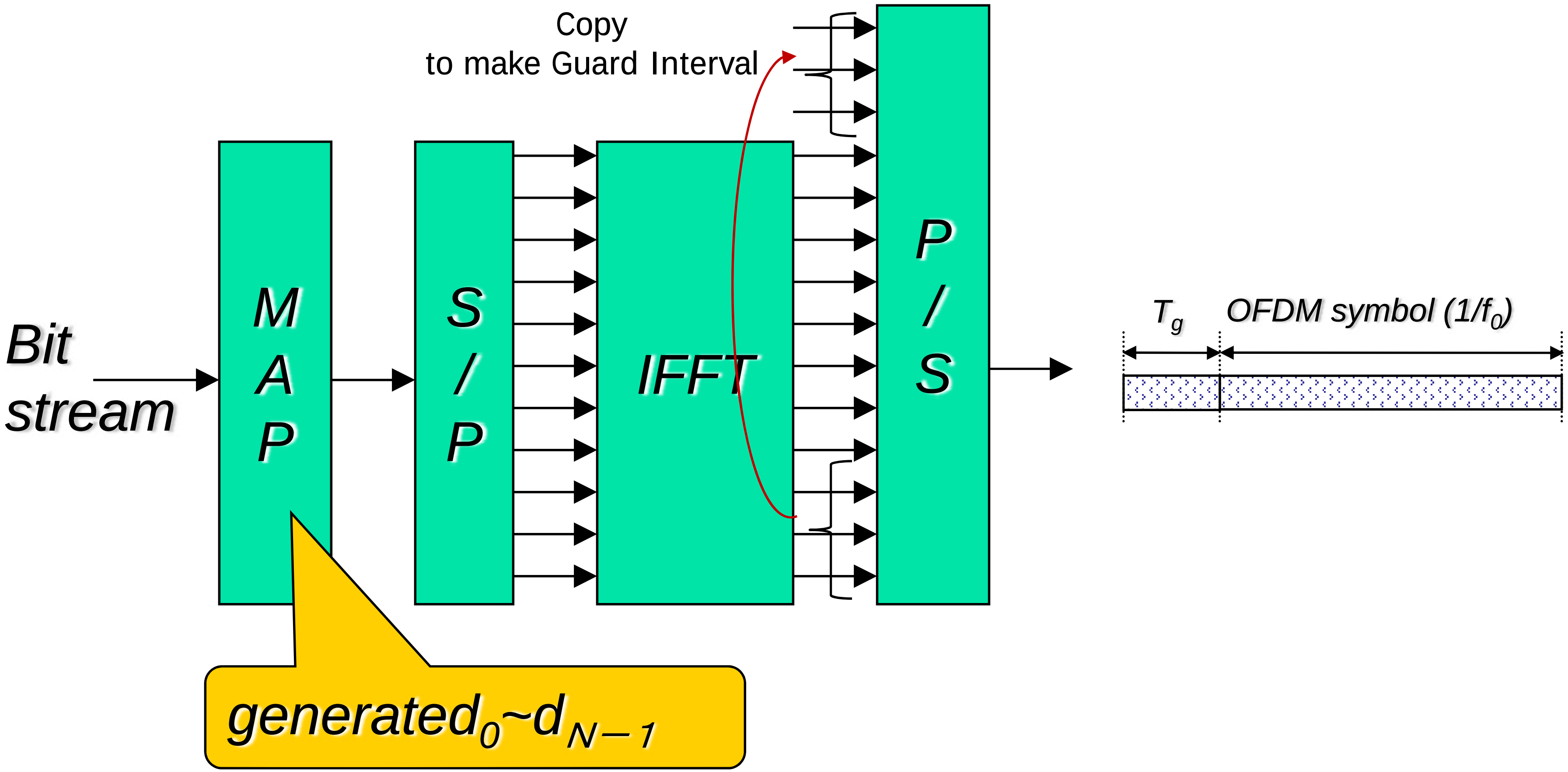
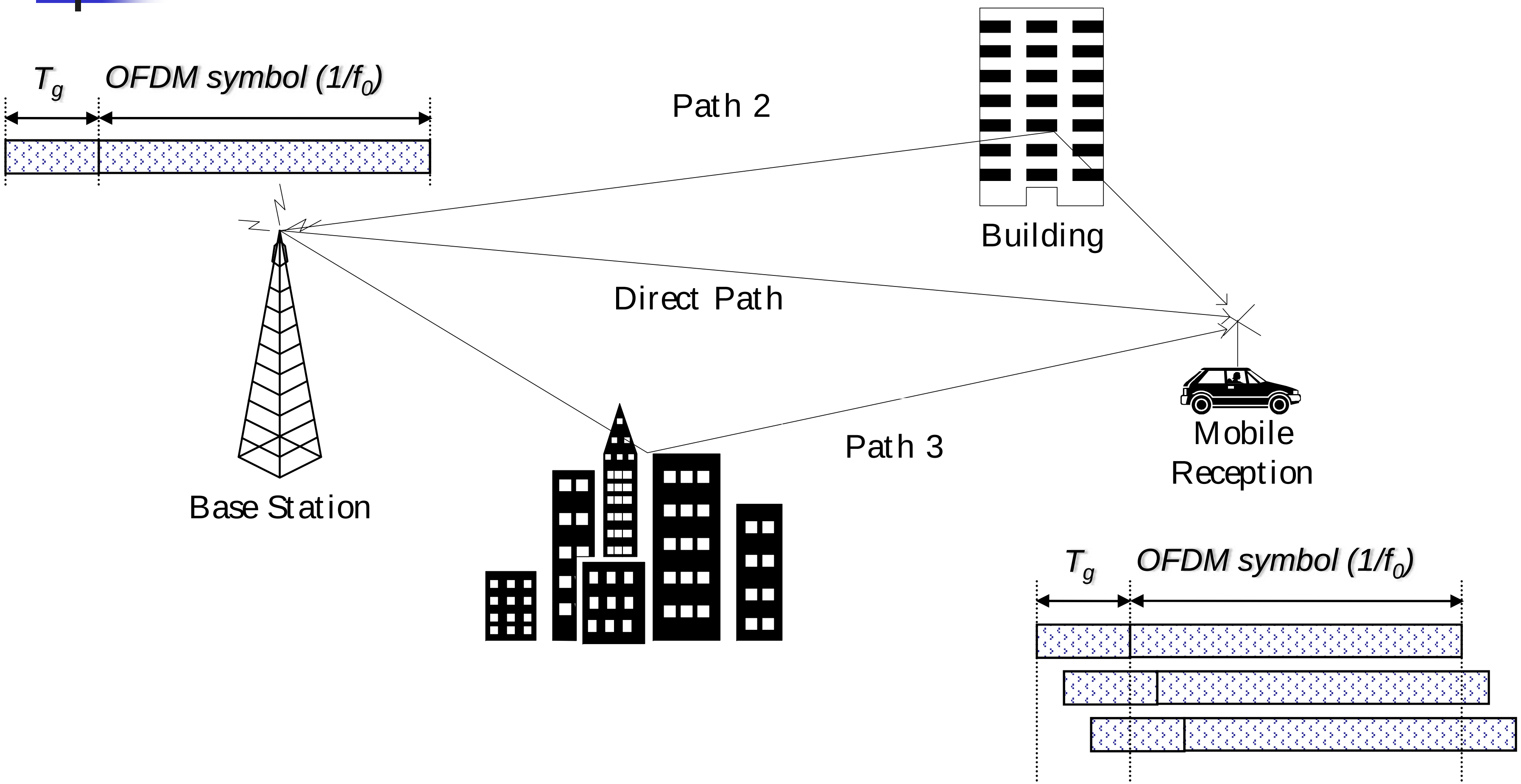
OFDM(2)
Matrix Based Simlation

Fire Tom Wada
Professor, Information Engineering, Univ. of the Ryukyus
Chief Scientist at Magna Design Net, Inc
wada@ie.u-ryukyu.ac.jp
http://www.ie.u-ryukyu.ac.jp/~wada/

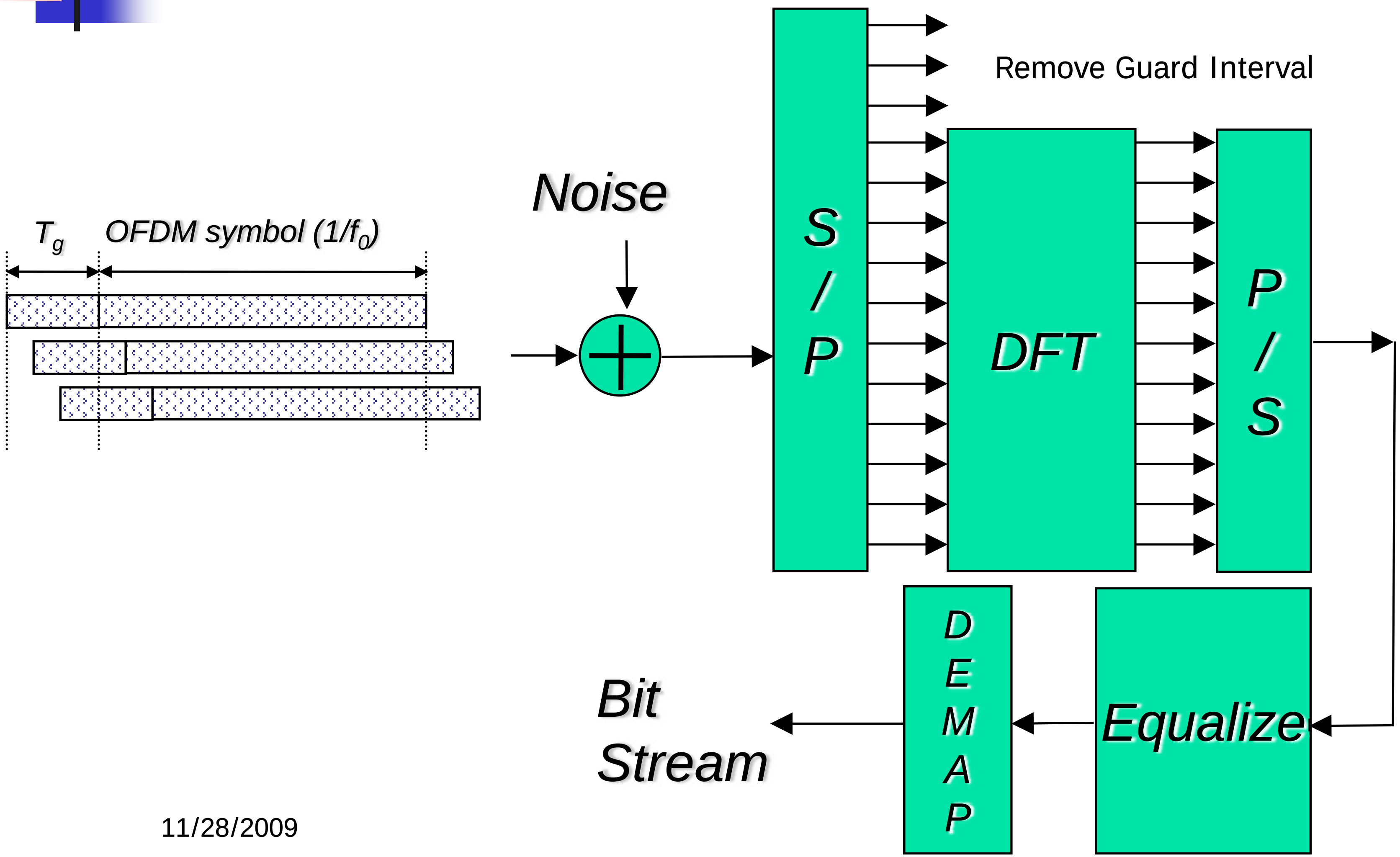
OFDM Modulator



Multi-path channel



OFDM Demodulator



FFT matrix

$$\begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix} = FFT\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix} = \frac{1}{\sqrt{M}}\left[\omega^{-(k-1)*(l-1)}; k-row, l-column\right]\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix}$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix} = \frac{1}{\sqrt{M}}\begin{pmatrix} \omega^0 & \omega^0 & \cdots & \omega^0 \\ \omega^0 & \omega^{-1} & \cdots & \omega^{-(M-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^0 & \omega^{-(M-1)} & \cdots & \omega^{-(M-1)*(M-1)} \end{pmatrix}\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix}$$

$$Here, \omega = e^{j\frac{2\pi}{M}}$$

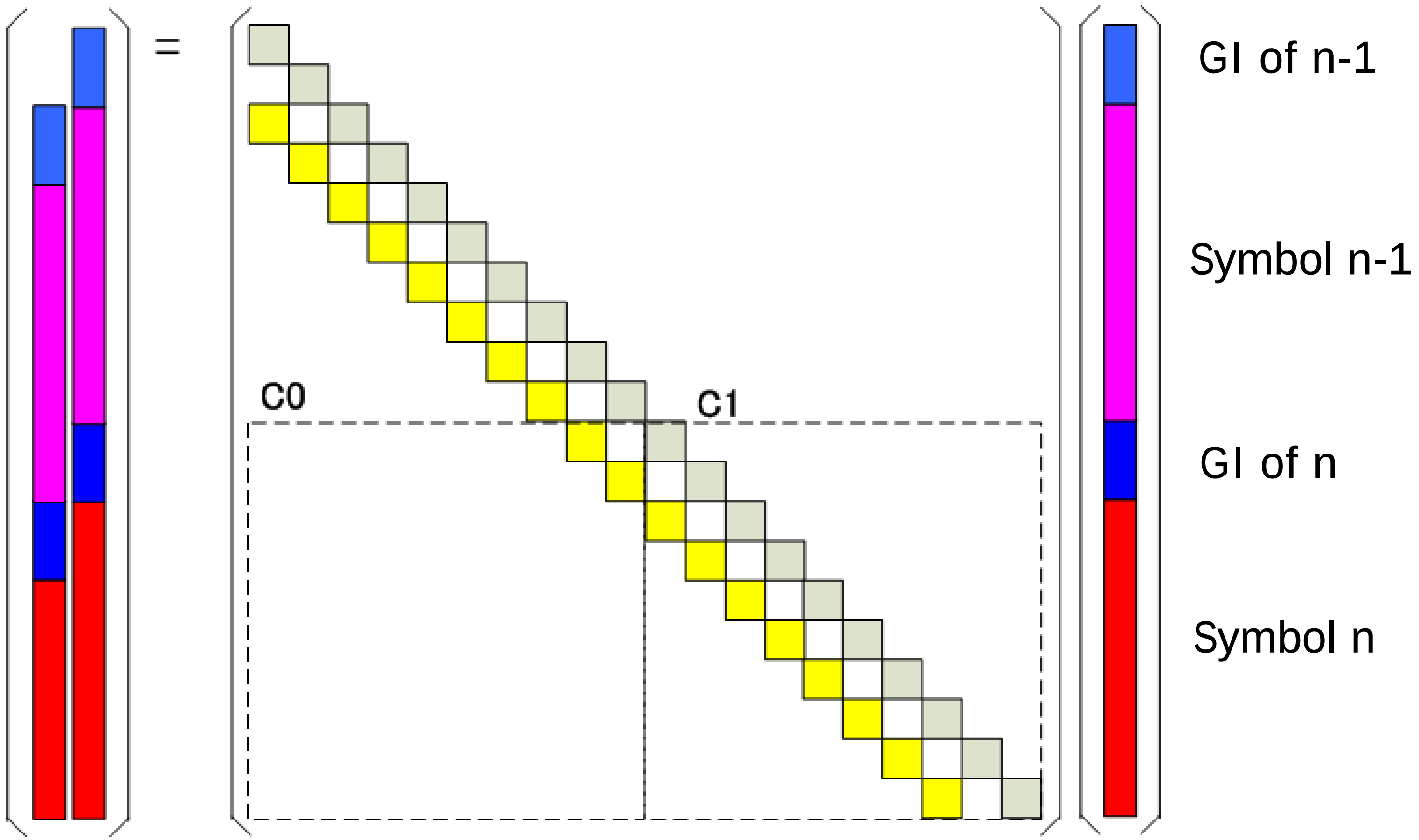
IFFT matrix

$$\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix} = IFFT \begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix} = \frac{1}{\sqrt{M}} \left[\omega^{(k-1)*(l-1)}; k-row, l-column \right] \begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix}$$

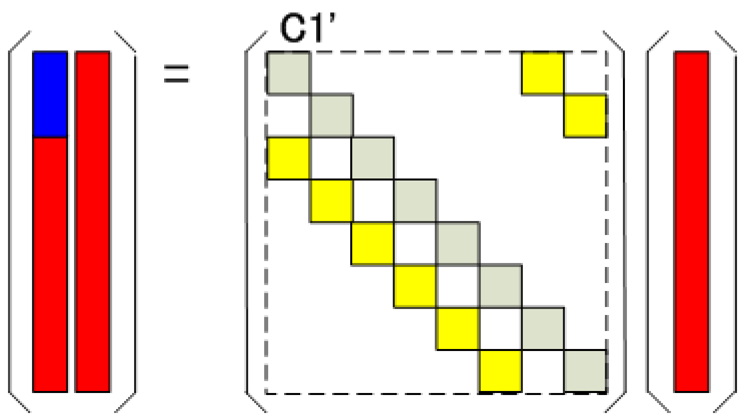
$$\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix} = \frac{1}{\sqrt{M}} \begin{pmatrix} \omega^0 & \omega^0 & \cdots & \omega^0 \\ \omega^0 & \omega^1 & \cdots & \omega^{(M-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^0 & \omega^{(M-1)} & \cdots & \omega^{(M-1)*(M-1)} \end{pmatrix} \begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix}$$

$$Here, \omega = e^{j\frac{2\pi}{M}}$$

Multi-path channel in Matrix

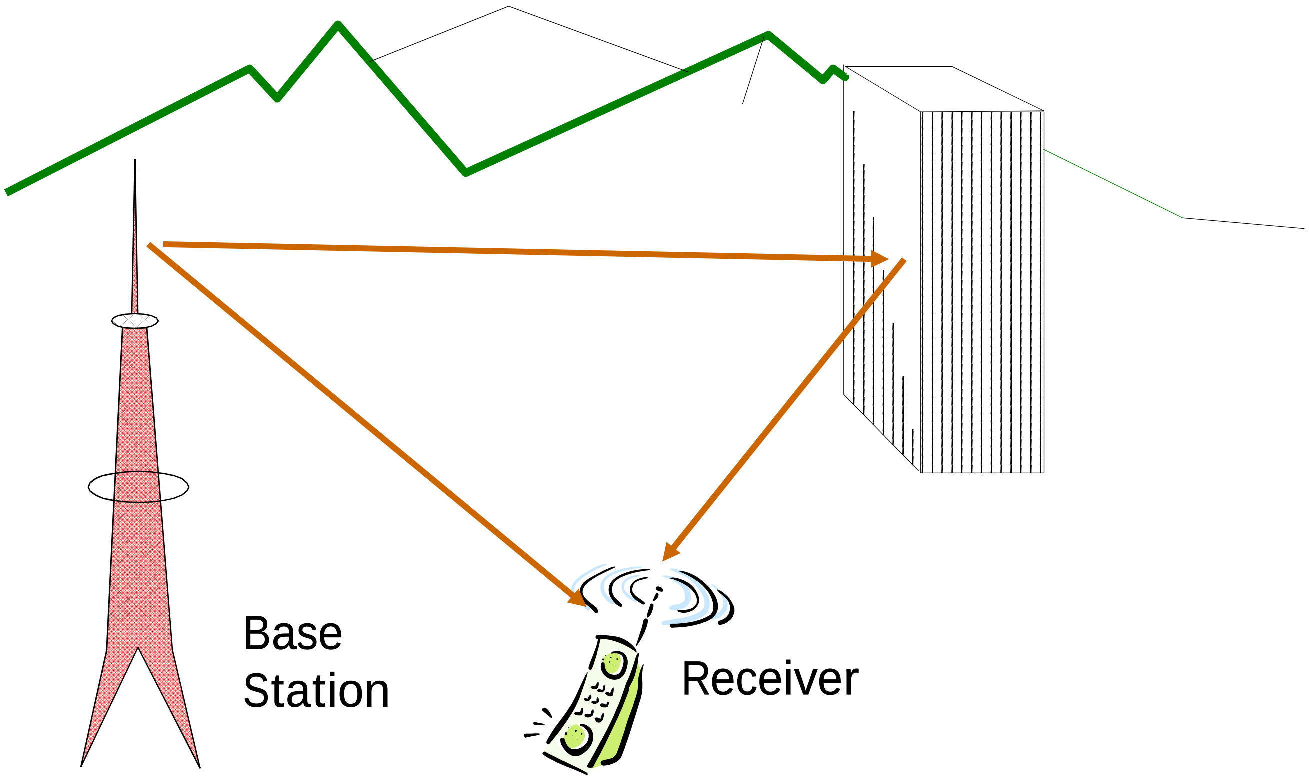


If Multi-path delay is small than GI length



- Channel Matrix is Cyclic Matrix!

Two path Multi path Channel Example



Channel Impulse Response = [1, 0.5 , 0, 0]

Two path Multi path Channel Example

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = FFT * Channel * IFFT * \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix}$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \omega^{-3} \\ 1 & \omega^{-2} & \omega^{-4} & \omega^{-6} \\ 1 & \omega^{-3} & \omega^{-6} & \omega^{-9} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0.5 \\ 0.5 & 1 & 0 & 0 \\ 0 & 0.5 & 1 & 0 \\ 0 & 0 & 0.5 & 1 \end{pmatrix} \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{1} & \omega^{2} & \omega^{3} \\ 1 & \omega^{2} & \omega^{4} & \omega^{6} \\ 1 & \omega^{3} & \omega^{6} & \omega^{9} \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix}$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = \begin{pmatrix} H(0) & 0 & 0 & 0 \\ 0 & H(1) & 0 & 0 \\ 0 & 0 & H(2) & 0 \\ 0 & 0 & 0 & H(3) \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix}$$

If time domain channel matrix is cyclic, Frequency Domain Channel Matrix is diagonal!

Additive Noise

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = FFT * \left[Channel * IFFT * \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} + \begin{pmatrix} noise(0) \\ noise(1) \\ noise(2) \\ noise(3) \end{pmatrix} \right]$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \omega^{-3} \\ 1 & \omega^{-2} & \omega^{-4} & \omega^{-6} \\ 1 & \omega^{-3} & \omega^{-6} & \omega^{-9} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0.5 \\ 0.5 & 1 & 0 & 0 \\ 0 & 0.5 & 1 & 0 \\ 0 & 0 & 0.5 & 1 \end{pmatrix} \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{1} & \omega^{2} & \omega^{3} \\ 1 & \omega^{2} & \omega^{4} & \omega^{6} \\ 1 & \omega^{3} & \omega^{6} & \omega^{9} \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} + \frac{1}{\sqrt{4}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \omega^{-3} \\ 1 & \omega^{-2} & \omega^{-4} & \omega^{-6} \\ 1 & \omega^{-3} & \omega^{-6} & \omega^{-9} \end{pmatrix} \begin{pmatrix} noise(0) \\ noise(1) \\ noise(2) \\ noise(3) \end{pmatrix}$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = \begin{pmatrix} H(0) & 0 & 0 & 0 \\ 0 & H(1) & 0 & 0 \\ 0 & 0 & H(2) & 0 \\ 0 & 0 & 0 & H(3) \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} + \begin{pmatrix} N(0) \\ N(1) \\ N(2) \\ N(3) \end{pmatrix}$$

How to recover sending signal from receiver signal. - EQUALIZE -

$$Ignore \quad Noise$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = FFT * Channel * IFFT * \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} = \begin{pmatrix} H(0) & 0 & 0 & 0 \\ 0 & H(1) & 0 & 0 \\ 0 & 0 & H(2) & 0 \\ 0 & 0 & 0 & H(3) \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix}$$

$$Then$$

$$\begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} = \begin{pmatrix} \frac{1}{H(0)} & 0 & 0 & 0 \\ 0 & \frac{1}{H(1)} & 0 & 0 \\ 0 & 0 & \frac{1}{H(2)} & 0 \\ 0 & 0 & 0 & \frac{1}{H(3)} \end{pmatrix} \begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix}$$

HW5

- Modify SCILAB program “MatrixOFDMSimulation1.sce” to measure Symbol Error Rate vs S/N ratio in M=16 OFDM with QPSK modulation
- You can create Matlab program if you like.
- Make Symbol Error Rate vs SN ratio
  - Vertical: SER in log scale
  - Horizontal: SN ratio 0dB, 1dB ... to 15dB
- Your report should contain your program and measured data in Graph.
- Dead Line : December End 2008
- Please submit to TA: Ikenoya-san
  - ike@lsi.ie.u-ryukyu.ac.jp