A Channel Estimation Using Least Square Fitting for OFDMA Downlink Partial Usage of Subchannels

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***Abstract*— Channel estimator is one of the key components of the receiver in wireless communication system. It estimates channel transfer function by using channel's information from pilots in transmitted signal, so that useful data can be recovered. The difficulties in channel estimation are caused by white Gaussian noise added to received signal and the complexity of channel itself as well as the system's structure in used. This study analyses IEEE 802.16e's DL PUSC (Downlink Partial Usage of Subchannels) system, and uses it as the platform to develop some proposed channel estimation methods. Two major conclusions were reached: the special system's structure limits the channel estimator and the proposed method using least square error fitting has very impressive performance and low complexity, hence it is very promising for realization.**

I. INTRODUCTION

In wireless communication system, when transmitted signals travel from the transmitter to the receiver, they suffer a lot of effects from the environment which is called wireless channel. At destination, they are impaired and no longer the transmitted signals. That is the reason we need channel estimator (CE) and equalizer to recover signal at the receiver. The purpose of CE is to find out the channel which impacted on signals. The more accurate the estimated channel is, the more correct signals are equalized. To estimate channel, transmitter puts some known data (named pilot, preamble or training, etc. depending on type of system) into specific positions; in other words, known data support to point out the channel function at their positions and from them, the receiver may work out channel at the rest. Besides that, CE depends on which system it belongs to because each system has its own transmit's method in fact. So far there is not many studies about CE have been reported for the IEEE 802.16 WiMAX (Worldwide Interoperability for Microwave Access), which is one of the newest wireless communication system. Typically, there are two main types: IEEE 802.16d (Fixed WiMAX) and 802.16e (Mobile WiMAX). Mobile WiMAX is a wireless solution that enables convergence of mobile and fixed broadband networks through a common wide area broadband radio access technology and flexible network architecture [1]. It has 4 physical layer specifications [2], in there, OFDMA (Orthogonal Frequency Division Multiple Access) is significantly improved for mobile multi-path non-line-of-sight environments. An OFDMA frame structure has two subframes, downlink (DL) and uplink (UL). Plus, there are many schemes for different multi-user treatments, such as PUSC (Partial Usage of Subchannels), FUSC (Full Usage of Subchannels) and AMC (Adaptive Modulation and Coding) mode. The objective of this paper is to investigate the performance of channel estimation methods in IEEE 802.16e OFDMA DL PUSC mode.

After the introduction, section 2 describes OFDM, OFDMA and basic blocks briefly in transmitting and receiving parts. Then, section 3 states the proposed methods for CE. Section 4 provides the simulation results and explanations. Section 5 compares the results with another group. And finally section 6 summarizes the paper with some conclusions.

II. SYSTEM DESCRIPTIONS

A. *OFDM description*

OFDM (Orthogonal Frequency Division Multiplexing) is a special form of multi-carrier modulation, a multiplexing technique that subdivides the bandwidth into multiple sub-carriers. In an OFDM system, the input data stream is divided into several parallel sub-streams of reduced data rate and each sub-stream is modulated and transmitted on a separate orthogonal subcarrier [1]. Only a small amount of data is carried on each subcarrier, so the influence of inter-symbol-interference (ISI) is significantly reduced. Furthermore, cyclic prefix (CP) can completely eliminate ISI as long as the CP duration is longer than the channel delay spreading. The CP is a copy of the last data samples of the block and appended to the beginning of this block, see fig. 1.

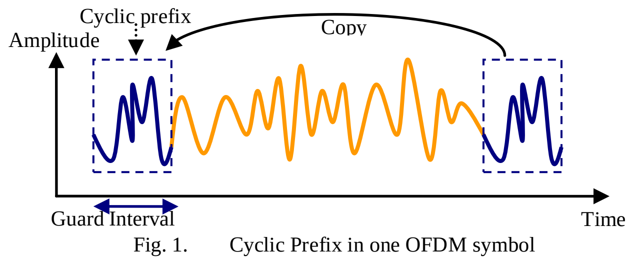

Fig. 1. Cyclic Prefix in one OFDM symbol

B. *OFDMA description*

Important terms in DL PUSC are stated as follows, see fig. 2.

Subcarrier: Bandwidth is divided into equal narrow subbands called subcarriers, includes: data subcarriers, pilot subcarriers, null subcarriers.
Subchannel: Active/used (data and pilot) subcarriers are grouped into subsets which contain some successive subcarriers called subchannels. A subchannel in a group contains 2 clusters (explained hereafter).
Slot: a slot is 1 subchannel by 2 OFDMA symbols.
Cluster: a cluster is 14 continuous subcarriers per 1 symbol or 28 subcarriers per slot.
Group: A group is made from continual logical clusters. Even group has 6 subchannel, odd group has 4 subchannels.

OFDMA symbol

OFDMA is the enhancement of existing OFDM technology. It divides an OFDM symbol into subchannels (groups of subcarriers); each subchannel is allocated to a different subscriber. Subscribers can be treated separately, independent of location as well as distance from the base station, interference and power requirements. Fig. 3 shows a spectrum of one symbol storing 2 subscribers' data.

Looking into this spectrum, obviously when we want to recover the channel transfer function, it can not be interpolated in whole spectrum like others [3]. This is a main characteristic of an OFDMA-based system affecting channel estimation methods. The advantages are that it's simple and fewer computations are required, but more difficult to estimate the channel because of the discontinuity.

The next subsection will be about transmitting and receiving parts in system.

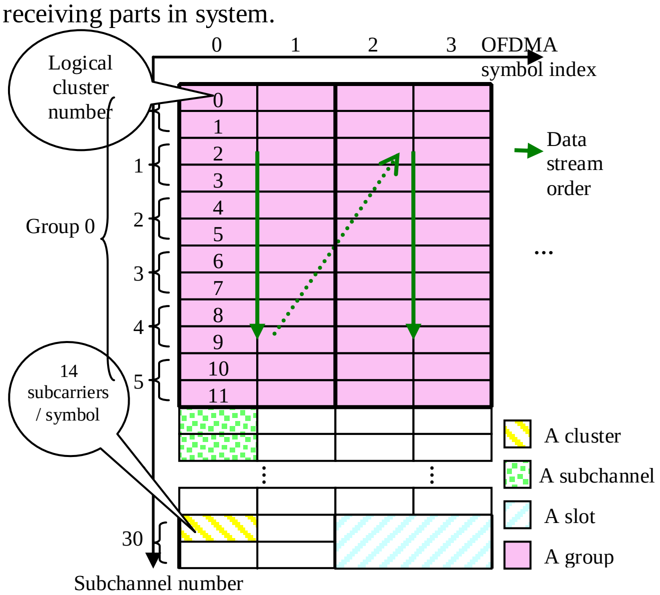

Fig. 2. OFDMA basic terms definition in DL PUSC mode

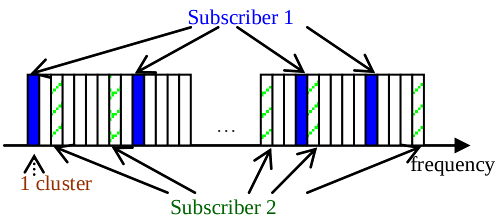

Fig. 3. Spectrum of 1 OFDMA symbol

C. *Transmitting part*

In general, OFDMA system has some fundamental parts in Transmitter, see fig. 4.
Modulation: supports 3 modes QPSK, 16 QAM and 64 QAM.
Basic permutation: rearranges subchannels in a group.
Pilot insertion: all pilot subcarriers have value (1 + 0j)*4/3 for downlink. Even and odd symbols are treated differently. The 5^{th} and 9^{th} subcarriers of a cluster in even symbols while the 1^{st} and the 13^{th} ones in odds are pilot positions in DL PUSC, see fig. 5.
Cluster permutation in 30 subchannels: After pilot insertion, all subcarriers in 30 subchannels will be allocated totally.
Subcarrier randomization: creates a set of weights in order to randomize all useful subcarriers.
IFFT: Subcarriers are swapped before taken IFFT.
Add GI (Guard Interval): GI may equal 1/4, 1/8, 1/16 or 1/32 of useful symbol length.

The signals will then go through channel models and come to receiving part.

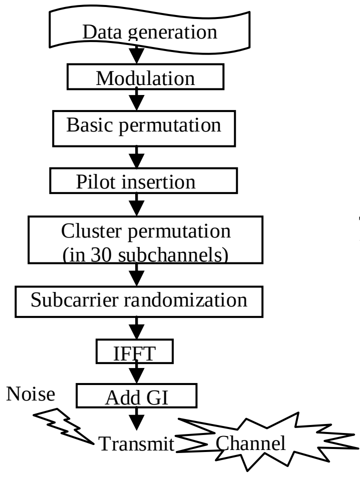

Fig. 4. Transmitting Part

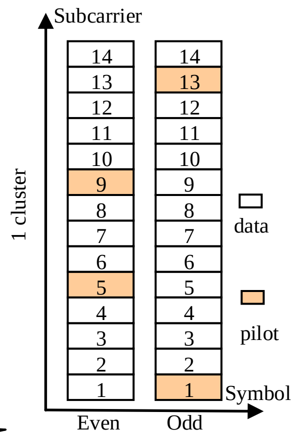

Fig. 5. Pilot pattern in DL PUSC mode

D. *Receiving part*

The receiving part will reverse all operations of the transmitting part except Channel Estimation for recovering the data, see fig. 6.

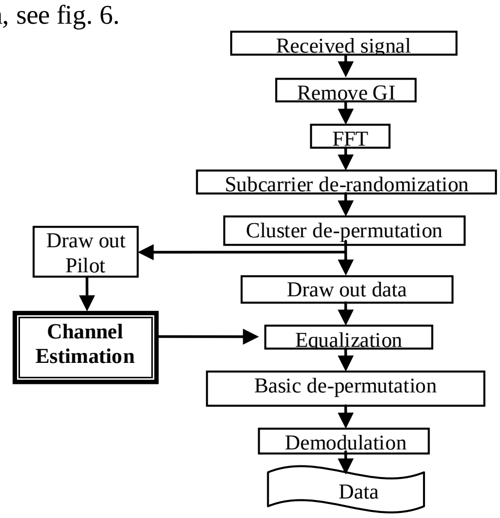

Fig. 6. Receiving part

III. Channel Estimation

In wireless system, the receiver needs a channel estimator to find out the channel transfer function. There are two methods in general: using preamble and pilots. The first method is adding training symbols before data transmission so that the receiver can estimate channel in the first time and consider channel will remain for the next moment when received data are useful information. This method is quite good when channel changes slowly, but if either the transmitter or the receiver is moving, Doppler Effect occurs, channel changes quickly and the accuracy of channel estimation degrades greatly. The second method assigns known data symbol called pilot into data stream with some kinds of pattern. By extracting pilots out of the received data, the channel transfer function can be estimated, usually the frequency response in OFDM system. This method can update channel on-the-go and adapt with the fast fading channel. In practical system, the combination of these two methods is preferred, for example we can see this in IEEE 802.11b/g and 802.16 systems. The proposed estimators are developed in sense of the second scheme.

In general, the CEs are considered consisting of two steps. Firstly, channel is calculated at known locations. Secondly, these values are used to interpolate the whole unknown channel.

A. *Common estimation methods*

Basically, we first estimate the channel in time dimension to increase number of pilots in each cluster from 2 to 4 subcarriers and then frequency dimension's estimation is getting easier.

Time estimation: At the 1st, 13th position in even symbols and the 5th, 9th position in odd symbols, pilot values will be estimated on neighbor pilot subcarriers. The middle subcarrier has average value while the border's has copy value of the nearest one, see fig. 7.

Frequency estimation: After time estimation, each cluster has 4 pilots and 10 unknown subcarriers. Some interpolation methods are investigated [4]:

- Linear interpolation (fig. 8a)
- Nearest neighbor interpolation (fig. 8b)
- Cubic-Spline interpolation (fig. 8c)
- Piecewise cubic Hermite interpolation (fig. 8d)

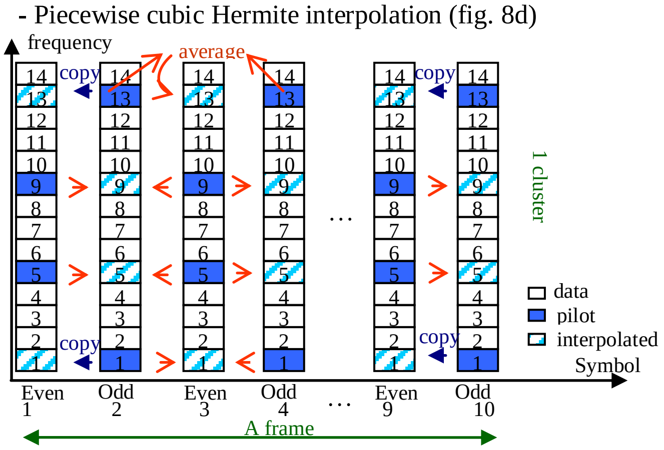

Fig. 7. Time estimation

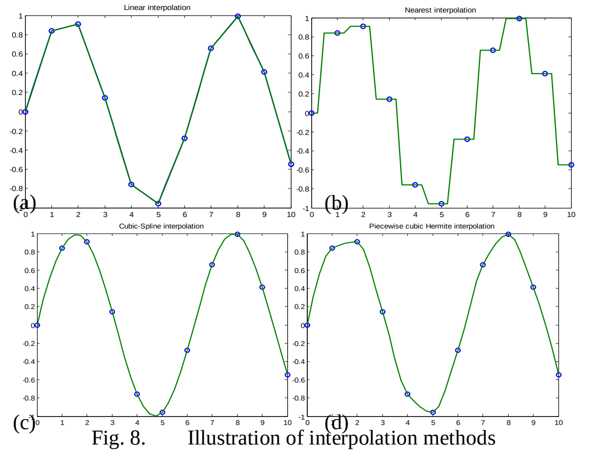

Fig. 8. Illustration of interpolation methods

(a) Linear interpolation; (b) Nearest interpolation

(c) Cubic-Spline interpolation; (d) Piecewise cubic Hermite interpolation

All of above interpolation methods are based on basic functions such as linear or cubic. One common point here is all of the interpolated lines going through their known data.

B. *Least square line fitting:*

We have known that our pilots suffer from not only channel transfer function but also noise. Those pilot values are approximately accurate and then it's better to use them approximately in interpolation. Common interpolation methods such as linear or spline used exactly pilot values to interpolate missing positions according to some equations. The proposed method uses reference values to approximate a reasonable line which is closest with all of them.

The proposed channel estimation method is based on the technique of curve fitting [5], for this case; it is the method of least square line fitting.

Considering a user has a packet of N symbols in DL PUSC system, for example N = 10.

Estimation in time:

In time axis, we have four streams which are related to subcarrier indexes: {1, 5, 9, 13} containing channel values at pilots, see fig. 7 for reference. Stream 1 and stream 13 have five pilots distributed differently in time: {2, 4, 6, 8, 10} while stream 5 and stream 9 have five pilots distributed in: {1, 3, 5, 7, 9}. So, in the sense of least square line fitting the estimation in time is finding four line equations in the form of the line (d) according to channel values at pilots and their positions in time.

$$(d): y = ax + b \qquad (1)$$

Where y is receive value and x is its positions in time axis, a and b are coefficients. The fitting line is found as follow:

We present the channel values at pilots by: $p_1, p_2, \ldots, p_5$, and their positions in time are: $x_1, x_2, \ldots, x_5$. The line (d) is called least square line fitting of the stream p_i when the sum of squared error S:

$$S = \sum_{i=1}^{5} (p_i - y_i)^2 = \sum_{i=1}^{5} (p_i - ax_i - b)^2 \qquad , y_i = ax_i + b \quad (2)$$

is minimized. That means we have to find the two coefficients {a, b} so that they can minimize S. {a, b} can be found by vanishing the partial derivatives: $\frac{\partial S}{\partial a}$ and $\frac{\partial S}{\partial b}$.

Therefore, the problem turns to solving the system of equations:

$$\begin{cases}\dfrac{\partial S}{\partial a}=0\\ \dfrac{\partial S}{\partial b}=0\end{cases} \Rightarrow \begin{cases} a=\dfrac{N\sum_{i=1}^{N}x_i y_i-\sum_{i=1}^{N}x_i\sum_{i=1}^{N}y_i}{\sum_{i=1}^{N}x_i^2 N-\left(\sum_{i=1}^{N}x_i\right)^2}\\ b=\dfrac{-\sum_{i=1}^{N}x_i\sum_{i=1}^{N}x_i y_i+\sum_{i=1}^{N}x_i^2\sum_{i=1}^{N}y_i}{\sum_{i=1}^{N}x_i^2 N-\left(\sum_{i=1}^{N}x_i\right)^2}\end{cases} \quad (3)$$

That solution is dealt by solving normal equation which sometime can be ill-conditioned. The preferable method is presenting this problem in matrix form and utilizes the QR-factorization [6].

We present this problem as:

$$p=\begin{bmatrix}p_1\\p_2\\ \vdots\\p_5\end{bmatrix}, A=\begin{bmatrix}x_1 & 1\\x_2 & 1\\ \vdots & 1\\x_5 & 1\end{bmatrix}, w=\begin{bmatrix}a\\b\end{bmatrix} \quad (4)$$

The coefficients {a, b} is obtained by solving equation:

$$A^T.Aw=A^T.y \Rightarrow w=(A^T A)^{-1}.A^T.y \quad (5)$$

This equation is considerable low complexity and easy to solve; about 602 additions and 468 multiplications in a calculation block of 10 symbols. When the least square line equations are found, the estimation in time is done by calculating all values of four streams indexed: {1, 5, 9, 13} using according equation and time position stream x_i = {1, 2, 3, …, 10}. Note that the channel values at pilots also changed.

Estimation in frequency:

After estimation in time, we have the packet of data like fig. 9. Now, in frequency axis, we have ten streams of data, each contains four known values. The estimation in frequency is carried-out exactly the same as the previous in time except that the procedure and the positions in frequency are unchanged for all streams from 1 to 10. Note that, all the values calculated in the time estimation step are changed.

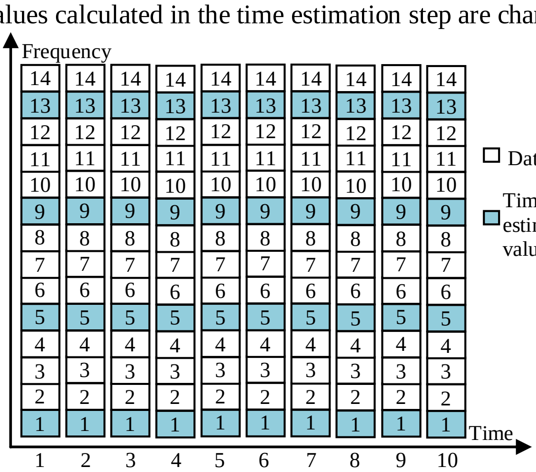

Fig. 9. After estimation in time dimesion

IV. Simulation Results

Following are the parameters used in the simulation:

- DL PUSC OFDMA 1024 FFT
- No preamble, no channel coding
- OFDM symbol time: Ts = 115.2µs;
- Useful symbol time: Tb = 102.4µs;
- Guard interval Tg =12.8µs;
- Carrier frequency 2.3GHz.
- Sampling frequency 10MHz

We exploit the research in 2 environments based on ITU (International Telecommunications Union) model [7]-[10].

1. ITU-Pedestrian A WiMAX channel model, speed 12 km/h, Doppler frequency 25.56 (Hz).
2. ITU-Vehicular A WiMAX channel model, speed 120 km/h. Doppler frequency 255.56 (Hz).

Table 1 lists all parameters of these channels using Jakes PSD (Power Spectral Density).

A. *Case 1*

Investigate all channel estimation methods in 64 QAM modulation mode with SNR (Signal to Noise Ratio) (dB) in x-axis and BER (Bit Error Rate) in y-axis, fig. 10 and fig. 11.

In all investigations, proposed channel estimation method brings the best results.

Linear estimation method is better than the Nearest, Spline and Pchip. This is because of "cluster-pilot" pattern, only 2 to 4 pilots for 12 data subcarriers therefore channel is quite flat in short interval.

TABLE I
PROFILES OF EXPERIMENT'S CHANNELS

1	Path Power (dB)	–3.9	–4.8	–8.8	–11.9	–11.7	–27.8
	Path Delay (µs)	0	0.2	0.8	1.2	2.3	3.7
2	Path Power (dB)	–3.1	–4.1	–12.1	–13.1	–18.1	–23.1
	Path Delay (µs)	0	0.31	0.71	1.09	1.73	2.51

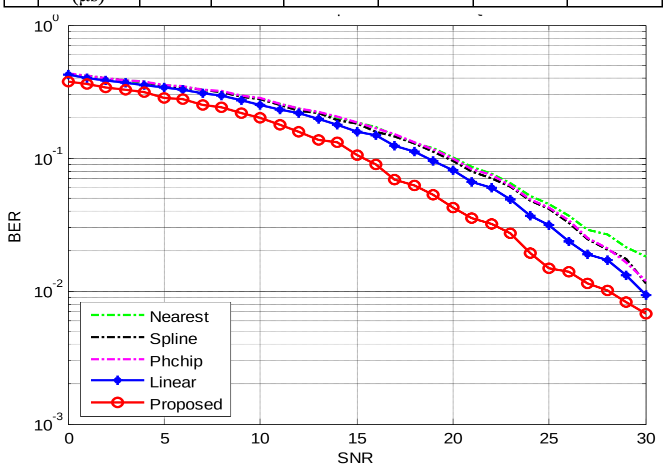

Fig. 10. Comparing all estimation methods in pedestrian B WiMAX channel, speed 12 km/h

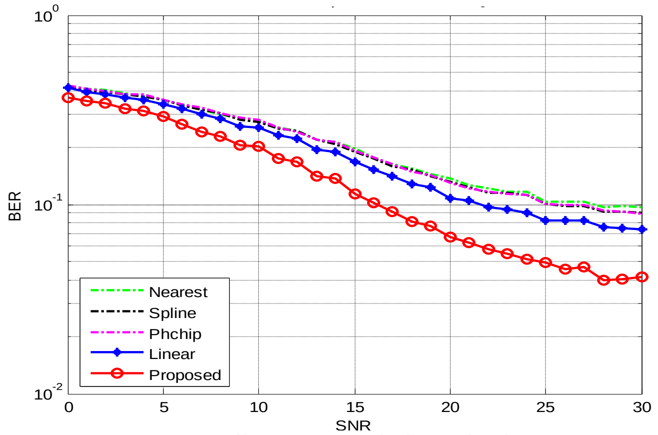

Fig. 11. Comparing all estimation methods in vehicular A WiMAX channel, speed 120 km/h

Channel is uneven and noise makes it become rough, sometime significantly bad. Hence, proposed least square line fitting method solves this problem; it keeps unevenness of flatness approximately because of using pilot approximately. It's better than linear estimation in all environments from pedestrian to vehicular cases; at least reducing 3 dB SNR at BER = 10^{-2} in pedestrian's and 4 dB SNR at BER = 10^{-1} in vehicular's when compared with linear estimation method.

B. *Case 2*

Investigate the performance of proposed method in all supported modulation modes: QPSK, 16 QAM and 64 QAM, see fig. 12, fig. 13.

With these simulation results, least square line fitting estimation method takes apparently good and stable effects in all modulation modes as well as various environments.

C. *General comments*

Why is proposed estimation method better than nearest, linear, spline, cubic and pchip? Fig. 14, 15 and 16 show the difference of channels in a cluster in 3-dimension taken from simulation under vehicular A WiMAX, speed 30 km/h, 20dB SNR and 64 QAM modulation modes.

Fig. 16 shows that the proposed estimated channel is flattened quite well accompany with the ideal channel while linear estimation is not similar.

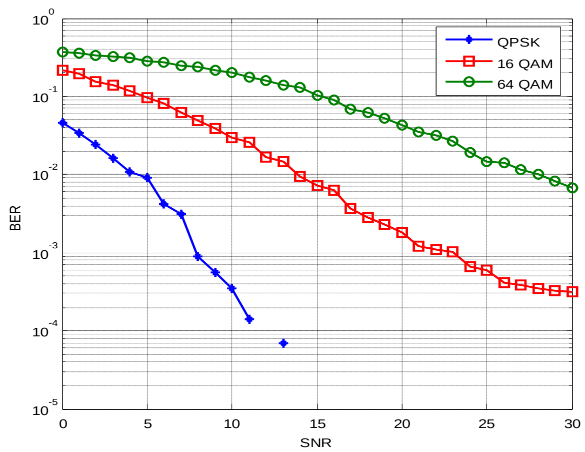

Fig. 12. Performance of all modulation modes when using proposed estimation methods in pedestrian B WiMAX channel, speed 12 km/h

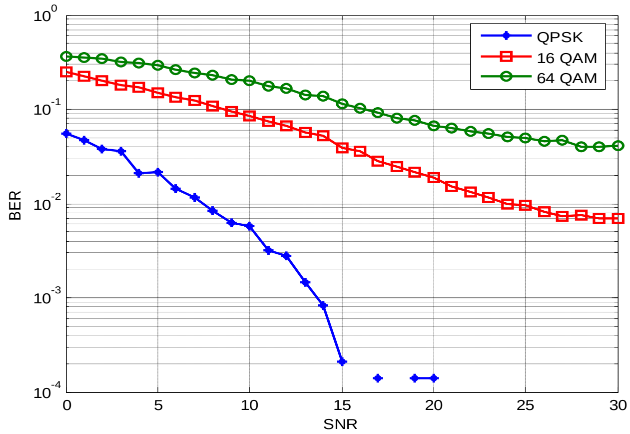

Fig. 13. Performance of all modulation modes when using proposed estimation methods in vehicular A WiMAX channel, speed 120 km/h

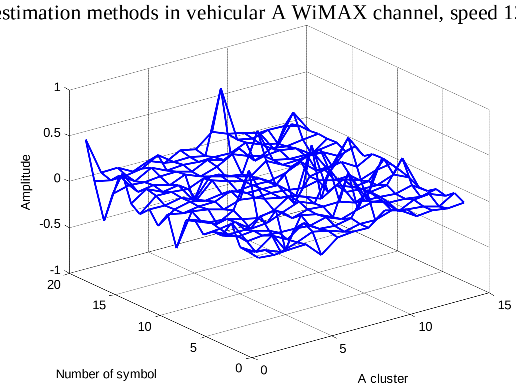

Fig. 14. The difference between desired channel and received channel

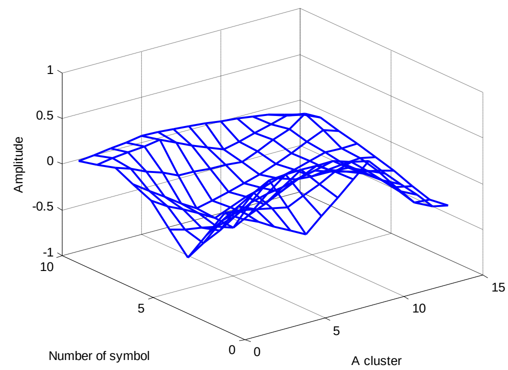

Fig. 15. The difference between desired channel and obtained channel after Linear Estimation

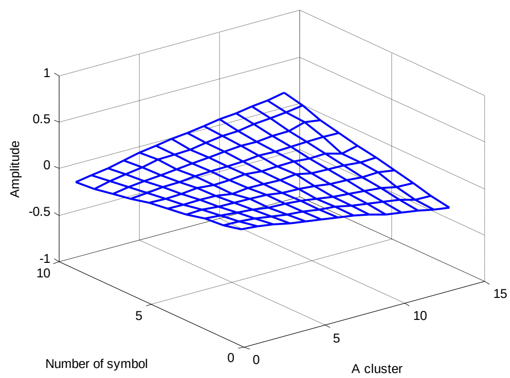

Fig. 16. The difference between desired channel and obtained channel after Proposed Estimation

V. Comparing With Other Research

Agilent Technologies, Inc. introduced the E8869 Mobile WiMAX Wireless Library (revised on 22nd August 2006). This product provides preconfigured simulation setups, signal sources and fully coded BER analysis for simulation of the circuitry used in mobile BWA designs [11].

In this technical report, it displayed a Downlink BER figure, fig. 17, using the following simulation parameters:

- DL PUSC FFT = 1024
- Carrier frequency Fc = 3407 MHz
- Bandwidth BW = 10 MHz; CP = 1/8
- ITU fading channel: Vehicular A, speed 60 km/h

For comparison, proposed Least Square Line Fitting estimation method was based on the above parameters to simulate and gained BERs versus E_b/N_0 (the ratio of Energy per Bit to the Spectral Noise Density) as shown in fig. 18.

These figures shows that in 3 modulation modes, Proposed Estimation Method is better than Agilent's results even in small range of E_b/N_0 (0 – 5 dB). Table 2 summarizes all comparisons. Though this comparison is not really equivalent, it boosts the Least Square Line Fitting to a promising channel estimation method.

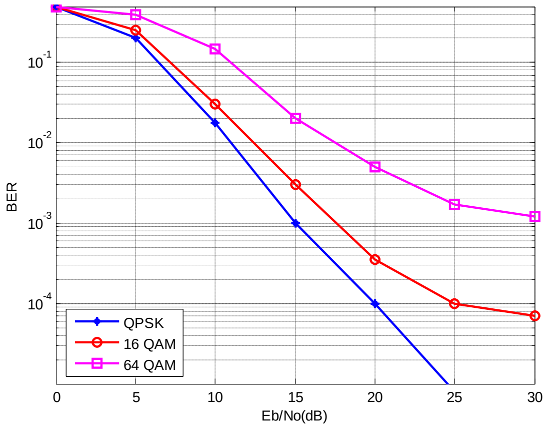

Fig. 17. BER of Agilent's products [10]

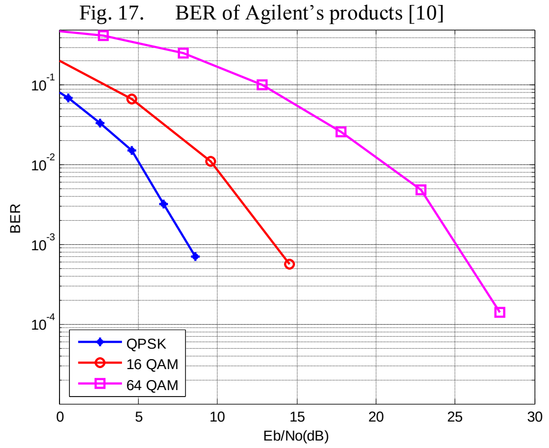

Fig. 18. BER of Proposed Channel Estimation

VI. Conclusions

This paper mentions about channel estimation limited in OFDMA-based physical layer, Downlink – PUSC mode of Mobile WiMAX 802.16e system. Two conclusions are stated.

Table II
Profiles of Experiment's Channels

E_b/N_0	Properties	**Agilent's**	**Proposed's**
	Channel coding	CC rate 1/2	CC rate 1/2
	Channel decoding	Soft decision (not mentioned in details)	Viterbi decoding
5 dB	QPSK	2 x10^{-1} dB BER	1.5 x 10^{-2} dB BER
	16 QAM	2.5 x 10^{-1} dB BER	6 x 10^{-2} dB BER
	64 QAM	4 x 10^{-1} dB BER	3.5 x 10^{-1} dB BER
15dB	QPSK	1 x 10^{-3} dB BER	0 dB BER
	16 QAM	3.5 x 10^{-3} dB BER	5 x 10^{-4} dB BER
	64 QAM	2 x 10^{-2} dB BER	5.5 x 10^{-2} dB BER
25dB	QPSK	0 dB BER	0 dB BER
	16 QAM	1 x 10^{-4} dB BER	0 dB BER
	64 QAM	1.8 x 10^{-3} dB BER	1 x 10^{-3} dB BER

The first is the pilot pattern in DL PUSC is quite different from other systems. Clusters are not continuous in the frequency axis; there are 2 pilot subcarriers in each cluster (12 data subcarriers). Therefore the channel estimation is limited from the pilots that reside inside the cluster over several symbol durations.

The second is that the proposed estimation method improves the system's performance impressively. This method is called the Least Square Line Fitting estimation. Briefly, from investigation results in section 4, the Least Square Line Fitting proved that it is the best channel estimation method in all cases from pedestrian to vehicular environment. Moreover, it is much simpler with fewer computations. Therefore, this is a promising method in reality.

All results were gained from computer simulation. Comparison with Agilent's, proposed channel estimation is better in most cases.

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