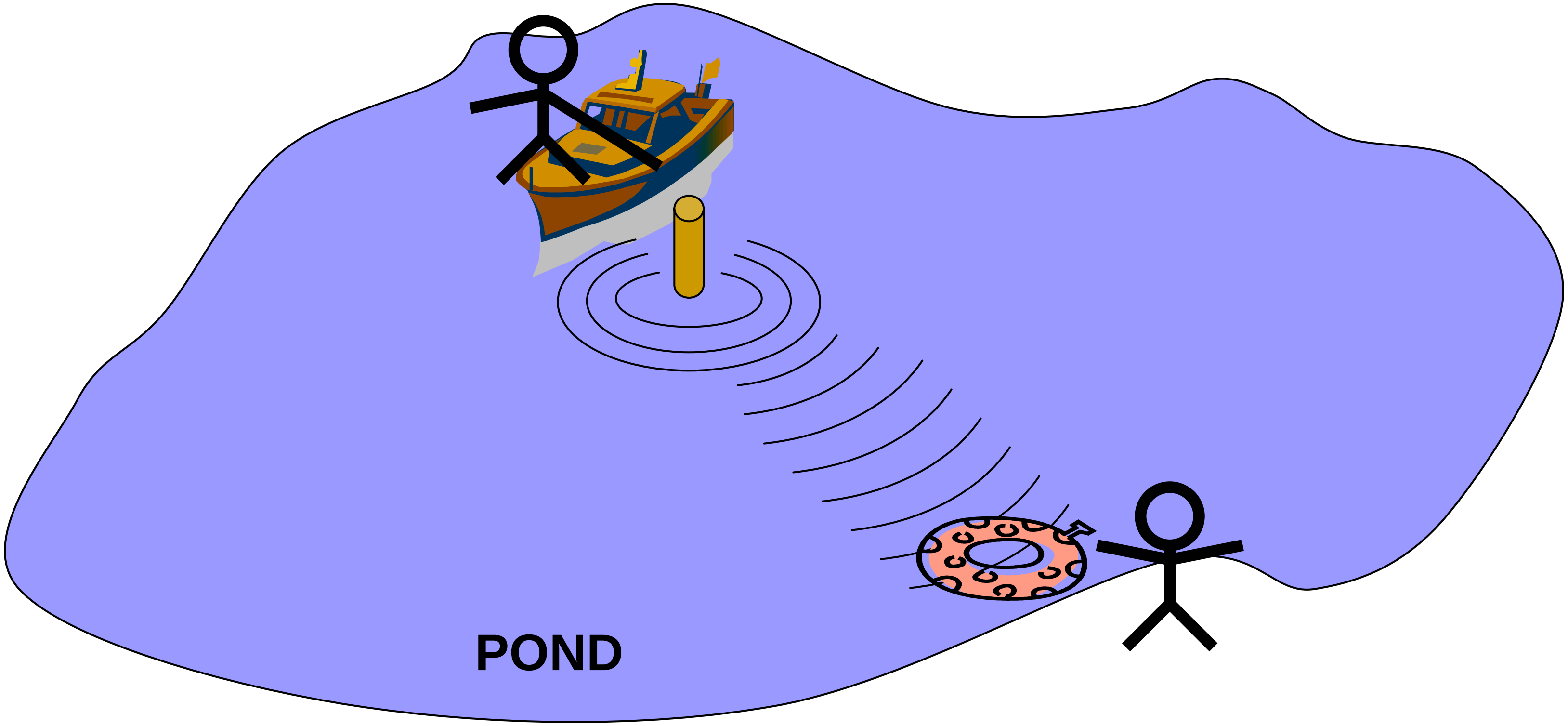
OUTLINE

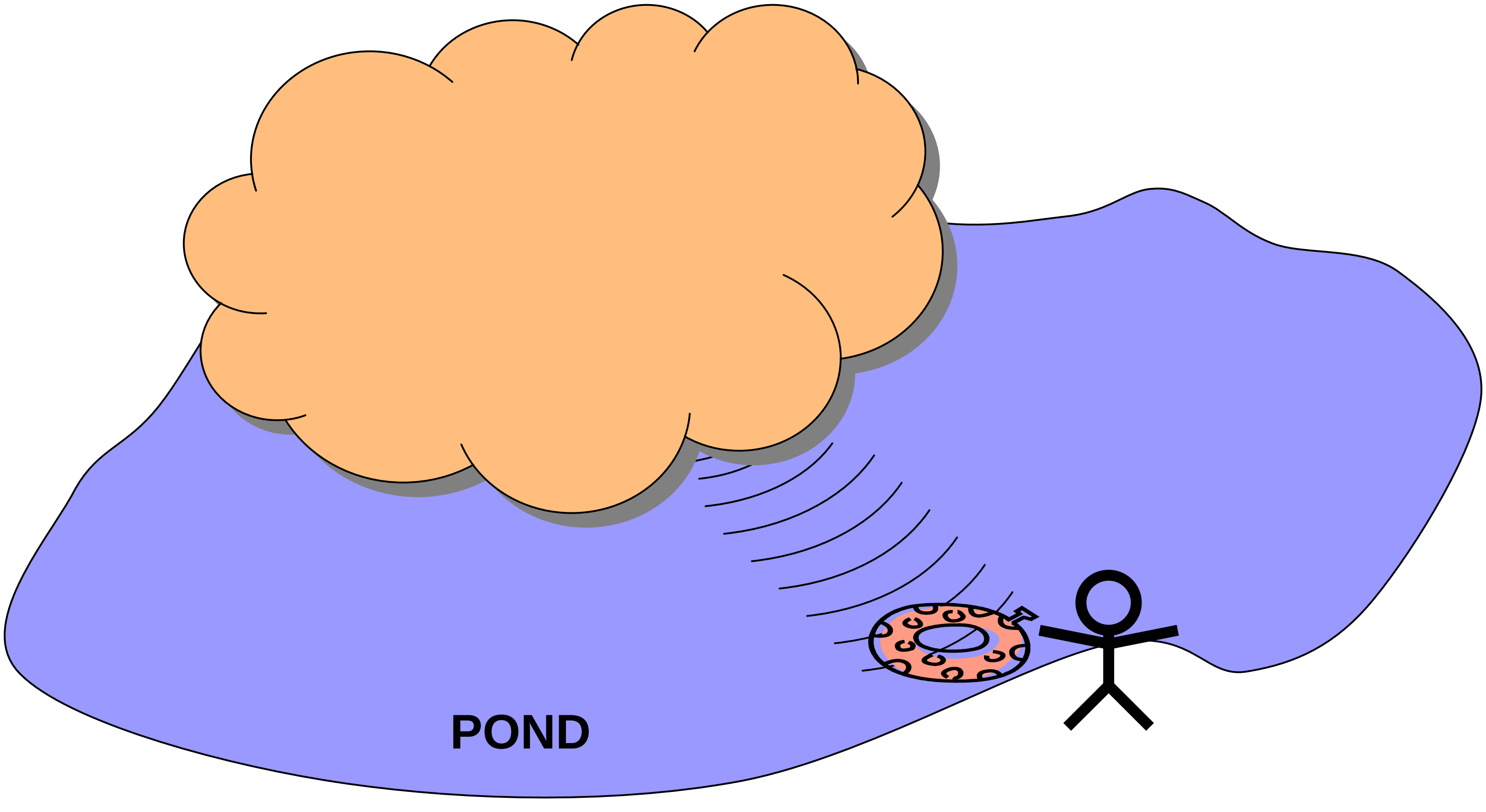
- Wave
- Sinusoidal signal review
- Complex exponential signal
- Complex amplitude (phasor)
- What is Spectrum

2 D wave



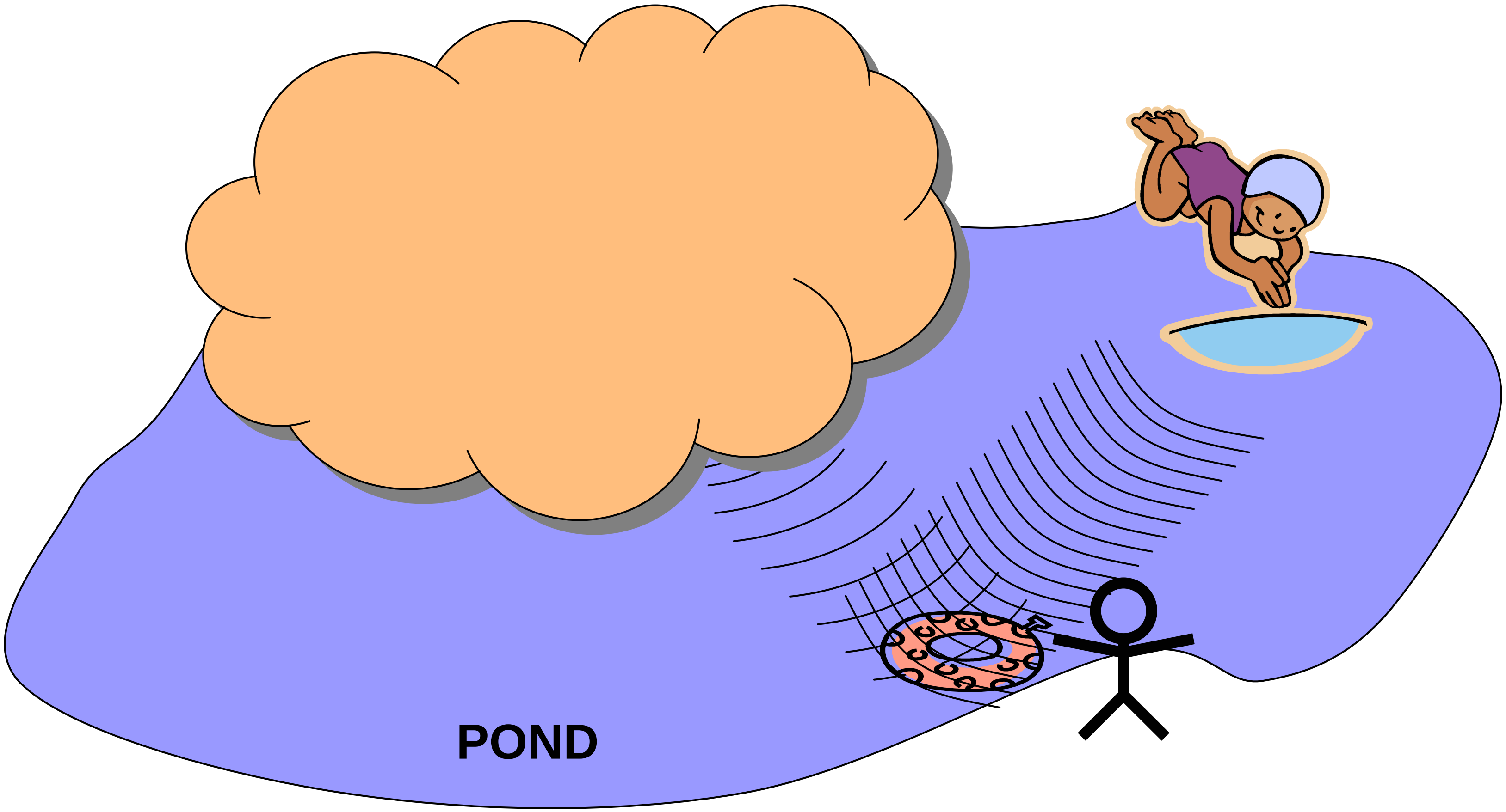
BOAT person can send message by making water wave.
LAND person can understand message by the movement of the float.

Even with FOG



LAND person just observe ONLY the float.

Even with NOISE



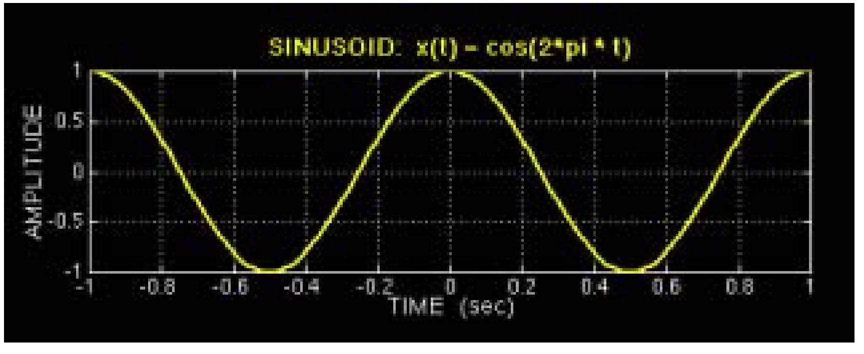
If main signal is enough stronger than the interference (noise),

LAND person can observe the message.

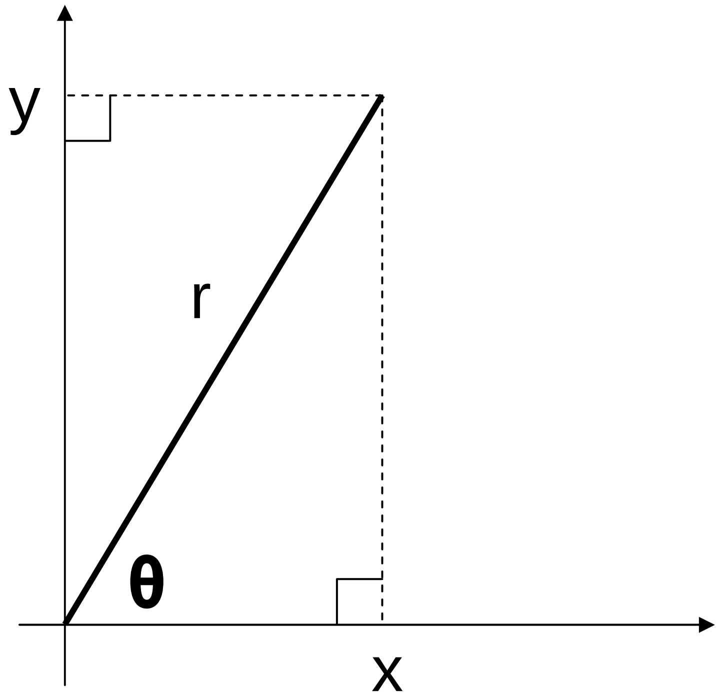
Typical wave "Sinusoids"

$$x(t) = A\cos(\omega_0 t + \phi)$$

- t : time
- A : amplitude
- $\boldsymbol{\omega}_0$: radian frequency
- $\boldsymbol{\Phi}$: phase-shift



Review of cosine and sine



$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$y = r\sin\theta$$

$$x = r\cos\theta$$

Period T_0 of sinusoids

$$x(t+T_0)=x(t)$$

$$A\cos(\omega_0(t+T_0)+\phi)=A\cos(\omega_0 t+\phi)$$

$$\cos(\omega_0 t+\omega_0 T_0+\phi)=\cos(\omega_0 t+\phi)$$

$$\omega_0 T_0=2\pi$$

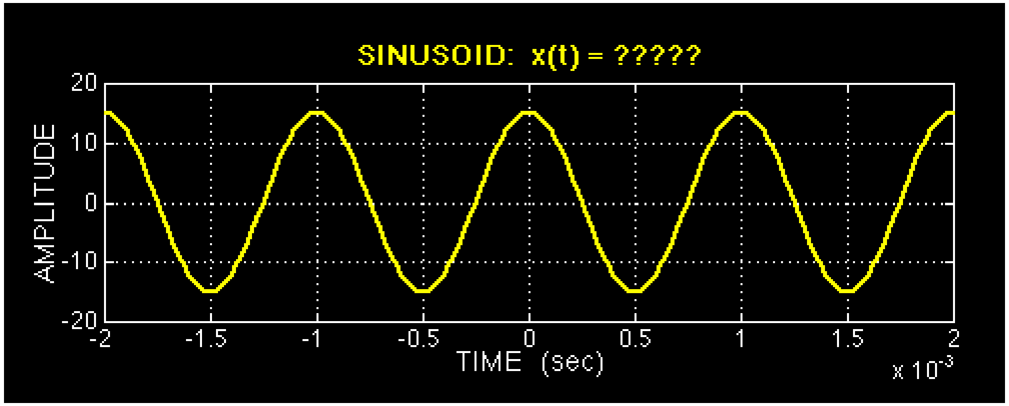
$$(2\pi f_0)T_0=2\pi$$

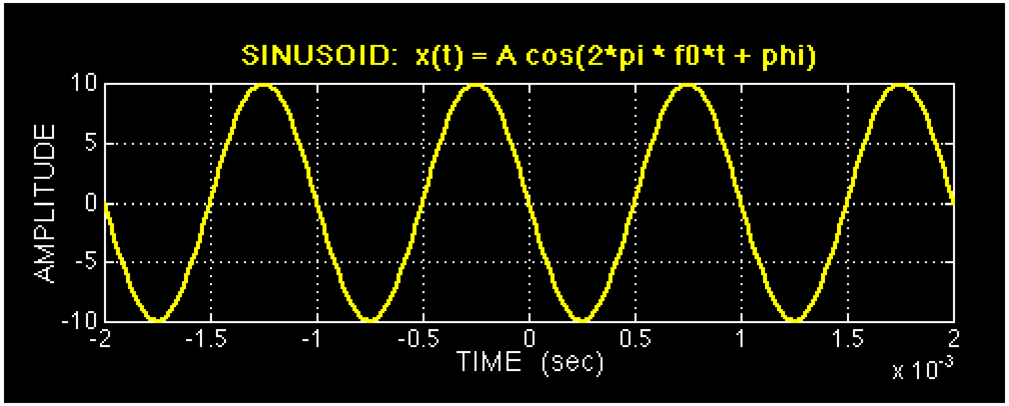
$$\Rightarrow$$

$$T_0=\frac{2\pi}{\omega_0}$$

$$T_0=\frac{1}{f_0}$$

QUIZ – what is the equation?-





Review of complex

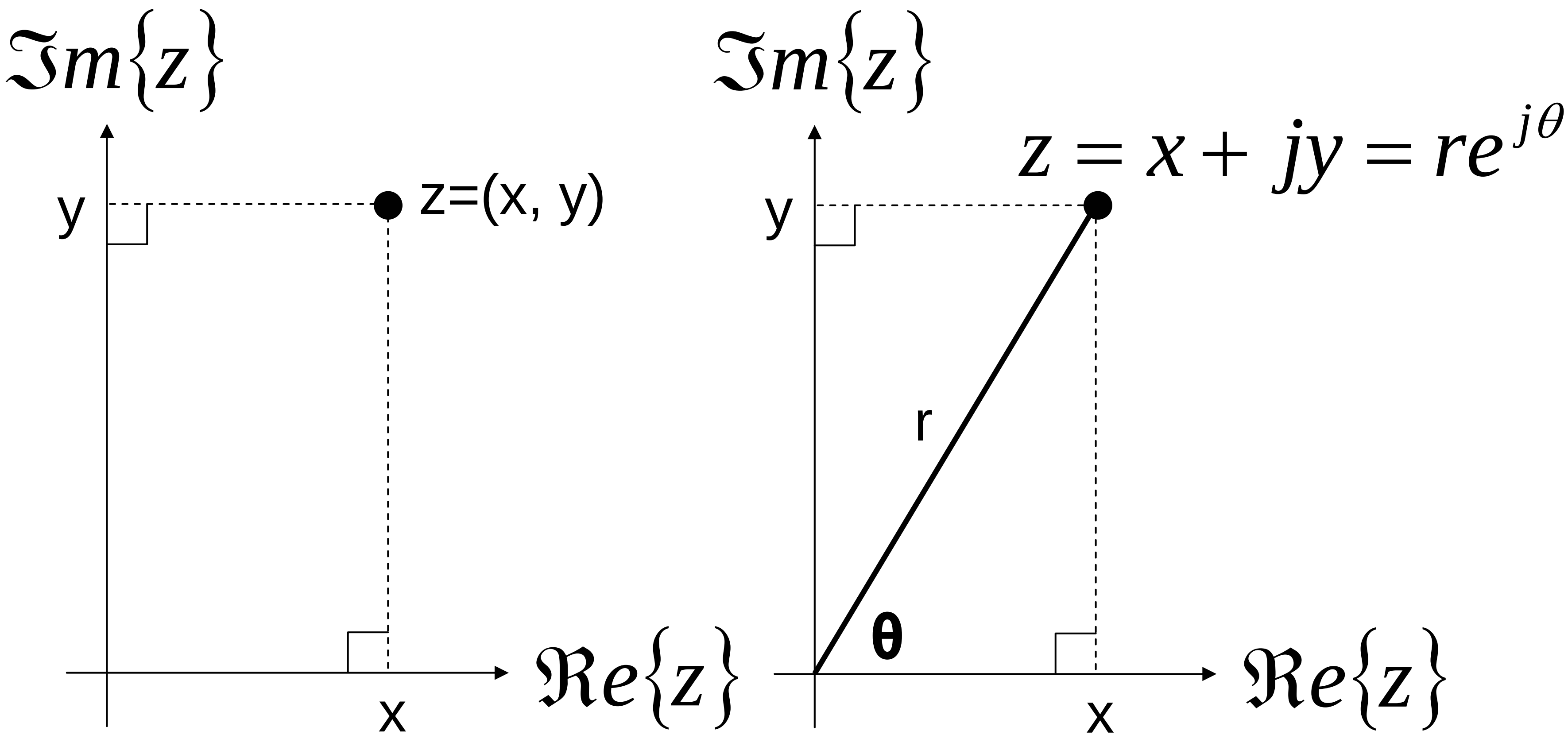
- Cartesian Form
  - Complex z=(x, y)
  - x = Real part of z :
  - y = Imaginary part of z :
  - z = x + j y
- Polar Form
  - Complex z=(r, **θ**)

$$x = \Re e\{z\}$$

$$y = \Im m\{z\}$$

$$j = \sqrt{-1}$$

Cartesian Form and Polar Form



Transform

- Polar to Cartesian

$$x = r\cos\theta$$

$$y = r\sin\theta$$

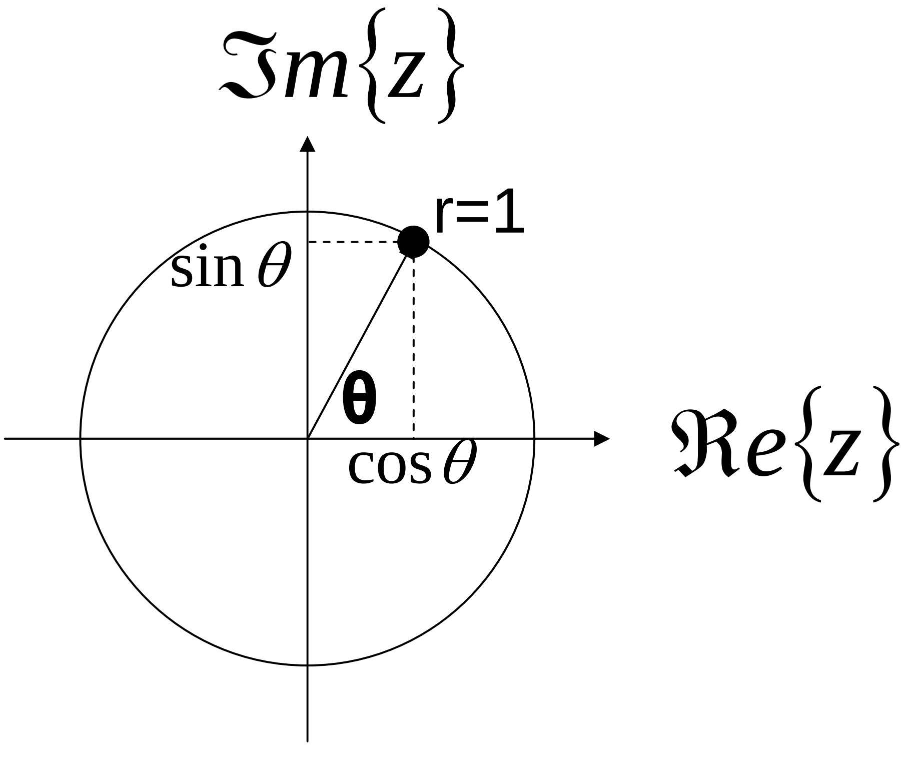
- Cartesian to Polar

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

Euler's law

$$e^{j\theta} = \cos\theta + j\sin\theta$$

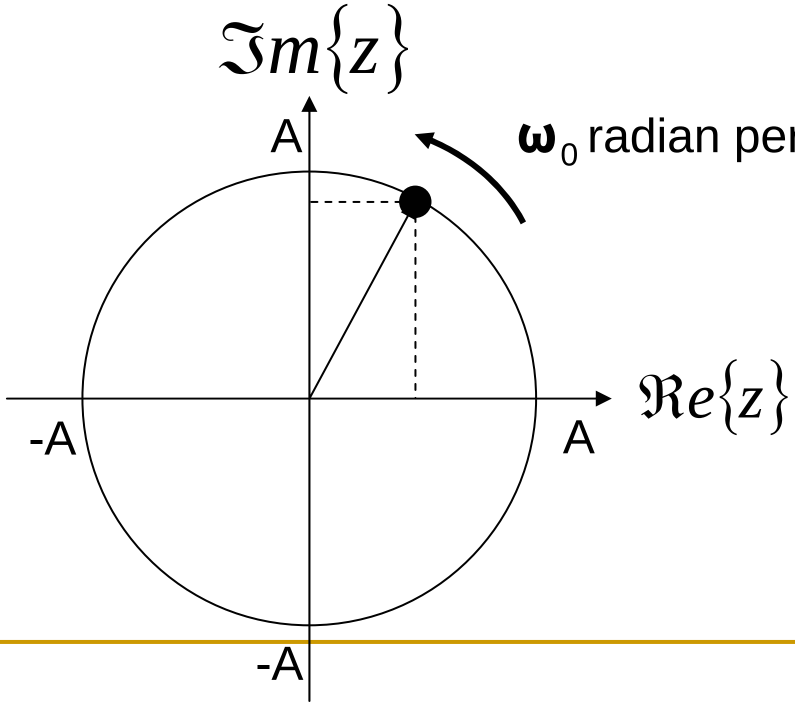


Complex exponential signal (Rotation Function)

$$\tilde{x}(t) = Ae^{j(\omega_0 t + \phi)}$$

$$= A\cos(\omega_0 t + \phi) + jA\sin(\omega_0 t + \phi)$$

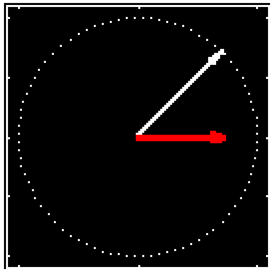
- Amplitude = A
- $\Theta = \omega_0 t + \Phi$
- Φ: phase-shift
- ω_0 : rotation speed
  - \+ : counterclockwise
  - \- : clockwise



The complex exponential signal is another representation of cosine signal

$$x(t) = \Re e\left\{Ae^{j(\omega_0 t+\phi)}\right\} = A\cos(\omega_0 t+\phi)$$

- Re means a Projection to X-axis

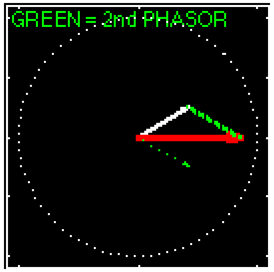


Inverse Euler's law

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$A\cos(\omega_0 t + \phi) = A\left\{\frac{e^{j(\omega_0 t+\phi)} + e^{-j(\omega_0 t+\phi)}}{2}\right\}$$

$$= \frac{\tilde{x}(t) + \tilde{x}^*(t)}{2} = \Re e\{\tilde{x}(t)\}$$



X* is conjugate of X.

Inverse Euler's law (2)

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

- Real signal is the combination of + and - frequency complex exponential signal.

$$A\cos(\omega_0 t + \phi) = A\left\{\frac{e^{j(\omega_0 t + \phi)} + e^{-j(\omega_0 t + \phi)}}{2}\right\}$$

Complex amplitude (Phasor)

$$x(t) = \Re e\left\{Ae^{j(\omega_0 t+\phi)}\right\} = A\cos(\omega_0 t + \phi)$$

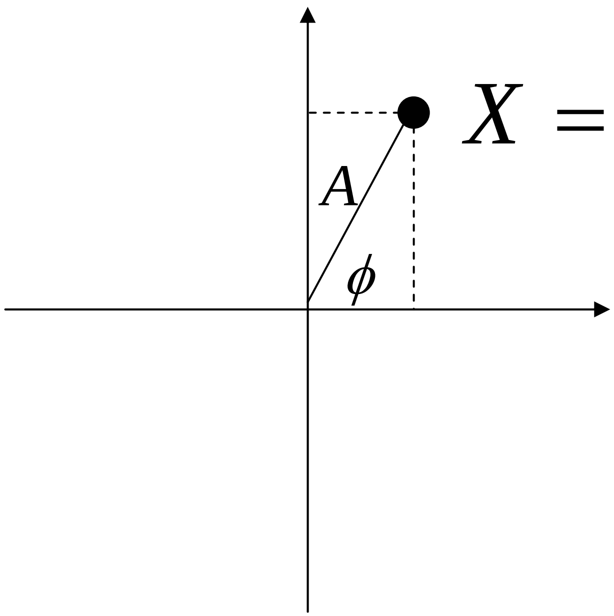
$$\tilde{x}(t) = Ae^{j(\omega_0 t+\phi)} = Ae^{j\phi} \cdot e^{j\omega_0 t}$$

$$X = Ae^{j\phi}$$

$$\tilde{x}(t) = Xe^{j\omega_0 t}$$

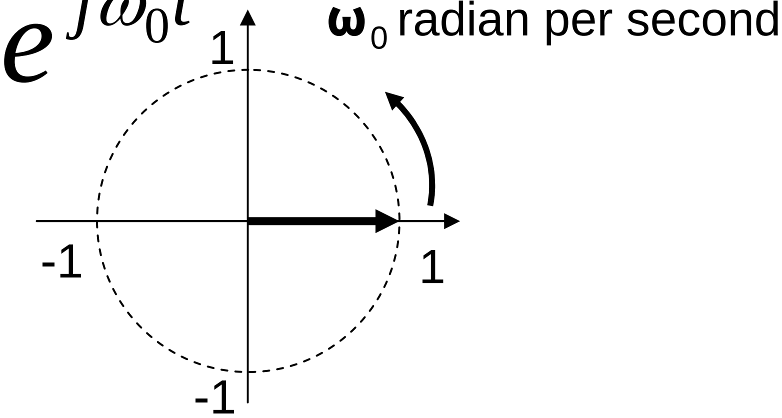
- X is called as Complex Amplitude or Phasor.

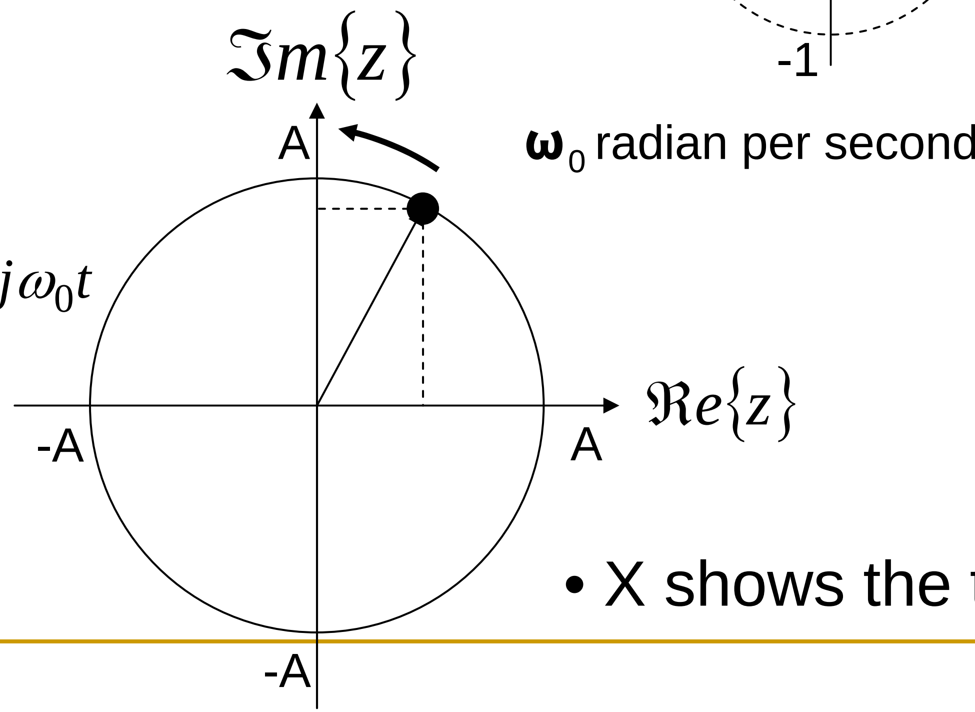
Complex amplitude (Phasor) -2-



$$X = Ae^{j\phi}$$

$$e^{j\omega_0 t}$$





$$\tilde{x}(t) = Xe^{j\omega_0 t}$$

- X shows the t=0 position.

Why Phasor is useful?

1. If you want to add two waves….

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

2. Make the complex exponential signal

$$x_1(t) = \Re e[1.7e^{j70\pi/180} \cdot e^{j2\pi(10)t}]$$

$$x_2(t) = \Re e[1.9e^{j200\pi/180} \cdot e^{j2\pi(10)t}]$$

Why Phasor is useful? (2)

3. Make the Phasor…

$$X_1 = 1.7e^{j70\pi/180} = 0.5814 + j1.597$$

$$X_2 = 1.9e^{j200\pi/180} = -1.785 - j0.6498$$

4. Add them…

$$X_3 = X_1 + X_2$$

$$= 0.5814 + j1.597 + (-1.785 - j0.6498)$$

$$= -1.204 + j0.9476$$

$$= 1.532e^{j141.79\pi/180}$$

Why Phasor is useful? (3)

5. Result

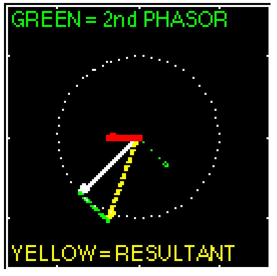
$$X_3 = 1.532 e^{j141.79\pi/180}$$

$$x_3(t) = \Re e[1.532 e^{j141.79\pi/180} \cdot e^{j2\pi(10)t}]$$

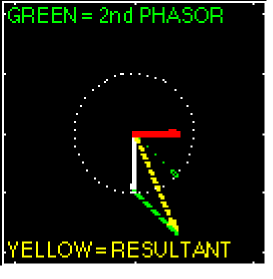
$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

$$= 1.532\cos(2\pi(10)(t + 0.0394))$$

2 different frequency composition



- 2 apart frequencies



- 2 close frequencies
- Beat

Sum of N cosine waves

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k)$$

$$X_k = A_k e^{j\phi_k}$$

$$x(t) = X_0 + \sum_{k=1}^{N} \Re e\left\{X_k e^{j2\pi f_k t}\right\}$$

$$= X_0 + \sum_{k=1}^{N} \left\{\frac{X_k}{2} e^{j2\pi f_k t} + \frac{X_k^*}{2} e^{-j2\pi f_k t}\right\}$$

- This is sum of 2N+1 different frequency signals.

What is Spectrum

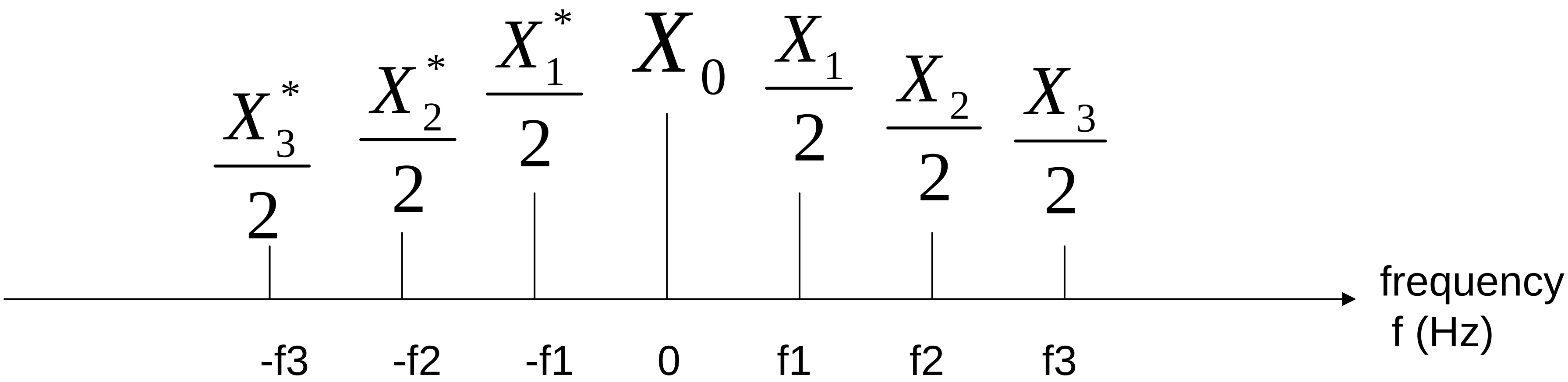
- Spectrum is the set of phasors for each frequencies.

$$\left\{ \left(X_0, 0 \right), \left(\frac{1}{2} X_1, f_1 \right), \left(\frac{1}{2} X_1^*, -f_1 \right), \left(\frac{1}{2} X_2, f_2 \right), \left(\frac{1}{2} X_2^*, -f_2 \right), \cdots \right\}$$

Spectrum plot

$$x(t) = X_0 + \sum_{k=1}^{N} \left\{ \frac{X_k}{2} e^{j2\pi f_k t} + \frac{X_k^*}{2} e^{-j2\pi f_k t} \right\}$$

N=3



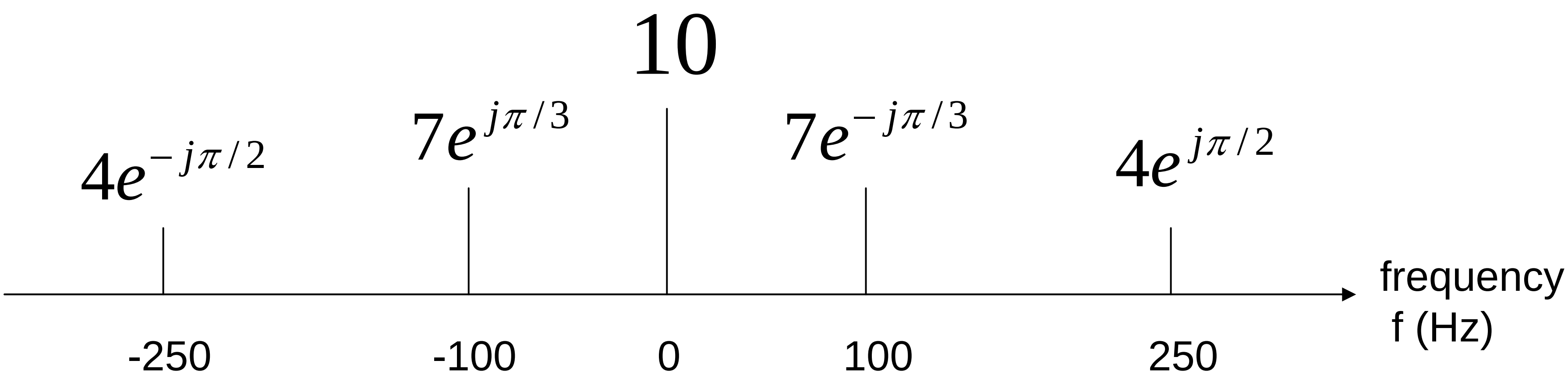
The line length is proportional to |phasor|.

Example

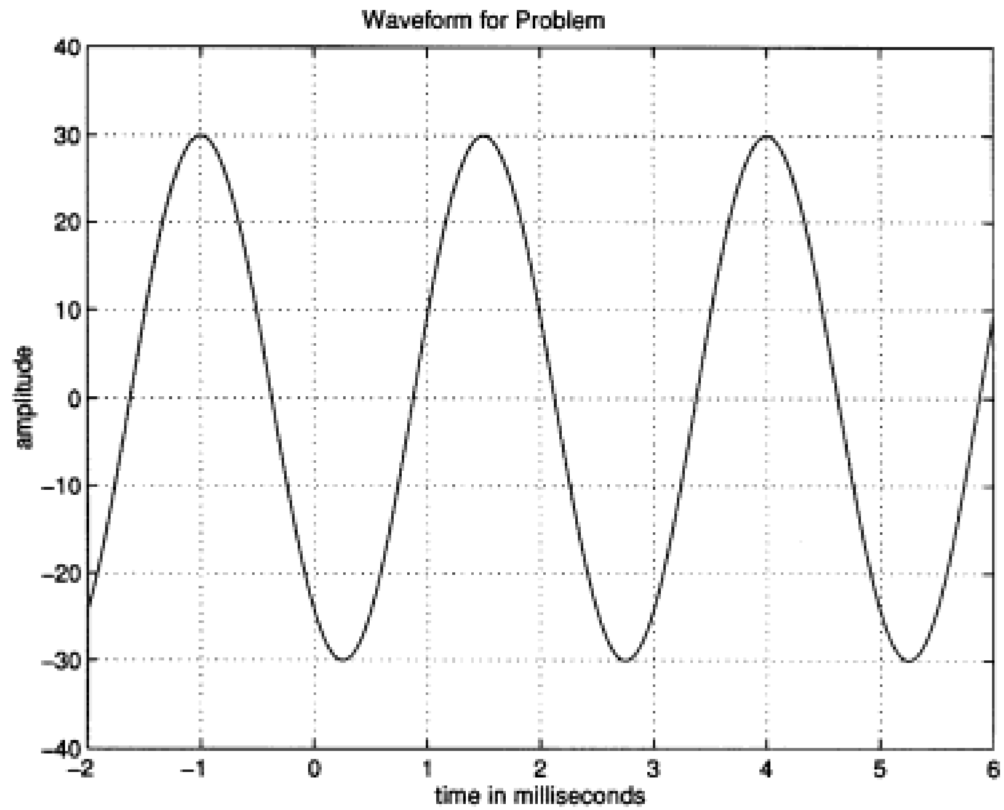
$$x(t) = 10 + 14\cos(200\pi t - \pi/3) + 8\cos(500\pi t + \pi/2)$$

$$= 10 + 7e^{-j\pi/3}e^{j2\pi(100)t} + 7e^{j\pi/3}e^{-j2\pi(100)t}$$

$$+ 4e^{j\pi/2}e^{j2\pi(250)t} + 4e^{-j\pi/2}e^{-j2\pi(250)t}$$



HW1-1



(a) The above figure shows a plot of a sinusoidal wave. From the plot, determine the values of A, ω_0, and $-\pi < \phi \leq \pi$ in the representation

$$x(t) = A\cos(\omega_0 t + \phi)$$

Where appropriate, be sure to indicate the units of the sinusoidal signal parameters.

(b) Determine a complex signal

$$z(t) = Ze^{j\omega_0 t}$$

such that $x(t) = \Re e\{z(t)\}$.

2.30

HW1-2

It is possible to rewrite the sinusoidal signal $x(t) = A\cos(\omega_\circ t + \phi)$ in the form:

$$x(t) = A\cos(\omega_\circ(t - t_1)) \tag{1}$$

(a) Determine a formula that gives the relationship between ϕ and t_1.

(b) When $x(t) = \sin(11\pi t)$ determine the value of t_1 that would be needed in the representation of equation (1).

(c) Prove that a peak of the cosine wave will always be at $t = t_1$.

2.68

HW1-3

A signal composed of sinusoids is given by the equation

$$x(t) = 20\cos(400\pi t - \pi/4) + 5\sin(800\pi t) - 6\cos(1200\pi t)$$

(a) Sketch the spectrum of this signal indicating the complex size of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex phasor value at the appropriate frequency.

(b) Is $x(t)$ periodic? If so, what is the period?

(c) Now consider a new signal $y(t) = x(t) + 10\cos(600\pi t + \pi/6)$. How is the spectrum changed? Is $y(t)$ still periodic? If so, what is the period?

(d) Finally, consider another new signal $w(t) = x(t) + 10\cos(1800t + \pi/6)$. How is the spectrum changed? Is $w(t)$ still periodic? If so, what is the period?

3.9

References

1. DSP First – A Multimedia Approach