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Full Soft-decision Decoding on Concatenated Convolutional Inner Code and Reed-Solomon Outer Code

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Abstract This paper presents a full soft-decision decoding solution for a concatenated channel coding stage, composed of Reed-Solomon code (RS) as outer code and Convolutional code (CC) as inner code, relying on Max-Log-Map algorithm (MXLP) for CC followed by soft information output and APC-SD for RS with soft decision decoding. The APC-SD algorithm is a gradient descent, iterative soft-decision algorithm using adaptive parity check matrix based on sorted reliability of incoming soft information proposed by Jingjiang in 2006. Using a statistics method, we also make a comparison for the reliability between the soft outputs of MXLP and demodulated 16QAM.

Keyword Concatenated Code, Full Soft-decision decoding , Max-Log-Map, Steepest gradient descent algorithm, Adaptive parity check matrix

1. Introduction

The channel coding stage in many communication systems such as ISDB-T system (the digital terrestrial broadcasting standard of Japan) is based on a concatenated code, composed of the CC as inner code and RS as outer code. The conventional decoder for such concatenated code scheme in practical systems consists of a Viterbi algorithm (VA) for CC and an algebraic hard decisions RS decoder via Berlekamp-Massey algorithm (BM), while VA transferring hard decisions to BM. Recently a turbo-like iterative decoding technique [1][2] was applied for decoding the concatenated codes and gives a good bit error rate (BER) performance after a reasonable iterations. This technique utilizes a feedback decoding flag signal affiliated to the decoded output of BM algorithm. Although this turbo-like iterative decoding brings forward a good BER performance, the fact of long iteration (feedback from output of RS decoder to input of CC decoder) is not favorable for a high throughput of real system. In addition, when soft information is available to RS decoder, BM can incur a significant BER performance loss compared to optimal soft decision decoding.

Decoder that operates on soft inputs and produce soft outputs instead of hard-decisions are called soft-input soft-output (SISO) decoder. The sign of the soft-information gives the hard-decision value, while the magnitude of it is called the reliability of the bit decision. In recent years, a number of SISO algorithms have been proposed for CC decoding such as maximum a posteriori type (MAP-type) decoding algorithm [3] and soft output Viterbi algorithm (SOVA) [4]. For RS decoder in recent 5 years, soft decoding such as Koetter-Vardy (KV) algorithm, box and match algorithm (BMA) and APC-SD algorithm are proposed and proved very strong decoding capability. However, the complexity of KV algorithm and BMA algorithm can be prohibitively large. Additionally, KV algorithm is a list-decoding based technique, which is not suitable for decoding of the generator polynomial based constructed RS codes employed in ISDB-T system.

In this paper, we present a full soft-decision decoding scheme for concatenated CC and RS codes. In order to transfer the soft information to RS decoder, the MXLP algorithm is applied for inner code. And then the APC-SD algorithm [5] is employed for decoding RS code using the soft output from CC decoder.

The rest of the paper is organized as follows: The system model is introduced in section 2. We review the algorithm of MXLP and APC-SD in section 3. Additionally, a variation of APC-SD is also investigated in this section. In sections 4, simulation results of proposed full soft-decision decoding compared with conventional decoding scheme are presented and a statistics comparison for the reliability of bit soft information between from MXLP and from demodulated soft output of channel is also shown. Conclusions are located in section 5.

2. System model

A block diagram of the basic system considered is given in Fig.1. After encoded by outer RS encoder and inner CC encoder, the information is modulated using 16QAM and then transmitted over AWGN channel.

In receiver side, demodulator of 16QAM (De-16QAM) is used for providing bit level soft information to MXLP. Consider a 16QAM constellation as Fig.2, each of I signal and Q signal is modulafed independently by two bits. It is assumed that at time k, $u_{k,1}$ and $u_{k,3}$ modulate the Q component denoted as Q_k and $u_{k,0}$ and $u_{k,2}$ modulate the I component denoted as I_k

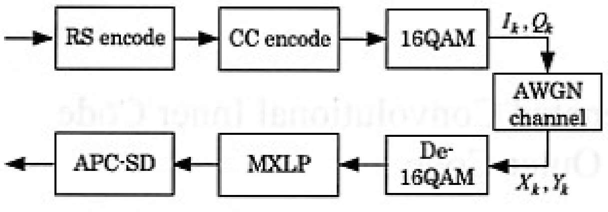


Fig.1. System Model

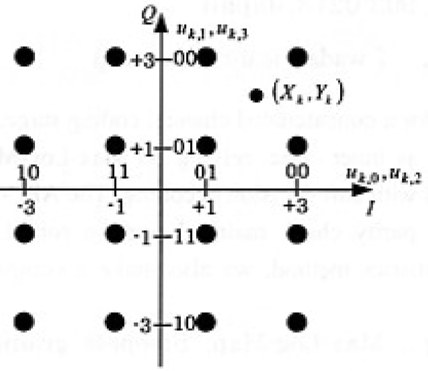


Fig.2. 16QAM constellation

respectively. Consider a AWGN channel outputs X_k and Y_k are equal to:

$$X_k = I_k + n_{ik}$$
$$Y_k = Q_k + n_{qk} \tag{1}$$

Where n_{ik} and n_{qk} are two uncorrelated Gaussian noises, with zero mean, variance σ^2. From observation of $\{X_k, Y_k\}$, the Logarithm of Likelihood Ratio (LLR) associated with each bit $u_{k,i}$ can be determined as (2) and used as a relevant soft decision information by MXLP decoder.

$$LLR(u_{ki}) = Log\frac{\Pr\{u_{ki} = 0 / X_k or Y_k\}}{\Pr\{u_{ki} = 1 / X_k or Y_k\}} \tag{2}$$

Using Bayes' rule, (2) is evaluated directly from following (3),

$$LLR(u_{ki}) = Log\frac{\sum_{i=1}^{2}\exp\left[-\frac{1}{2\sigma^2}(A_k - B_{i0})^2\right]}{\sum_{i=1}^{2}\exp\left[-\frac{1}{2\sigma^2}(A_k - B_{i1})^2\right]} \tag{3}$$

Where, A_k represent X_k or Y_k depending on evaluated bit. $B_{i,0}$ and $B_{i,1}$ respectively represent the realization of constellation symbols I_k or Q_k conditionally on $u_{k,i} = 0$ and $u_{k,i} = 1$.

3. A brief review of Max-Log-Map and APC-SD algorithm

3.1. Max-Log-Map algorithm

For our proposed decoding scheme, we apply Max-Log-Map algorithm, which propagates approximations to logarithms of probabilities used in MAP algorithm, for decoding CC code and also providing LLR soft output for RS soft decoder.

We do not discuss the theory of MAP-type algorithm. Reader can refer to [3]. To illuminate the working condition of our system model, we explain our evaluating method of branch metric of MXLP here. In the model, channel noise is assumed unavailable. Consider a trellis structure of a rate 1/2 constituent decoder. At any decoding time, as step k, a branch metric $r_k(p,q)$ between the state p and q is defined in the log domain as follows correlation formula,

$$\log r_k(p,q) = y_{k,0}x_{k,0} + y_{k,1}x_{k,1} \tag{4}$$

Where $x_{k,0}$ and $x_{k,1}$ are modulated symbol of the trellis output of the transition from state p to q at k step (with 0 mapped +1 and 1 mapped -1), $y_{k,0}$ and $y_{k,1}$ are two received bit level LLR soft information at k step corresponding to $x_{k,0}$ and $x_{k,1}$ respectively.

3.2. APC-SD algorithm

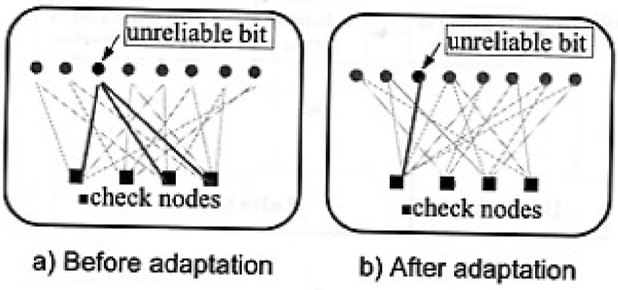
Let n denote the number of code-symbols and k denote the number of data symbols in a RS code, defined over a Galois Field with 2^q elements. This RS(n , k) has equivalent binary image expansion RS(N , K) at bit level in GF(2), where $N = n \times q$ and $K = k \times q$. Additionally, the binary expansion RS(N , K) hold an binary parity check matrix as well with size of $(N-K) \times N$ in GF(2).

Consider initial LLR information of each bit of RS code as vector $\vec{L} = [L^0(c_1), L^0(c_2), \cdots, L^0(c_N)]$ and binary parity check matrix of RS code H_b with element H_{jp} for $j = 1, \cdots N-K$ and $p = 1, \cdots N$. Here we can define a tanh operator $v : [-\infty, +\infty] \rightarrow [-1, +1]$ as a mapping from the LLR domain to tanh domain:

$$v(\mathrm{L}) = \tanh\left(\frac{\mathrm{L}}{2}\right) = \frac{e^{\mathrm{L}} - 1}{e^{\mathrm{L}} + 1} \tag{5}$$

After transform the LLR information of $\vec{L}$ to tanh domain using v operator, we get a tanh measure of $\vec{L}$ as (6),

$$\vec{T}^{(l)} = [\mathrm{T}_1, \mathrm{T}_2, \cdots \mathrm{T}_N] = [v(\mathrm{L}^{(l)}(\mathrm{c}_1)), v(\mathrm{L}^{(l)}(\mathrm{c}_2)), \cdots, v(\mathrm{L}^{(l)}(\mathrm{c}_N))] \tag{6}$$



a) Before adaptation b) After adaptation

Fig.3. Parity check matrix adaptation

Now let us take insight on following cost function J, which characterize the reliability of the vector $\vec{T}^{(l)}$ with a particular parity check matrix H_b,

$$J(H_b,\vec{T}) = -\sum_{j=1}^{(N-K)} \prod_{\substack{p=1\\ H_{jp}=1}}^{N} T_p \tag{7}$$

It is obvious that the cost function J has a minimal value $-(N-K)$ when a valid codeword is reached. Therefore the decoding problem can be interpreted as searching the most probable minimum cost potential vertex given the initial point observed from the channel. It is quite natural to apply the steepest gradient descent algorithm i.e. message passing decoding for the optimization problem.

However, the MDS property of RS code leads to a high density of 1's in the H_b matrix and hence to short cycles, As a result, the binary matrix H_b is not suitable for running a message passing algorithm. The main contribution of the adaptive parity check soft decoding (Jingjiangs' algorithm i.e. APC-SD) is the insight that message passing algorithm will run effectively on high density parity check matrices, if cycles are eliminated within the sub-graph corresponding to the low reliability received bits. The APC-SD algorithm consists of two main steps. One is parity matrix adaptation, in this step, one need first order the LLR values with increasing reliability. Then reduce to unit weight using Gaussian elimination for adapting parity check matrix, $(N-K)$ columns of H_b corresponding to $(N-K)$ low-reliability bits. Fig.3 shows after adaptation, the sub-graph corresponding to the low reliability becomes sparse. The APC-SD algorithm is diagrammaticly represented in Fig.4. The gradient descent updating rule can be written as,

$$\overrightarrow{T^{(l+1)}} \leftarrow \overrightarrow{T^{(l)}} - \alpha\nabla J(H_b{}^{(l)},\overrightarrow{T^{(l)}}) \tag{8}$$

Where, α is a damping coefficient, and

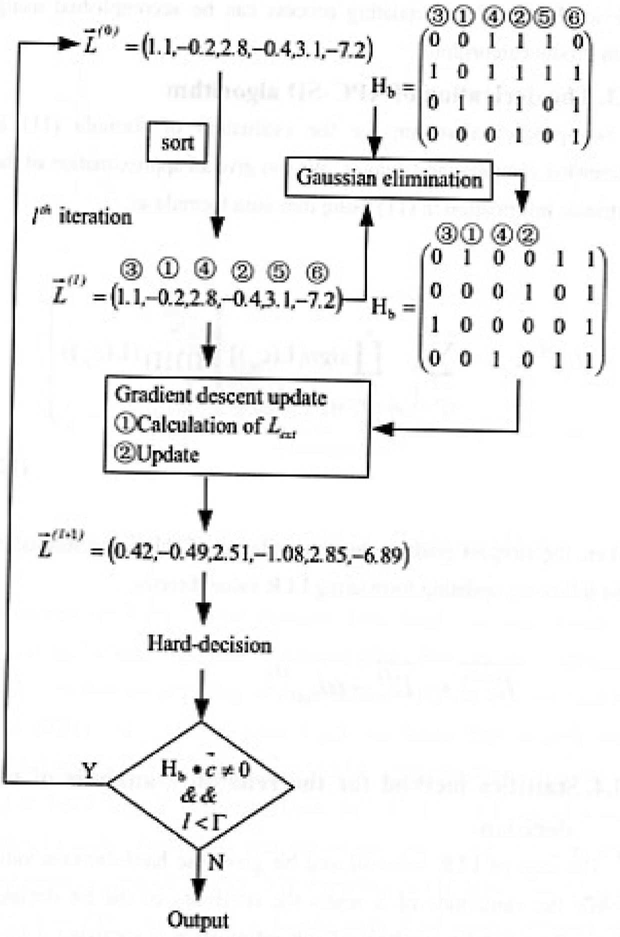


Fig.4. Diagram of APC-SD algorithm

$$\nabla J(H_b,\vec{T}) = [\frac{\partial J(H_b,\vec{T})}{\partial T_1},\cdots,\frac{\partial J(H_b,\vec{T})}{\partial T_N}], \tag{9}$$

Here,

$$\frac{\partial J(H_b,\vec{T})}{\partial T_i} = -\sum_{\substack{j=1\\ H_{ji}^{(l)}=1}}^{(N-K)} \left(\prod_{\substack{p=1\\ p\neq i, H_{jp}^{(l)}=1}}^{N} T_p^{(l)} \right) \tag{10}$$

In order to guarantee the value of T_p confined to $T_p \in [-1,+1]$, the following modification for formula (8) is needed,

$$\overrightarrow{T^{(l+1)}} \leftarrow v\left(v^{-1}\left(T_i^{(l)}\right) - \alpha\left[-\sum_{\substack{j=1\\ H_{ji}^{(l)}=1}}^{(n-k)} v^{-1}\left(\prod_{\substack{p=1\\ p\neq i, H_{jp}^{(l)}=1}}^{n} T_p^{(l)} \right)\right]\right) \tag{11}$$

Obviously, the above updating process can be accomplished using sum-product algorithm.

3.3. The derivation of APC-SD algorithm

Sum-product algorithm for the evaluation of formula (11) is somewhat computation complex. We can give an approximation of the extrinsic information in (11) using min-sum formula as,

$$L_{ext}^{(l)}(c_j)=\sum_{\substack{i=1\\ H_{ij}^{(l)}=1}}^{(n-k)}\left(\prod_{\substack{p=1\\ p\neq j,H_{ip}^{(l)}=1}}^{n} sign[\mathrm{L}(\mathrm{c}_p)]\right)\left(\min_{\substack{p=1\\ p\neq j,\\ H_{ip}^{(l)}=1}}^{n}(\mathrm{L}(c_p))\right) \tag{12}$$

Then, the steepest gradient descent updating of (11) is substituted by the following updating form using LLR value directly,

$$\overrightarrow{L^{(l+1)}}\leftarrow\overrightarrow{L^{(l)}}-\alpha L_{ext}^{(l)} \tag{13}$$

3.4. Statistics method for the reliability analysis of soft decision

The sign of LLR value of one bit gives the hard-decision value, while the magnitude of it scales the reliability of the bit decision. Recently, reliability analysis of soft information is receiving a lot of attention from researches since the wide application of soft decision decoding [7] and another area. The reliability of soft information has an effect on BER performance of a soft decision decoding. In this paper, the reliability of soft output is compared between MXLP and demodulated 16QAM based on simple statistics methods in order to explicate the BER performance of proposed full soft decision decoding.

As depicted in Fig.5.a, The number with N×M of incoming soft information is separated to M packet with N elements. The reliabilities in each packet are sorted from small to large and are made a survey for statistical bit error rate purposes by sorted position. Consider two soft information group A and B with different reliability and same bit error rate: If the packet sample number (M) of incoming soft information and element number (N) of one packet is enough large and A is more reliable than B, there exist a division point in the N position that the decision bit error rate in left of the point of A is greater than B, smaller in right of the point.

Additionally, we can also divide the sorted packet (Sample packet number is M packet) with N elements to unreliable part and reliable part as Fig.5.b. It is known that for the two soft information A and B, A is more reliable than B if the probability of decision error happened in reliable part of A is smaller than B.

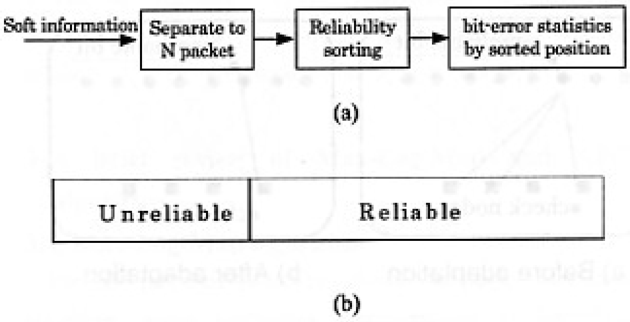


Fig.5. Reliability statistics comparison

4. Simulation results

In this section, simulation results for APC-SD algorithm of RS codes and for proposed full soft decision decoding of concatenated RS and CC codes are presented. As the need of analysis, the reliability comparison of MXLP output and demodulated 16QAM output is also shown here.

4.1. BER performance

Tab.1. simulation parameter

Channel	*AWGN*
Modulation	*16QAM*
CC code	*Nonsystematic and non-punctured with Rate=1/2, Constraint length=7, G1=171, G2=133*
RS code	*(204,188) in* $GF(2^8)$
System model	*Fig.1*

Tab.1. shows our simulation parameter. For explanation, the following notations will be used in the legends. BM refers to the conventional algebraic hard decision decoding for RS codes. Min-Sum(N) refers to the APC-SD soft decision decoding using min-sum algorithm for the steepest gradient descent updating. N refers to the maximum number of iterations of iterative decoding while aided with BM for speed up stopping criterion. MXLP+BM refers to engaging Max-Log-Map decoding for CC codes and BM for RS code, in the same way, MXLP+Min-Sum indicates that MXLP for CC and APC-SD decoding using min-sum algorithm for RS codes. Up to our best knowledge, we first present results of APC-SD decoding for RS with a bandwidth efficient modulation.

Fig.6 shows that APC-SD soft decoding for RS code achieves very good BER performance over BM hard decision decoding, about 1dB coding gain at BER=10^{-4} over AWGN channel with 16QAM.

Fig.7 indicates our proposed full soft decision decoding performance for concatenated RS and CC code. It is shown that about 0.35dB coding gain is achieved over MXLP+BM. Since the error correcting capacity of MXLP is almost same with Viterbi decoding, it is reasonable that proposed full soft decision decoding also obtains

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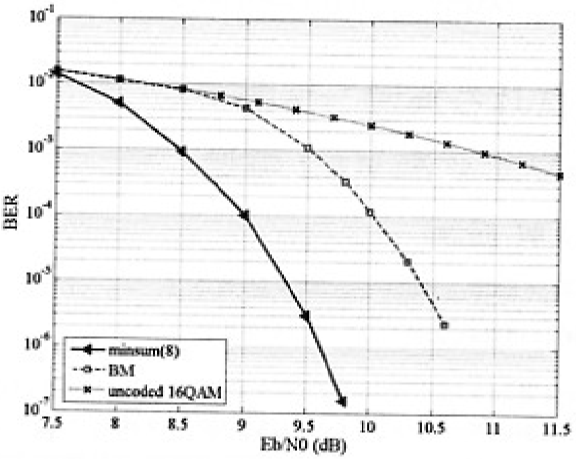

Fig.6. BER performance of APC-SD for RS with 16QAM over AWGN channel

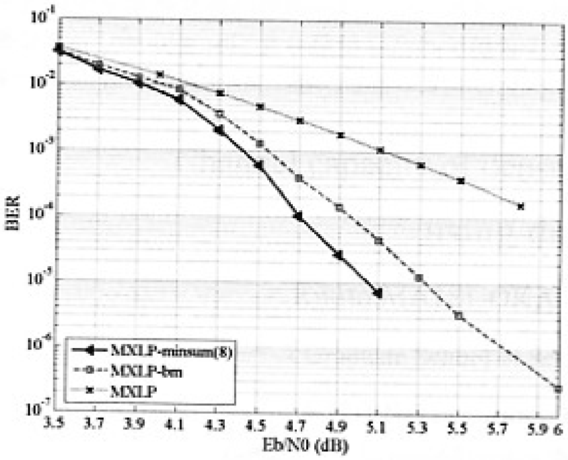

Fig.7. BER performance of proposed full soft decision decoding with 16QAM over AWGN channel

0.35dB gain at BER=10^{-4} over conventional VA+BM decoding solution. Based on our simulation result, the BER performance of the proposed full soft decoding scheme is not in expectation that ACP-SD achieved while using directly demodulated channel soft information. One factor for the appearance may be the concatenated code is not a capacity approaching code. Another factor for it is that if the soft information is not in expected reliable shall also bring the performance loss of APC-SD soft decision decoding.

4.2. Reliability analysis

Tab.2 and Fig.8 represent statistical results of our proposed method in section 3.4 of the reliability comparison between MXLP and demodulated 16QAM. In our statistics method, the packet sample number M=1e5 and element number in one packet N=1632. In addition, all both of the comparison is done under same bit error rate for the soft output of MXLP and de-16QAM. For Fig.7, x axis represents reliability sorted position from small to large within a packet and y axis represents statistical BER. Both the two statistics data show that the reliability of demodulated 16QAM is more reliable than MXLP. In addition, from Fig.8 we know that actually the reliability of soft information from MXLP is also somewhat reliable.

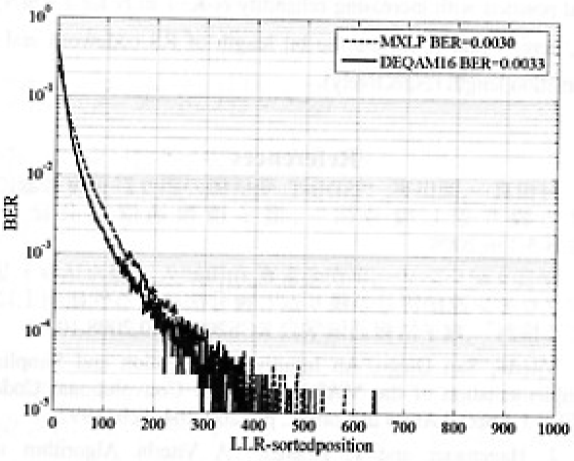

Fig.8. Reliability analysis of MXLP and DeQAM16

For Tab.2, the length of unreliable part is set to N-K for bit level RS(N,K) code. The reason that we choose this length relies on the performance analysis of APC-SD algorithm in [6]. Based on [6], we know that APC-SD algorithm do idle work if the error happened on reliable part, where the length of unreliable part is equal to N-K. In the table, P(R|▲,BER) represents the probability of decision error happened in reliable part conditionally on one bit error rate BER for a soft output ▲ and Ave. # represents the average number of bit error in reliable part.

Tab.2. Reliability analysis investigating reliable part

BER	MXLP=0.0048 DEMOD=0.0054		MXLP=0.0030 DEMOD=0.0033	
	P(R|▲,BER)	Ave. #	P(R|▲,BER)	Ave.#
MXLP	*0.0331*	*6.6295*	*0.0116*	*5.9862*
De-16QAM	*0.0131*	*2.0716*	*8.6e-4*	*2.0349*

5. Conclusions

We have shown a full soft decision decoding scheme for concatenated RS and CC code. About 0.35dB coding gain at BER=10^{-4} is achieved comparing to conventional VA+BM decoding solution on 16QAM AWGN channel. Using statistics method, we also given the reliability comparison between MXLP and demodulated QAM soft information. Combining our simulation results as Tab.2, Fig.6 and Fig.7 with the performance analysis of APC-SD algorithm proposed in [6], it is main drawback that APC-SD algorithm for RS decoding work not well if bit error happened in reliable part (from the sorted position with increasing reliability N-K+1 to N for a RS(N,K) code, here N and K denote the bit length of RS codeword and bit information length respectively).

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