



OFDM(2)

Matrix Based Simulation

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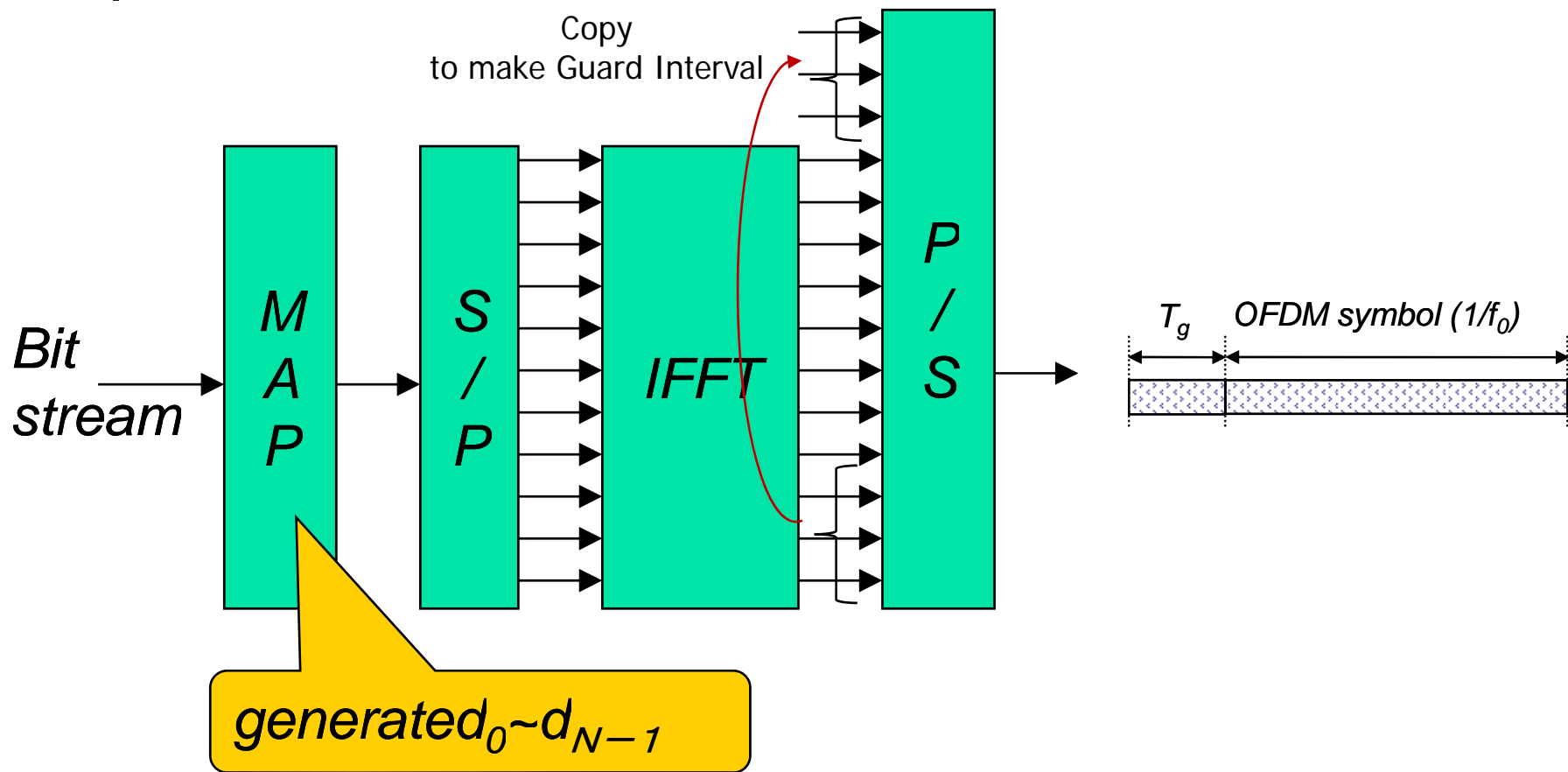
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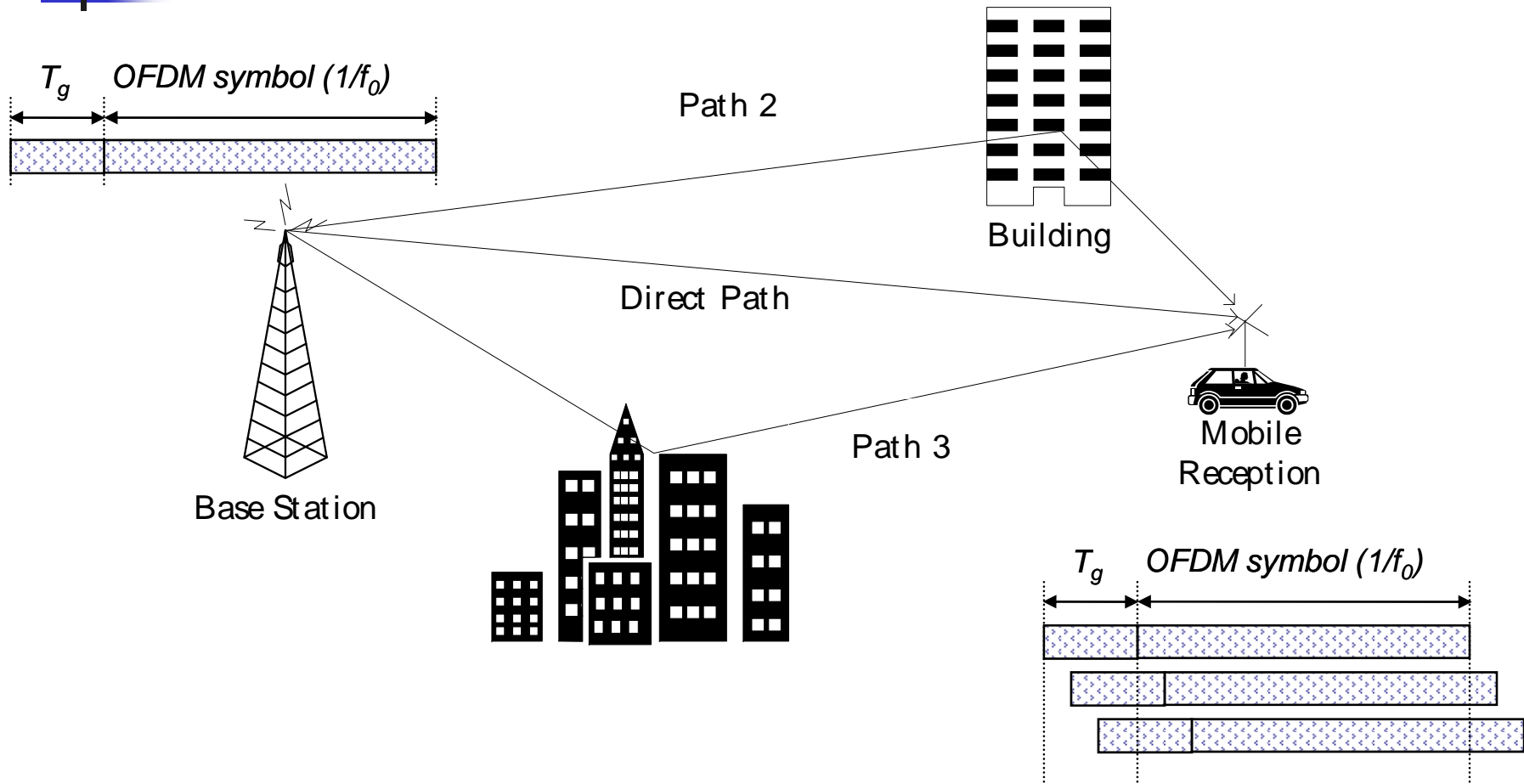
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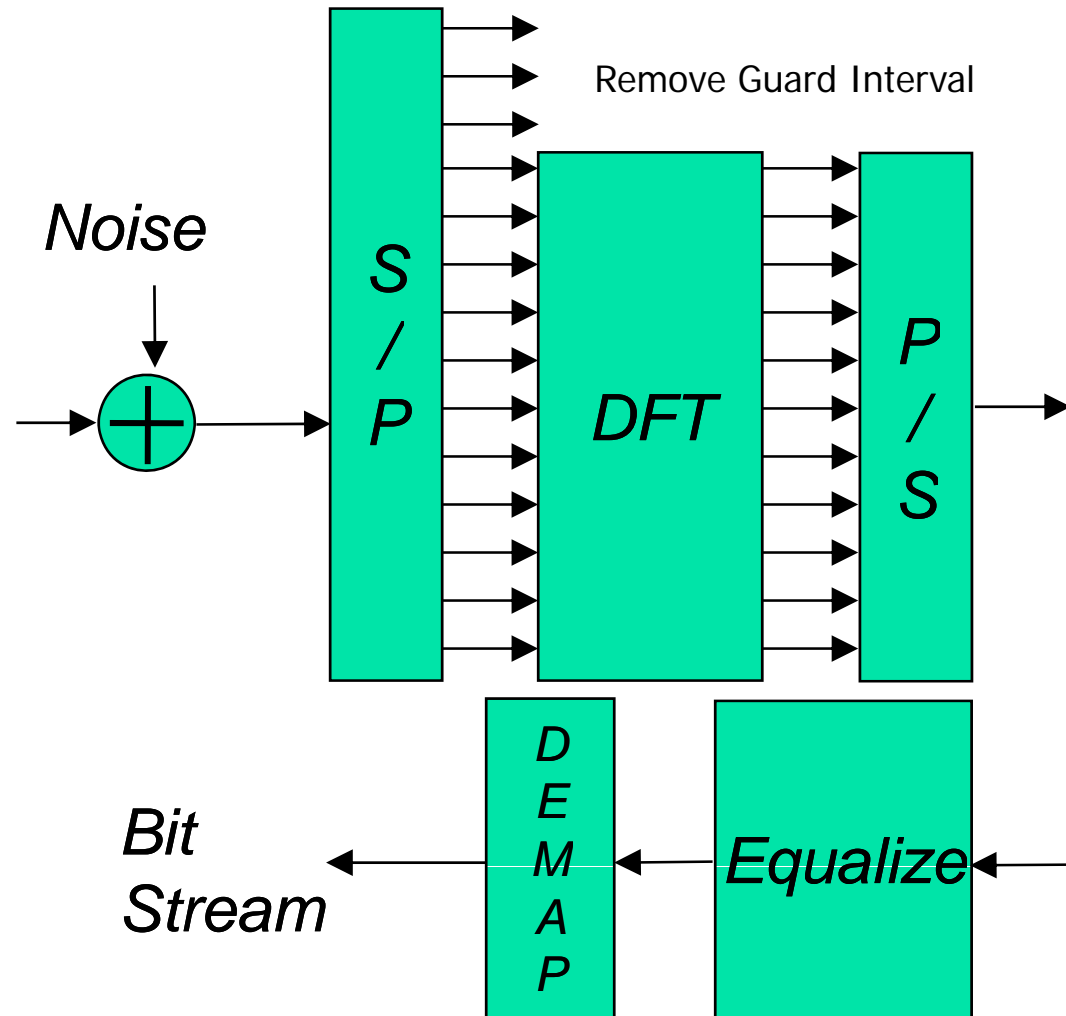
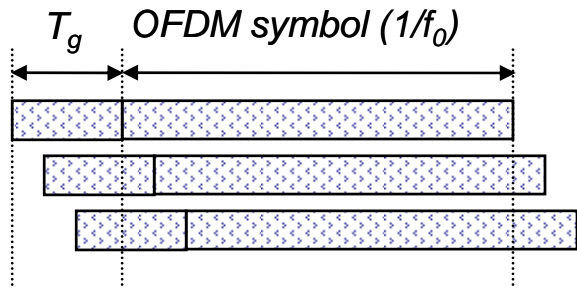
OFDM Modulator



Multi-path channel



OFDM Demodulator





FFT matrix

$$\begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix} = FFT \begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix} = \frac{1}{\sqrt{M}} \left[\omega^{-(k-1)*(l-1)}; k - \text{row}, l - \text{column} \right]$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix} = \frac{1}{\sqrt{M}} \begin{pmatrix} \omega^0 & \omega^0 & \dots & \omega^0 \\ \omega^0 & \omega^{-1} & \dots & \omega^{-(M-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^0 & \omega^{-(M-1)} & \dots & \omega^{-(M-1)*(M-1)} \end{pmatrix} \begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix}$$

$$\text{Here, } \omega = e^{j\frac{2\pi}{M}}$$



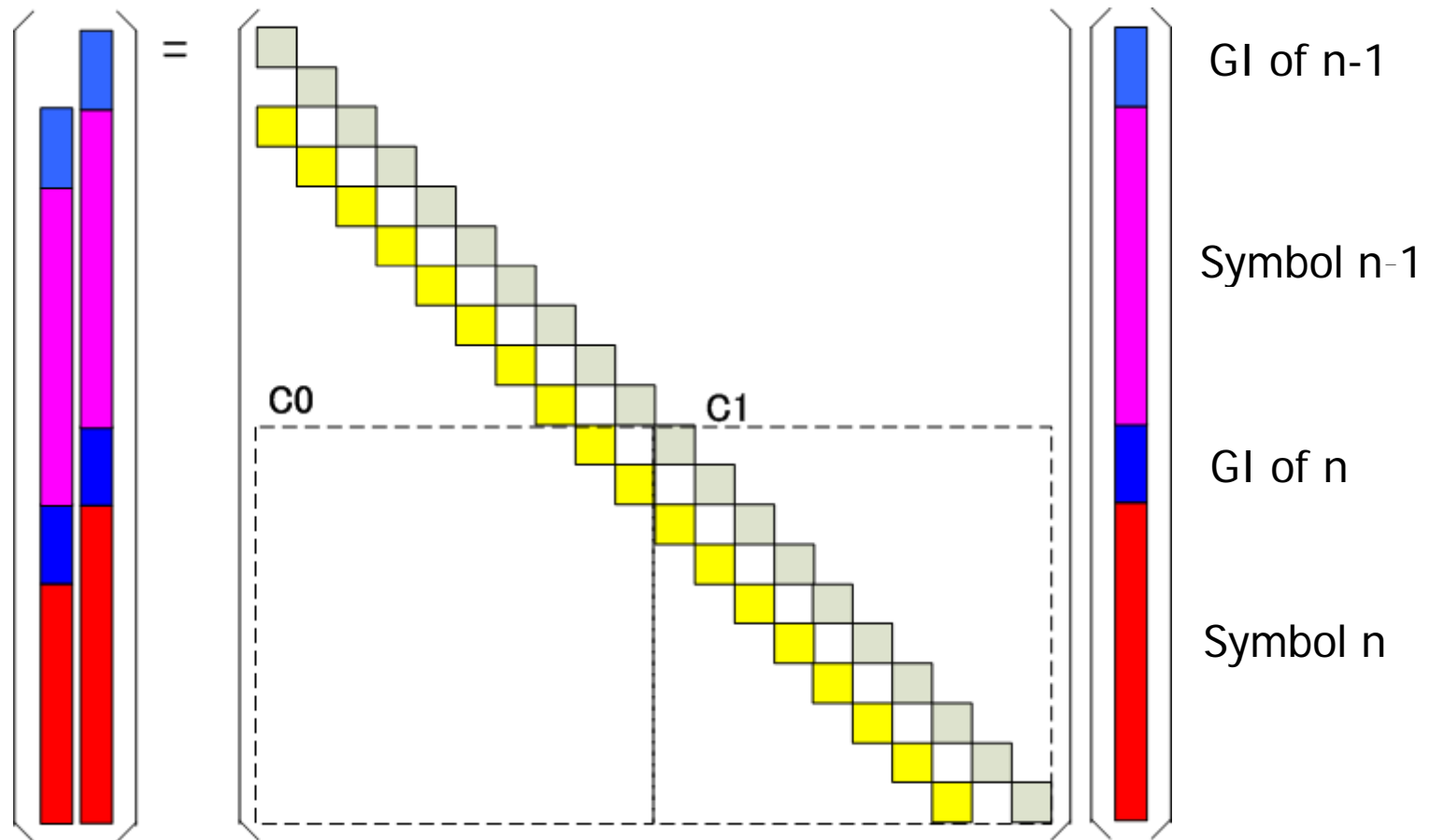
IFFT matrix

$$\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix} = \text{IFFT} \begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix} = \frac{1}{\sqrt{M}} \left[\omega^{(k-1)*(l-1)}; k - \text{row}, l - \text{column} \right]$$

$$\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix} = \frac{1}{\sqrt{M}} \begin{pmatrix} \omega^0 & \omega^0 & \dots & \omega^0 \\ \omega^0 & \omega^1 & \dots & \omega^{(M-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^0 & \omega^{(M-1)} & \dots & \omega^{(M-1)*(M-1)} \end{pmatrix} \begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix}$$

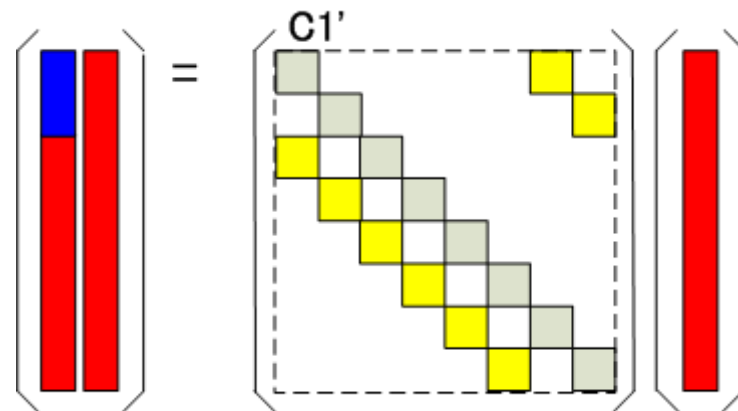
Here, $\omega = e^{j\frac{2\pi}{M}}$

Multi-path channel in Matrix



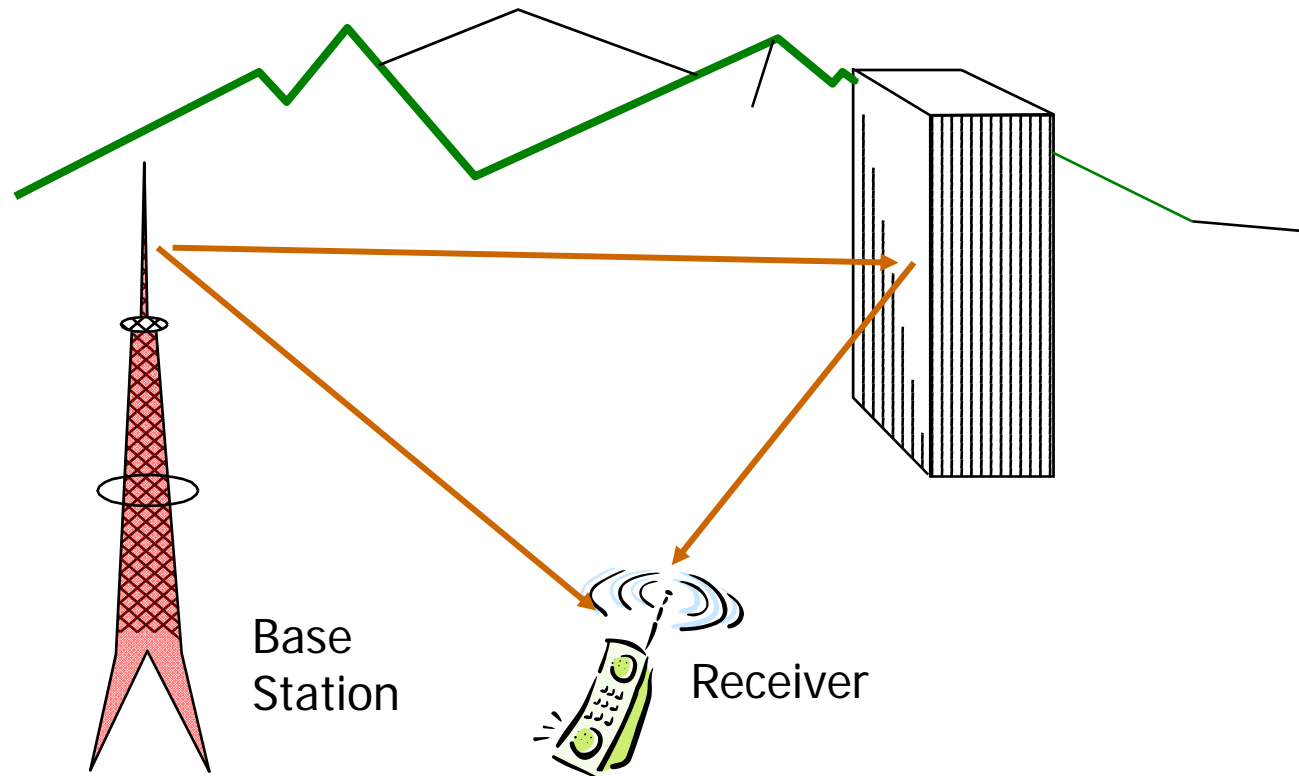


If Multi-path delay is small than GI length



- Channel Matrix is Cyclic Matrix!

Two path Multi path Channel Example



Channel Impulse Response = $[1, 0.5, 0, 0]$

Two path Multi path Channel Example

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = FFT * Channel * IFFT * \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix}$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \omega^{-3} \\ 1 & \omega^{-2} & \omega^{-4} & \omega^{-6} \\ 1 & \omega^{-3} & \omega^{-6} & \omega^{-9} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0.5 \\ 0.5 & 1 & 0 & 0 \\ 0 & 0.5 & 1 & 0 \\ 0 & 0 & 0.5 & 1 \end{pmatrix} \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix}$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = \begin{pmatrix} H(0) & 0 & 0 & 0 \\ 0 & H(1) & 0 & 0 \\ 0 & 0 & H(2) & 0 \\ 0 & 0 & 0 & H(3) \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix}$$

If time domain channel matrix is cyclic, Frequency Domain Channel Matrix is diagonal!



Additive Noise

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = FFT * \left[Channel * IFFT * \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} + \begin{pmatrix} noise(0) \\ noise(1) \\ noise(2) \\ noise(3) \end{pmatrix} \right]$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \omega^{-3} \\ 1 & \omega^{-2} & \omega^{-4} & \omega^{-6} \\ 1 & \omega^{-3} & \omega^{-6} & \omega^{-9} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0.5 \\ 0.5 & 1 & 0 & 0 \\ 0 & 0.5 & 1 & 0 \\ 0 & 0 & 0.5 & 1 \end{pmatrix} \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^1 & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} + \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \omega^{-3} \\ 1 & \omega^{-2} & \omega^{-4} & \omega^{-6} \\ 1 & \omega^{-3} & \omega^{-6} & \omega^{-9} \end{pmatrix} \begin{pmatrix} noise(0) \\ noise(1) \\ noise(2) \\ noise(3) \end{pmatrix}$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = \begin{pmatrix} H(0) & 0 & 0 & 0 \\ 0 & H(1) & 0 & 0 \\ 0 & 0 & H(2) & 0 \\ 0 & 0 & 0 & H(3) \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} + \begin{pmatrix} N(0) \\ N(1) \\ N(2) \\ N(3) \end{pmatrix}$$



How to recover sending signal from receiver signal. - EQUALIZE -

Ignore Noise

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = FFT * Channel * IFFT * \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} = \begin{pmatrix} H(0) & 0 & 0 & 0 \\ 0 & H(1) & 0 & 0 \\ 0 & 0 & H(2) & 0 \\ 0 & 0 & 0 & H(3) \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix}$$

Then

$$\begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} = \begin{pmatrix} \frac{1}{H(0)} & 0 & 0 & 0 \\ 0 & \frac{1}{H(1)} & 0 & 0 \\ 0 & 0 & \frac{1}{H(2)} & 0 \\ 0 & 0 & 0 & \frac{1}{H(3)} \end{pmatrix} \begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix}$$



SysArch2008 Midterm Report

- Modify SCILAB program "MatrixOFDMSimulation1.sce" to measure Symbol Error Rate vs S/N ratio in M=16 OFDM with QPSK modulation
- Make Symbol Error Rate vs SN ratio
 - Vertical: SER in log scale
 - Horizontal: SN ratio 0dB, 1dB ... to 15dB
- Your report should contain your program and measured data in Graph.
- Dead Line : December End 2008
- Please submit to TA: Kenta-san
 - kentan@lsi.ie.u-ryukyu.ac.jp