $C'(k)$ by using (10).

PNS algorithm can be processed briefly as following steps.

- Averaged CPE component (phase rotation factor) r_{cpe} can be calculated by (6) and (7).
- CPE is able to be compensated from received OFDM symbol by using (8).
- After cancelling CPE, estimating $\tilde{\sigma}^2$ of ICI and AWGN power by (13), $\tilde{Q}(l-k)$ by (15), and γ by (17).
- Calculating $C'(k)$ by inserting $\tilde{Q}(l-k)$ and $\tilde{\sigma}^2$ into (12)
- Desired values of received signal can be obtained by (12) or by feedback process (17). $\tilde{Q}(l-k)$ is updated to $\hat{Q}(l-k)$, the received signal are then optimized.

Whole received symbols are estimated by iterating previous steps. Figure 2 and Figure 3 show the receiving process using PNS algorithm. At first, we estimate CPE component and supress it by averaging method. Next, we calculate γ and $\tilde{\sigma}^2$. After that, in order to compensate ICI component, MMSE equalization is processed by using γ and $\tilde{\sigma}^2$ that are calculated previous stage.

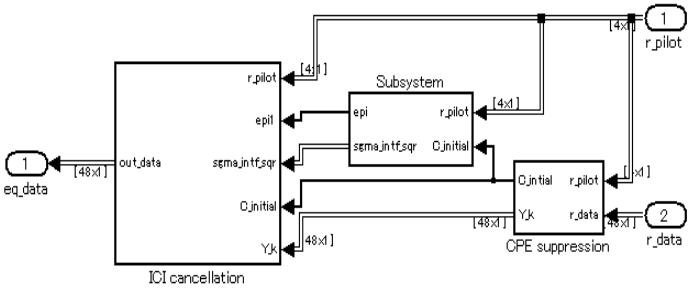


Figure 2. Simulink designed Receiver with PNS Algorithm

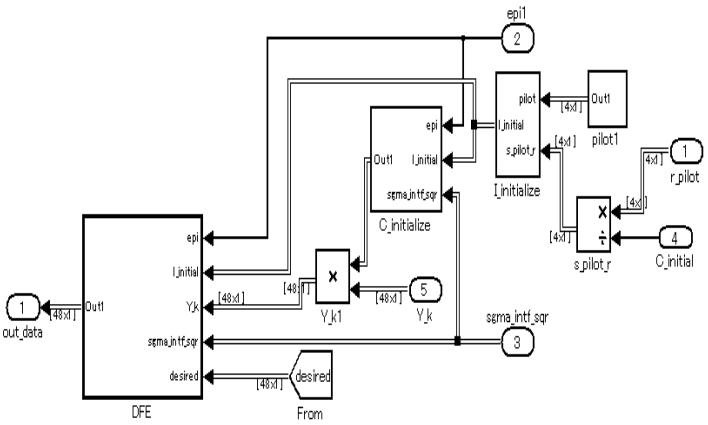


Figure 3. Details of ICI cancellation part

IV. SIMULATION RESULT

To analyse and compare the system performance before and after applying PNS, some computer simulations examined by Simulink. Simulation parameters are as following Table 1.

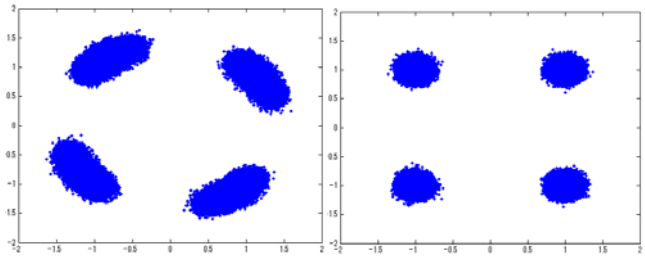
Table 1. SIMULATION PARAMETERS

Conditions	
Carrier Modulation	QPSK, 16QAM
IFFT/FFT Size	64
Channel	AWGN
Data Format	IEEE 802.11a WLAN.
Phase Noise	-15dBc/-20dBc, cut-off=10kHz

From those simulation parameters, Table 1, system performances were analysed in the OFDM system including PNS algorithm. In IEEE 802.11a, there are 64(N=64) subcarriers per symbol. It is divided as data subcarriers

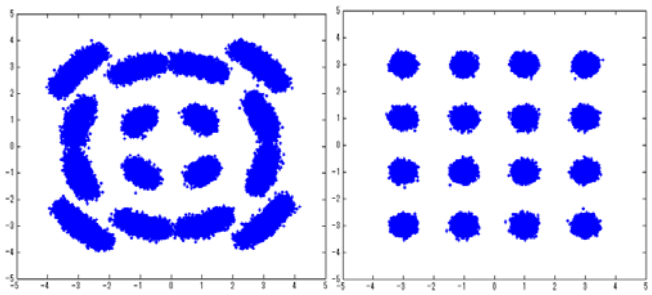
$N_D = 48$, pilot subcarriers $N_P = 4$ and null subcarriers $N_N = 12$. In those simulations, signal that is modulated with two different modulation models, i.e. QPSK and 16QAM, is propagated through AWGN channel and mixed with phase noise effect in receiver oscillator.

Figure 4 and 5 show the constellations of the system. (a) is without compensation and (b) is applied into PNS algorithm. In this simulation, we fixed as phase noise level is -15dBc, cut off frequency is the same 100kHz and signal to noise ratio (SNR) is 20dB in each modulation simulation. Compared with (a) and (b), CPE phase rotation and ICI component can be compensated in Figure 4.(b) and Figure 5.(b). It also seems that noise amplification can be greatly reduced. According to those results, PNS algorithm is effective.



(a) Without PNS algorithm **(b)** With PNS algorithm

Figure 4. Constellation of QPSK (phase noise = -15dBc, SNR=20dB)



(a) Without PNS algorithm **(b)** With PNS algorithm

Figure 5. Constellation of 16QAM (phase noise = -15dBc, SNR=20dB)

Figure 6 and 7 show bit error rate (BER) versus signal to SNR calculation result of each modulation. In this simulation, we consider two phase noise level; -15dBc and -20dBc, and cut off frequency is the same 100kHz. In case of QPSK, Figure 6 shows that phase noise is almost compensated by using PNS algorithm. When phase noise is -15dBc, with PNS algorithm system is better almost 4.8dB than without PNS algorithm system at BER=10^{-4}. And when phase noise is -20dBc, with PNS algorithm system is better about 1dB than without PNS algorithm. In 16QAM modulation, Figure 7 shows that without PNS algorithm system occurs error floor at BER=10^{-4}, but with PNS algorithm system satisfied BER=10^{-4} under SNR=14dB when phase noise is -15dBc. When phase noise is -20dBc, with PNS algorithm system shows about 4.5dB improvement than without PNS algorithm system.

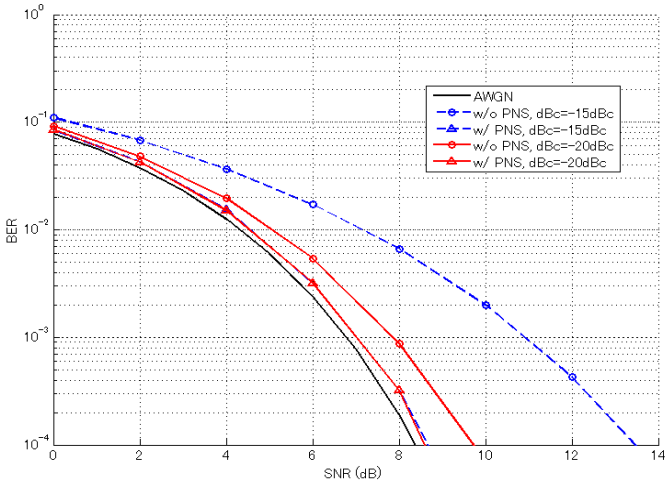


Figure 6. Result of BER performance on QPSK

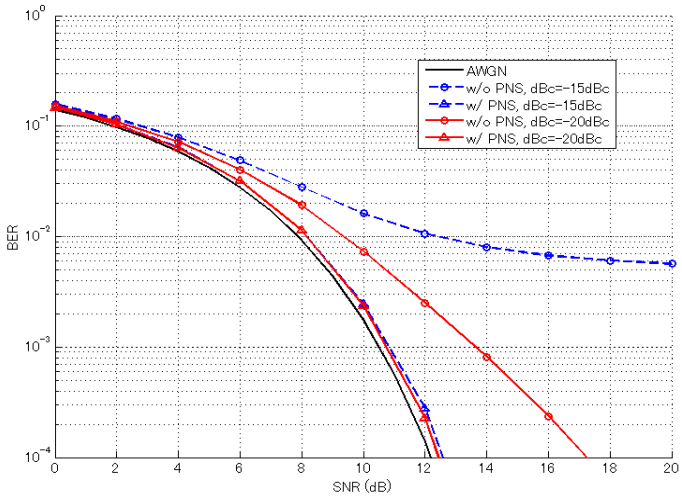


Figure 7. Result of BER performance on 16QAM

V. Conclusions

In this paper, we have analysed phase noise effect which is caused by transceiver oscillator in WLAN OFDM system. Phase noise effect can be divided two components. One is CPE, and the other is ICI. Then, we compensated its effect by using PNS algorithm. After that, we simulated its system performance by using Simulink in order to convert FPGA level. In this simulation, QPSK and 16QAM are used. When modulation is QPSK and phase noise is -15dBc, with PNS system is better almost 4.8dB than without PNS system and when phase noise is -20dBc, with PNS system is about 1dB improvement at BER=10^{-4}. In 16QAM modulation, when phase noise is -15dBc, with PNS system showed better performance than without one. And when phase noise is -20dBc, about 4.5dB improvement was shown in with PNS system. According to those simulation results, PNS algorithm can compensate phase noise effects in each modulation.

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