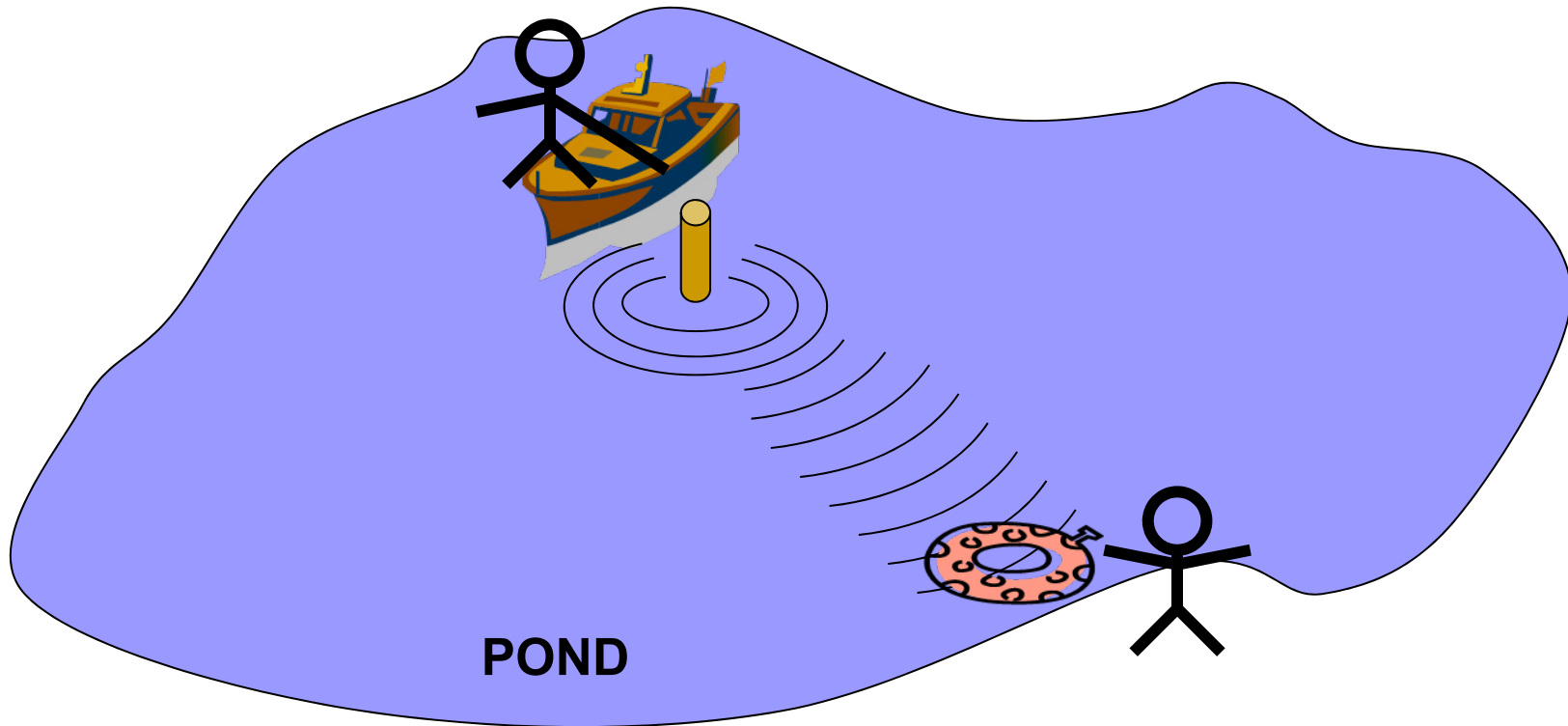


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# OUTLINE

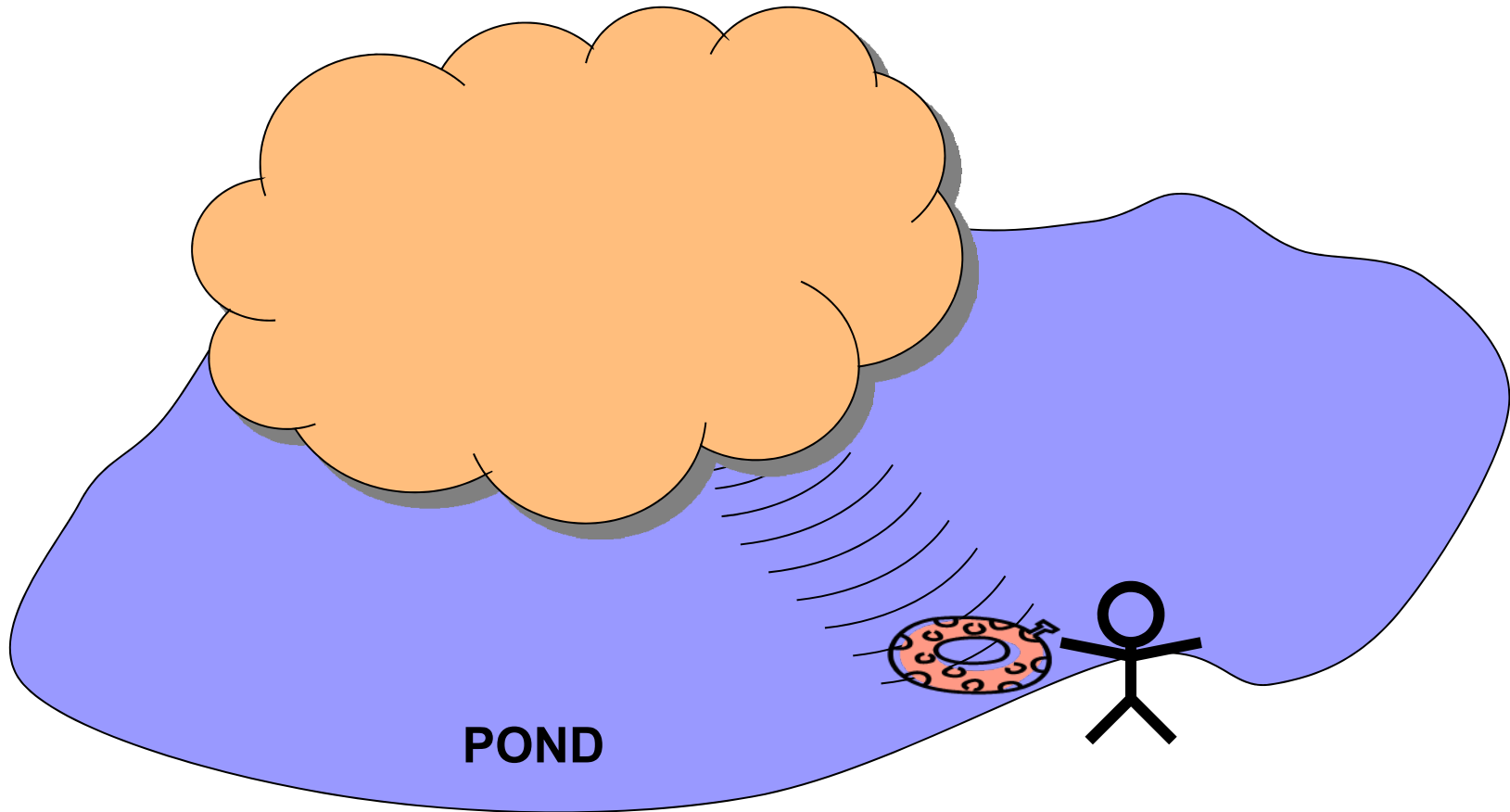
- Wave
- Sinusoidal signal review
- Complex exponential signal
- Complex amplitude (phasor)
- What is Spectrum

## 2 D wave



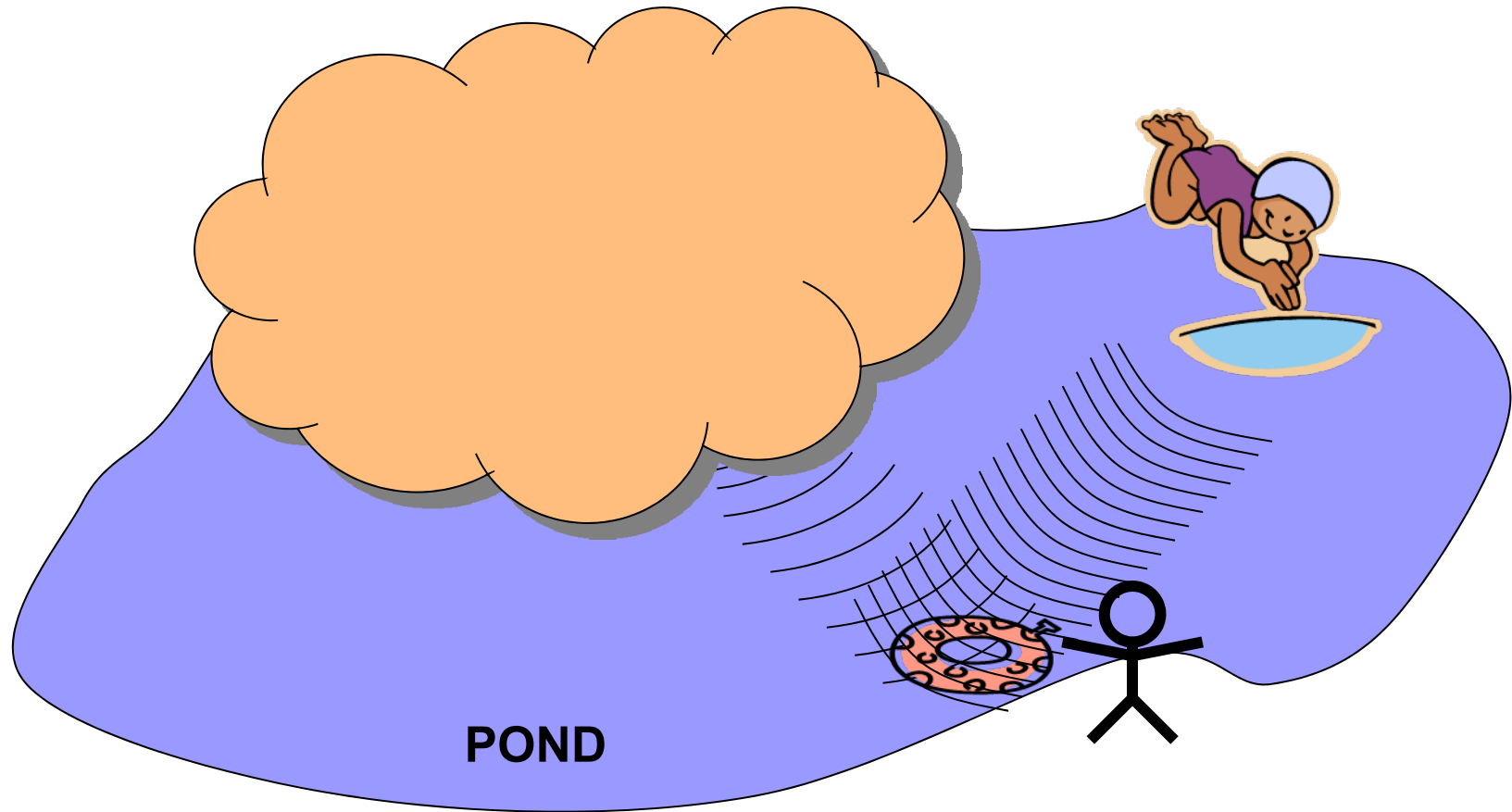
**BOAT** person can send message by making water wave.  
**LAND** person can understand message by the movement of the float.

# Even with FOG



**LAND person just observe ONLY the float.**

# Even with NOISE

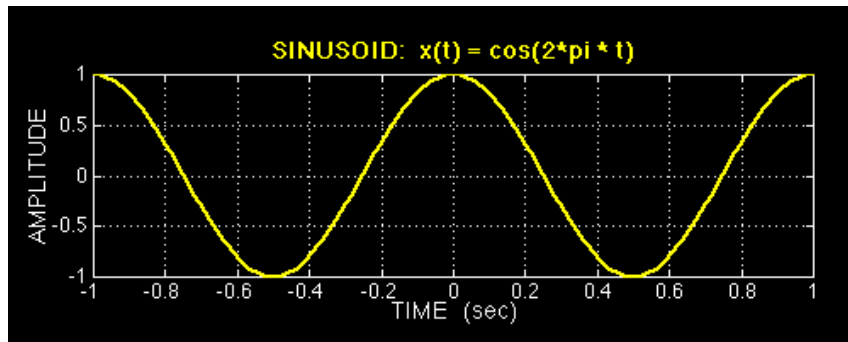


**If main signal is enough stronger than the interference (noise),  
LAND person can observe the message.**

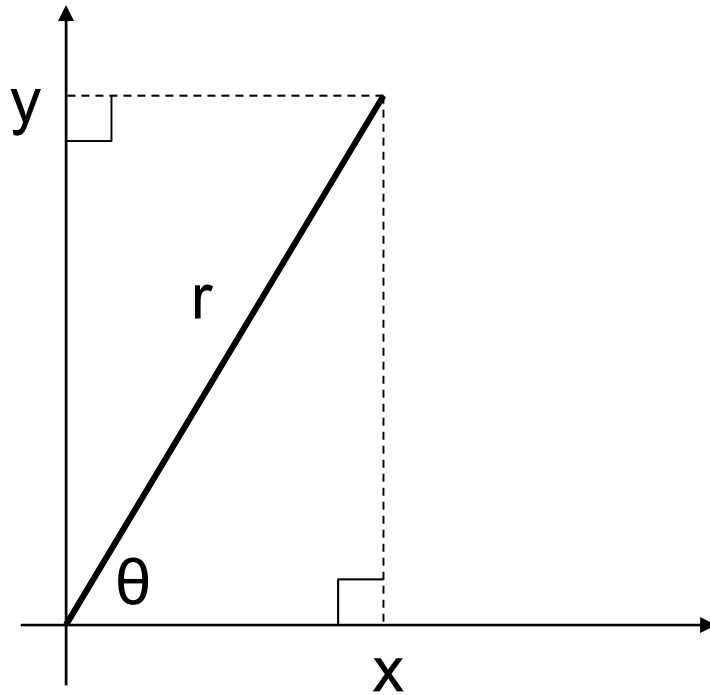
# Typical wave “Sinusoids”

$$x(t) = A \cos(\omega_0 t + \phi)$$

- $t$  : time
- $A$  : amplitude
- $\omega_0$  : radian frequency
- $\Phi$ : phase-shift

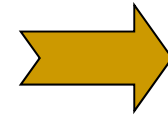


# Review of cosine and sine



$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$



$$y = r \sin \theta$$

$$x = r \cos \theta$$

# Period $T_0$ of sinusoids

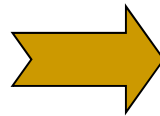
$$x(t + T_0) = x(t)$$

$$A \cos(\omega_0(t + T_0) + \phi) = A \cos(\omega_0 t + \phi)$$

$$\cos(\omega_0 t + \omega_0 T_0 + \phi) = \cos(\omega_0 t + \phi)$$

$$\omega_0 T_0 = 2\pi$$

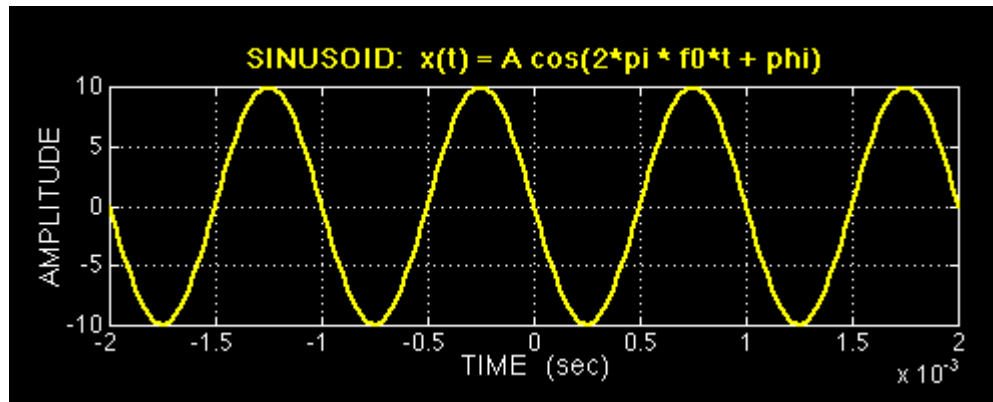
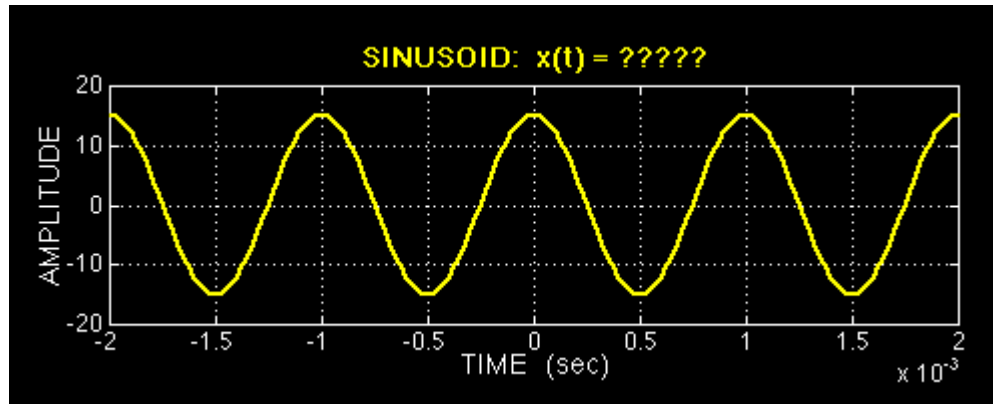
$$(2\pi f_0) T_0 = 2\pi$$



$$T_0 = \frac{2\pi}{\omega_0}$$

$$T_0 = \frac{1}{f_0}$$

# QUIZ – what is the equation?-





# Review of complex

## ■ Cartesian Form

□ Complex  $z=(x, y)$

□  $x =$  Real part of  $z$  :

□  $y =$  Imaginary part of  $z$  :

□  $z = x + j y$

## ■ Polar Form

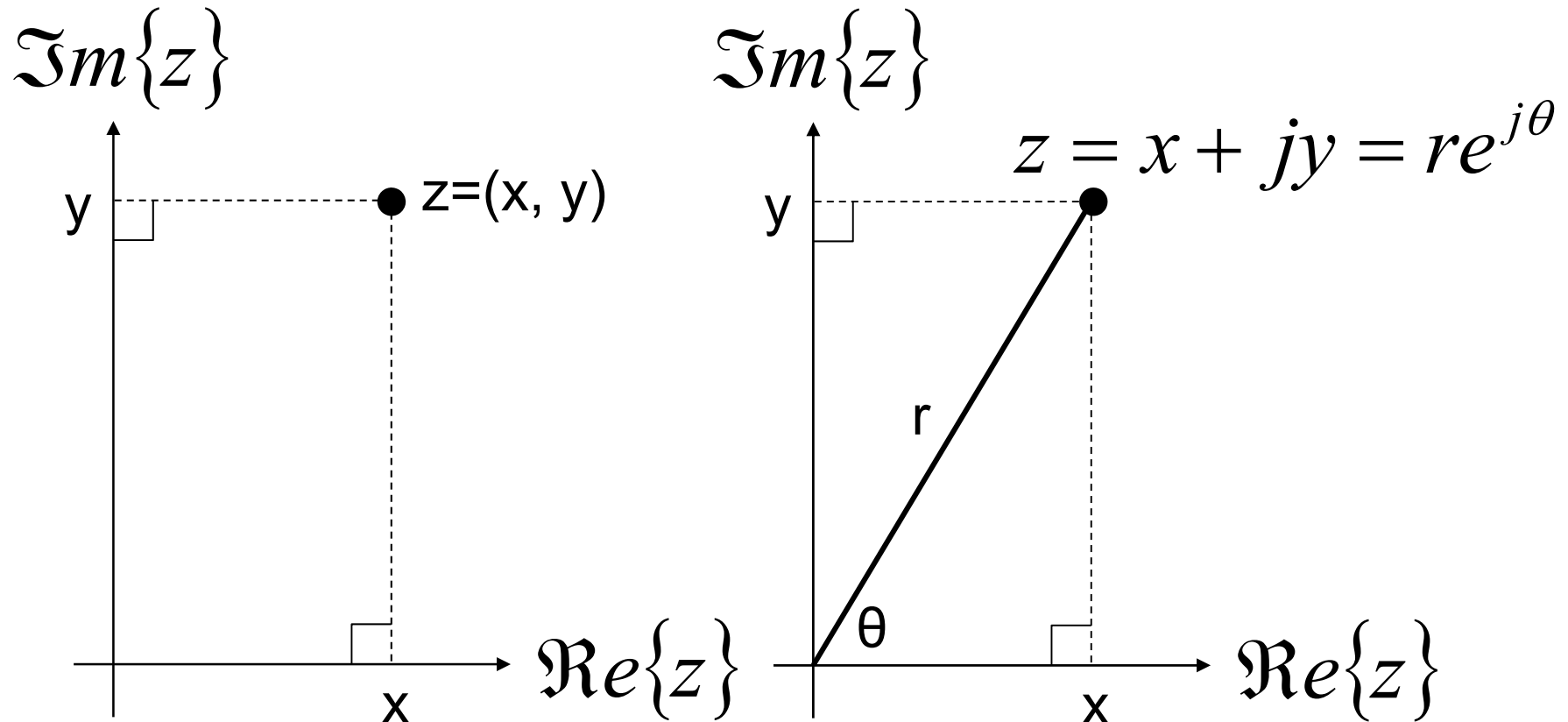
□ Complex  $z=(r, \theta)$

$$x = \Re\{z\}$$

$$y = \Im\{z\}$$

$$j = \sqrt{-1}$$

# Cartesian Form and Polar Form



# Transform

- Polar to Cartesian

$$x = r \cos \theta$$

$$y = r \sin \theta$$

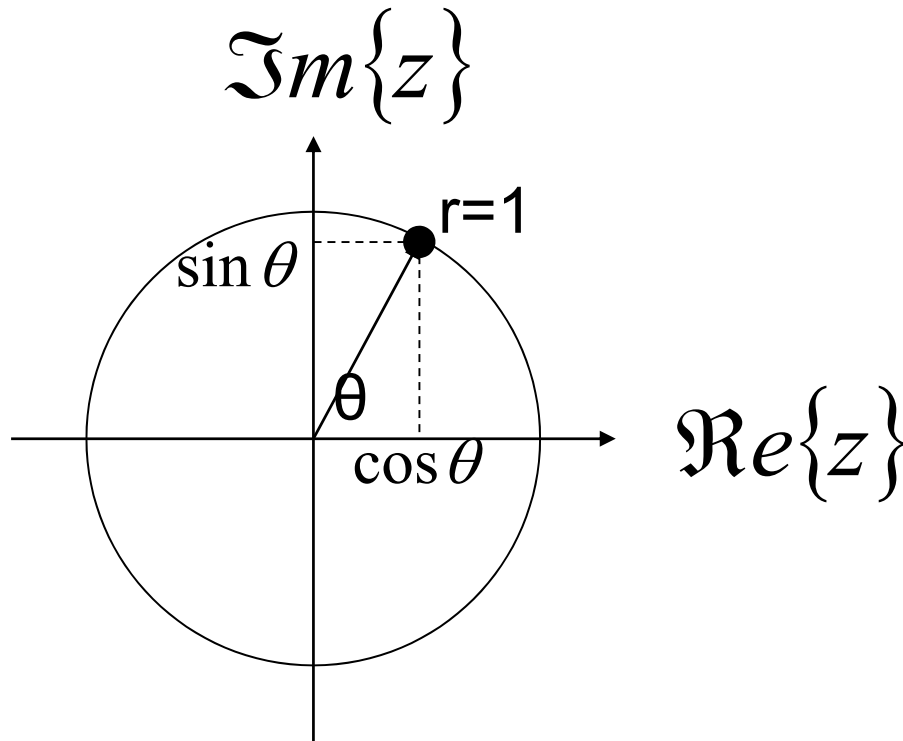
- Cartesian to Polar

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$

# Euler's law

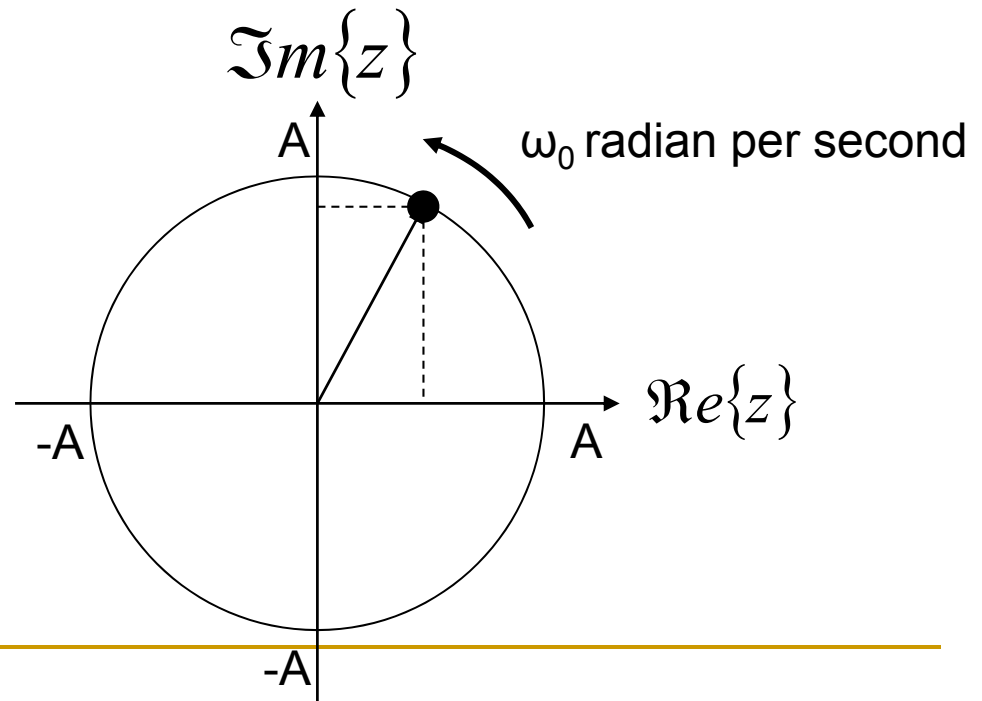
$$e^{j\theta} = \cos \theta + j \sin \theta$$



# Complex exponential signal (Rotation Function)

$$\begin{aligned}\tilde{x}(t) &= Ae^{j(\omega_0 t + \phi)} \\ &= A \cos(\omega_0 t + \phi) + jA \sin(\omega_0 t + \phi)\end{aligned}$$

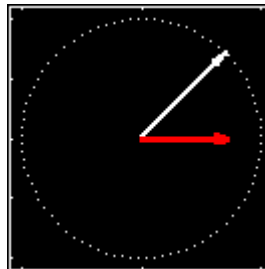
- Amplitude =  $A$
- $\Theta = \omega_0 t + \Phi$
- $\Phi$ : phase-shift
- $\omega_0$ : rotation speed
  - + : counterclockwise
  - - : clockwise



The complex exponential signal is another representation of cosine signal

$$x(t) = \Re\{Ae^{j(\omega_0 t + \phi)}\} = A \cos(\omega_0 t + \phi)$$

- Re means a Projection to X-axis



# Inverse Euler's law

$$\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$A \cos(\omega_0 t + \phi) = A \left\{ \frac{e^{j(\omega_0 t + \phi)} + e^{-j(\omega_0 t + \phi)}}{2} \right\}$$

$$= \frac{\tilde{x}(t) + \tilde{x}^*(t)}{2} = \Re\{\tilde{x}(t)\}$$



**X\* is conjugate of X.**

# Inverse Euler's law (2)

$$\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

- Real signal is the combination of + and - frequency complex exponential signal.

$$A \cos(\omega_0 t + \phi) = A \left\{ \frac{e^{j(\omega_0 t + \phi)} + e^{-j(\omega_0 t + \phi)}}{2} \right\}$$



# Complex amplitude (Phasor)

$$x(t) = \Re\{Ae^{j(\omega_0 t + \phi)}\} = A \cos(\omega_0 t + \phi)$$

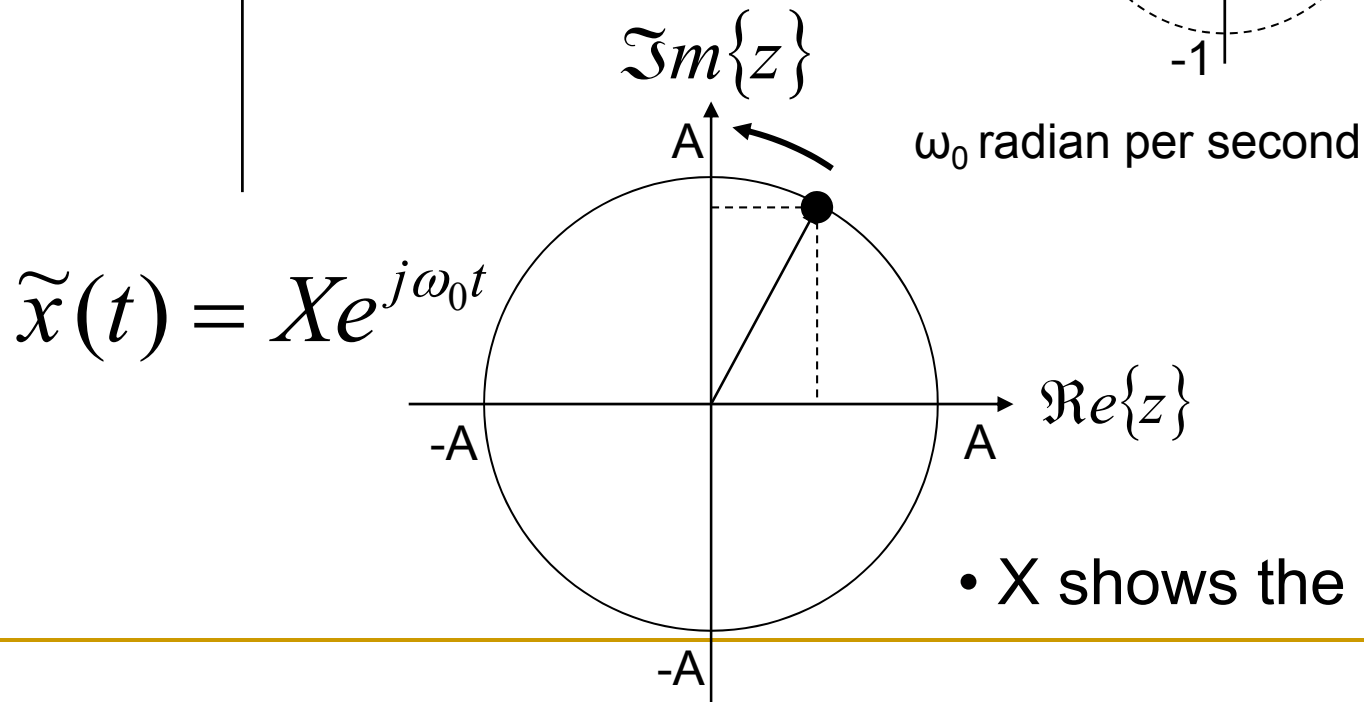
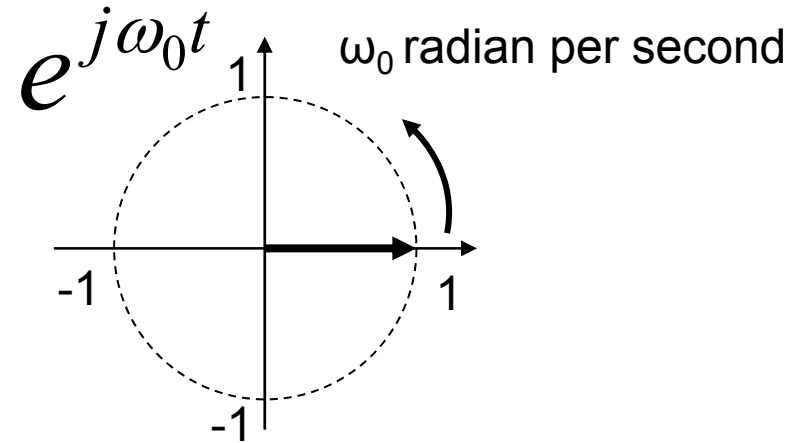
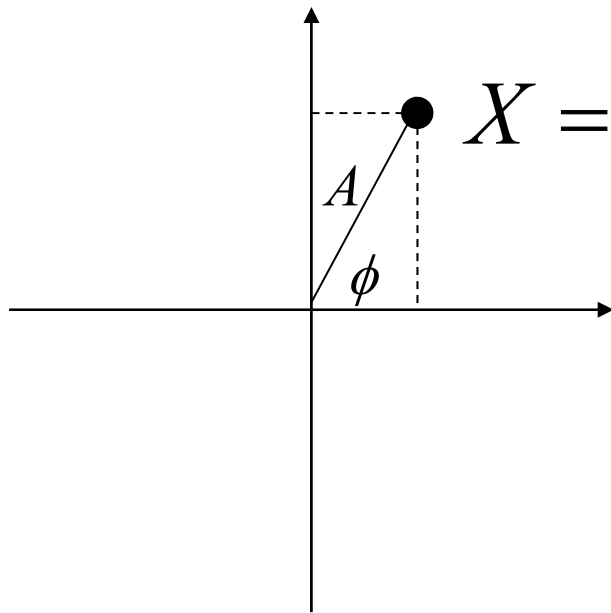
$$\tilde{x}(t) = Ae^{j(\omega_0 t + \phi)} = Ae^{j\phi} \cdot e^{j\omega_0 t}$$

$$X = Ae^{j\phi}$$

$$\tilde{x}(t) = Xe^{j\omega_0 t}$$

- $X$  is called as Complex Amplitude or Phasor.

# Complex amplitude (Phasor) -2-



- $X$  shows the  $t=0$  position.

# Why Phasor is useful?

1. If you want to add two waves....

$$x_1(t) = 1.7 \cos(2\pi(10)t + 70\pi / 180)$$

$$x_2(t) = 1.9 \cos(2\pi(10)t + 200\pi / 180)$$

2. Make the complex exponential signal

$$x_1(t) = \Re[1.7e^{j70\pi/180} \cdot e^{j2\pi(10)t}]$$

$$x_2(t) = \Re[1.9e^{j200\pi/180} \cdot e^{j2\pi(10)t}]$$

# Why Phasor is useful? (2)

3. Make the Phasor...

$$X_1 = 1.7e^{j70\pi/180} = 0.5814 + j1.597$$

$$X_2 = 1.9e^{j200\pi/180} = -1.785 - j0.6498$$

4. Add them...

$$X_3 = X_1 + X_2$$

$$= 0.5814 + j1.597 + (-1.785 - j0.6498)$$

$$= -1.204 + j0.9476$$

$$= 1.532e^{j141.79\pi/180}$$

# Why Phasor is useful? (3)

## 5. Result

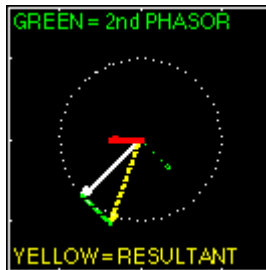
$$X_3 = 1.532e^{j141.79\pi/180}$$

$$x_3(t) = \Re[1.532e^{j141.79\pi/180} \cdot e^{j2\pi(10)t}]$$

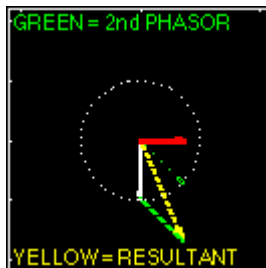
$$= 1.532 \cos(2\pi(10)t + 141.79\pi / 180)$$

$$= 1.532 \cos(2\pi(10)(t + 0.0394))$$

# 2 different frequency composition



- 2 apart frequencies



- 2 close frequencies
- Beat

# Sum of N cosine waves

$$x(t) = A_0 + \sum_{k=1}^N A_k \cos(2\pi f_k t + \phi_k)$$

$$X_k = A_k e^{j\phi_k}$$

$$x(t) = X_0 + \sum_{k=1}^N \Re\{X_k e^{j2\pi f_k t}\}$$

$$= X_0 + \sum_{k=1}^N \left\{ \frac{X_k}{2} e^{j2\pi f_k t} + \frac{X_k^*}{2} e^{-j2\pi f_k t} \right\}$$

- This is sum of  $2N+1$  different frequency signals.

# What is Spectrum

- Spectrum is the set of phasors for each frequencies.

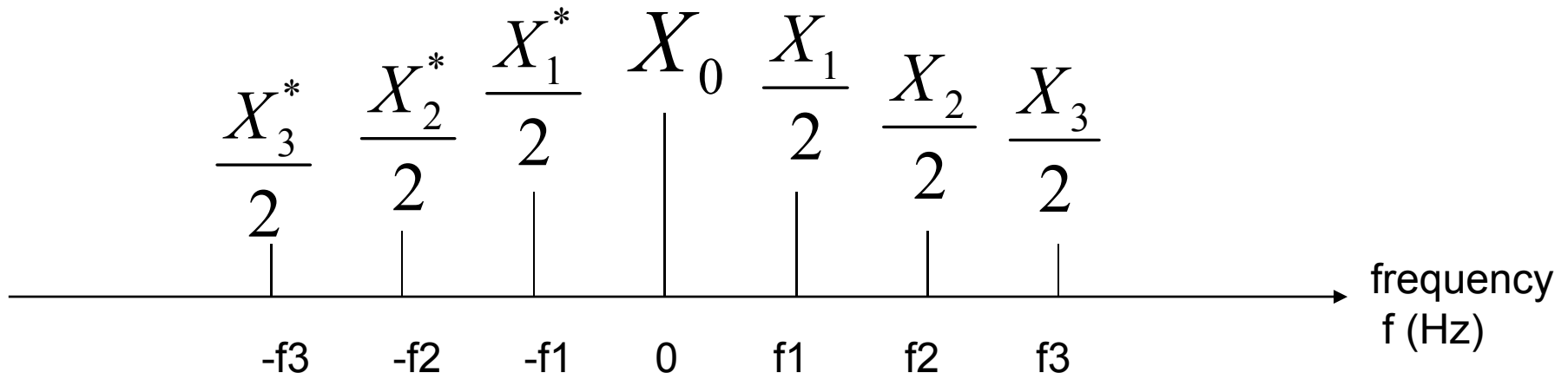
$$\left\{ (X_0, 0), \left( \frac{1}{2} X_1, f_1 \right), \left( \frac{1}{2} X_1^*, -f_1 \right), \left( \frac{1}{2} X_2, f_2 \right), \left( \frac{1}{2} X_2^*, -f_2 \right), \dots \right\}$$



# Spectrum plot

$$x(t) = X_0 + \sum_{k=1}^N \left\{ \frac{X_k}{2} e^{j2\pi f_k t} + \frac{X_k^*}{2} e^{-j2\pi f_k t} \right\}$$

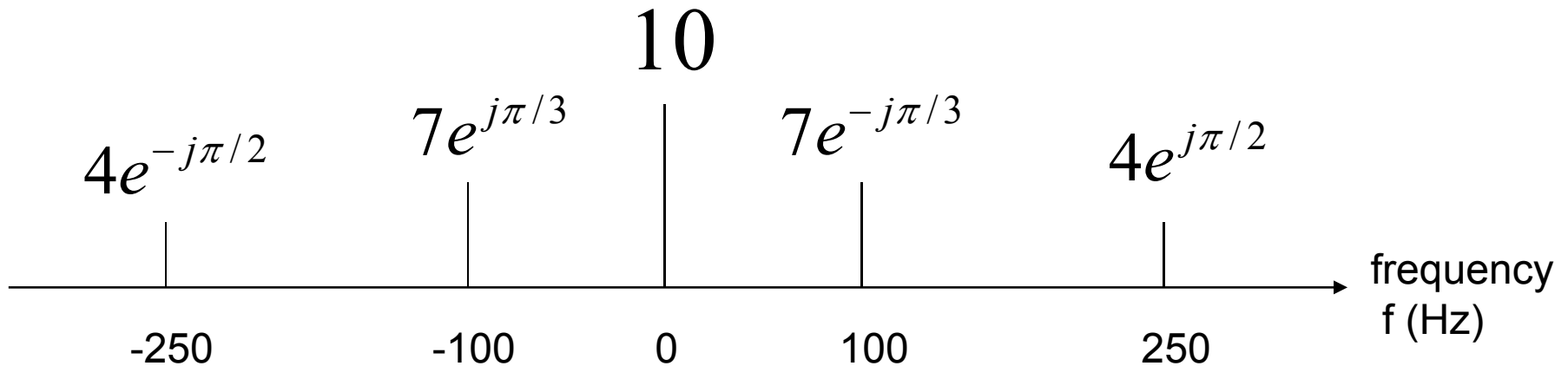
**N=3**



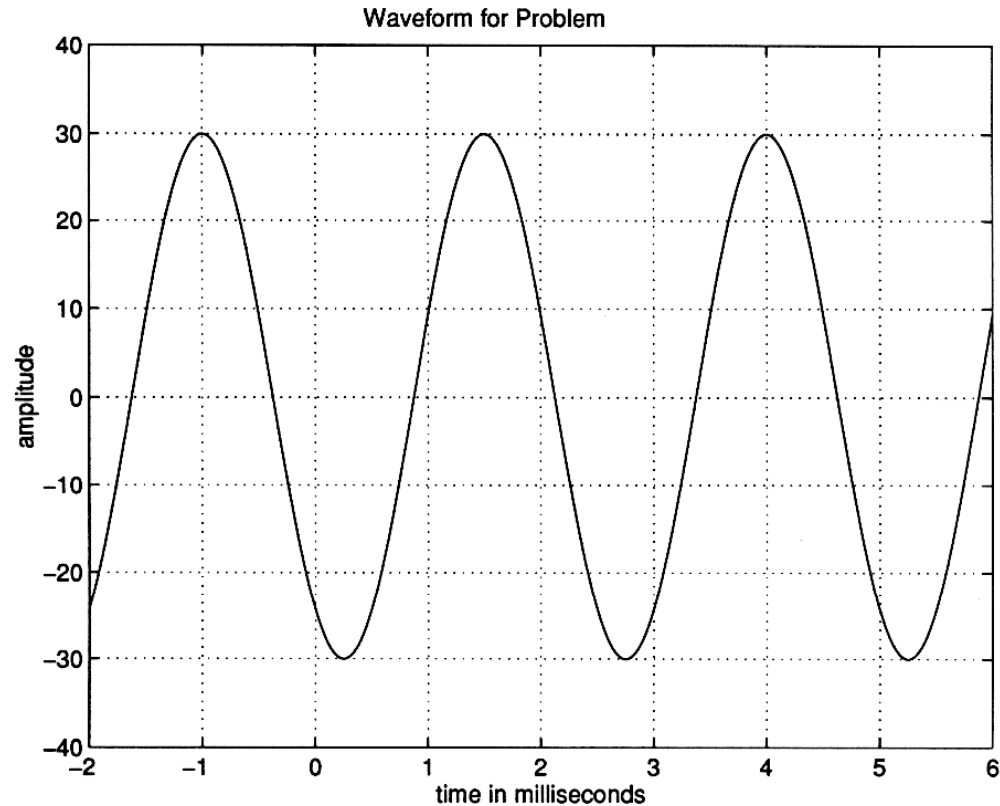
The line length is proportional to |phasor|.

# Example

$$\begin{aligned}x(t) &= 10 + 14 \cos(200\pi t - \pi / 3) + 8 \cos(500\pi t + \pi / 2) \\ &= 10 + 7e^{-j\pi/3} e^{j2\pi(100)t} + 7e^{j\pi/3} e^{-j2\pi(100)t} \\ &\quad + 4e^{j\pi/2} e^{j2\pi(250)t} + 4e^{-j\pi/2} e^{-j2\pi(250)t}\end{aligned}$$



# HW1-1



- (a) The above figure shows a plot of a sinusoidal wave. From the plot, determine the values of  $A$ ,  $\omega_0$ , and  $-\pi < \phi \leq \pi$  in the representation

$$x(t) = A \cos(\omega_0 t + \phi)$$

Where appropriate, be sure to indicate the units of the sinusoidal signal parameters.

- (b) Determine a complex signal

$$z(t) = Z e^{j\omega_0 t}$$

such that  $x(t) = \Re\{z(t)\}$ .

2.30

# HW1-2

It is possible to rewrite the sinusoidal signal  $x(t) = A \cos(\omega_o t + \phi)$  in the form:

$$x(t) = A \cos(\omega_o(t - t_1)) \quad (1)$$

- (a) Determine a formula that gives the relationship between  $\phi$  and  $t_1$ .
- (b) When  $x(t) = \sin(11\pi t)$  determine the value of  $t_1$  that would be needed in the representation of equation (1).
- (c) Prove that a peak of the cosine wave will always be at  $t = t_1$ .

2.68

# HW1-3

A signal composed of sinusoids is given by the equation

$$x(t) = 20 \cos(400\pi t - \pi/4) + 5 \sin(800\pi t) - 6 \cos(1200\pi t)$$

- (a) Sketch the spectrum of this signal indicating the complex size of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex phasor value at the appropriate frequency.
- (b) Is  $x(t)$  periodic? If so, what is the period?
- (c) Now consider a new signal  $y(t) = x(t) + 10 \cos(600\pi t + \pi/6)$ . How is the spectrum changed? Is  $y(t)$  still periodic? If so, what is the period?
- (d) Finally, consider another new signal  $w(t) = x(t) + 10 \cos(1800t + \pi/6)$ . How is the spectrum changed? Is  $w(t)$  still periodic? If so, what is the period?

## 3.9

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# HW submission

- Submit your Home Work to Ikenoya-san
- [ike@lsi.ie.u-ryukyu.ac.jp](mailto:ike@lsi.ie.u-ryukyu.ac.jp)
- Subject: SysArch09-HW1
- Show your Student ID and Your Name

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# References

1. DSP First – A Multimedia Approach