SYSTEM ARCHITECTURE
ADVANCED SYSTEM ARCHITECTURE
BATEMAN
Chapter1: Background material

2013/Fall-Winter Term
Monday 12:50

Room# 1-322 or 5F Meeting Room

Instructor: Fire Tom Wada, Professor

Chapter1: Background material

1.1 Time/frequency representation of digital signals

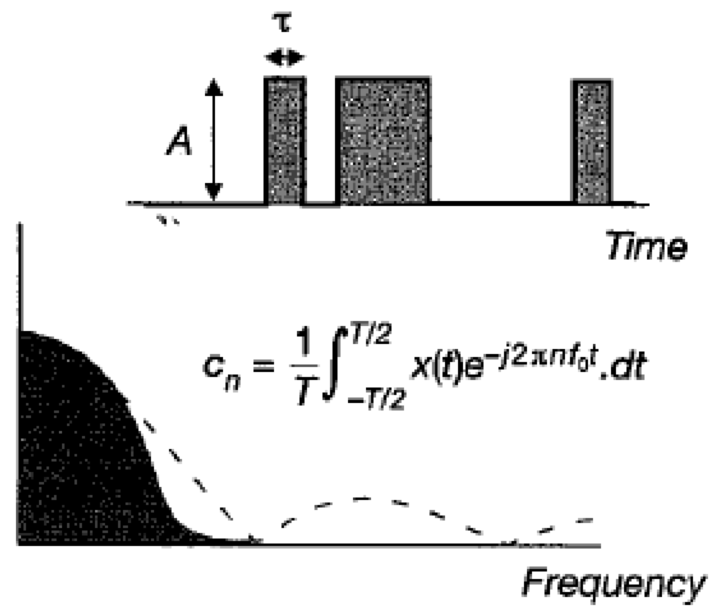
1.2 Trigonometric relationships

1.3 Communications networks and signaling protocols

1.4 Definition of terms

1.1 Time/frequency representation of digital signals

- Signal is represented either in time domain or in frequency domain.
- Fourier series representation (for periodic signals)
- Fourier transform (for non periodic signals)

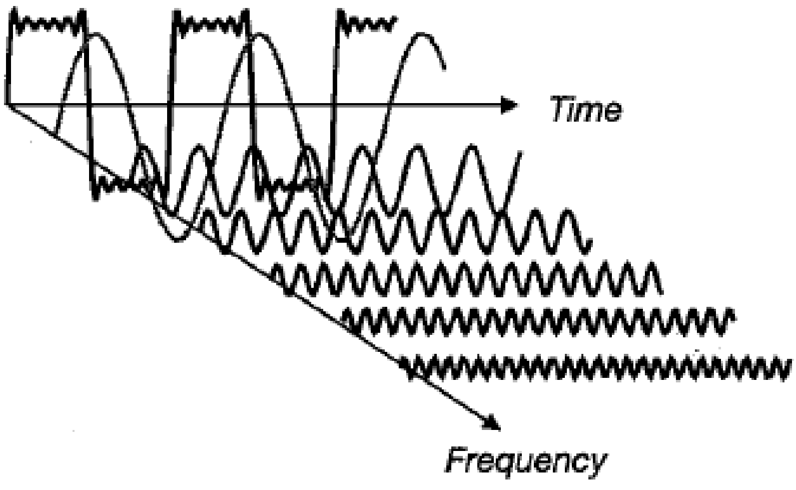


Fourier series

Fourier series can represent any periodic time domain signal by a summation of harmonically related sinewaves.

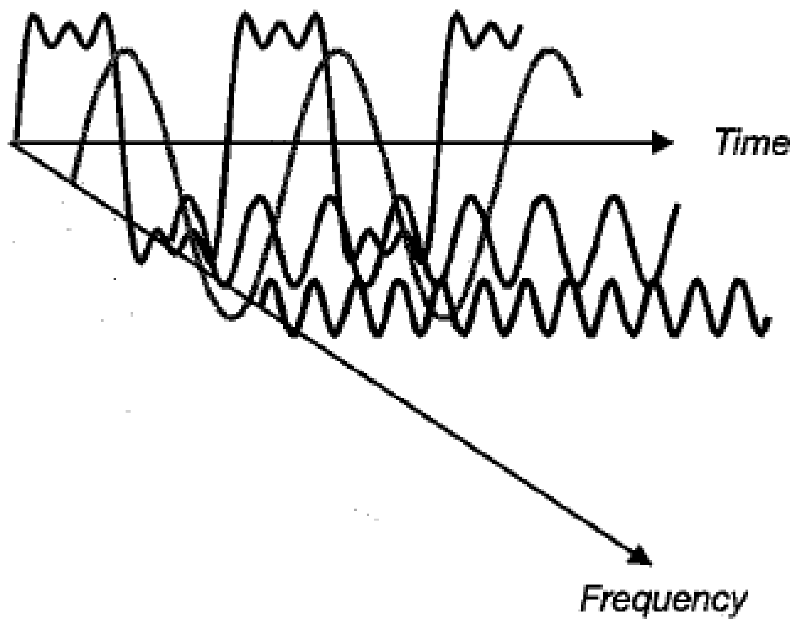
For example, the square wave (equivalent 1,0,1,0,1,0, . . .) data signal shown here can be constructed from sinewaves of descending amplitudes, spaced, in this example, at odd multiples of the *fundamental frequency* of the square wave.

If we wished to represent the 1,0,1,0,1,0, . . . pattern perfectly, an infinite number of sinusoidal components would be required, implying that we need an infinite channel bandwidth!



Only lower 3 components

The output response of a channel passing only the first three frequency components of the square wave is shown here and clearly demonstrates the change caused by restricted bandwidth on the time domain waveform. This example also shows, however, that the 1,0,1,0,1,0, . . . data stream can still easily be detected without all the constituent frequency components of the original square wave, and in fact correct demodulation is possible if only the *fundamental component* is passed by the channel.



Fourier series expansion

If x(t) has period of $T=1/f_0$, (and if x(t) is real signal)

$$x(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t))$$

$where$ $$a_0 = \frac{1}{T}\int_0^T x(t)dt \qquad = average\ of\ signal$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos(2\pi n f_0 t)\,dt$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin(2\pi n f_0 t)\,dt$$

Fourier series expansion
- complex exponential signal -

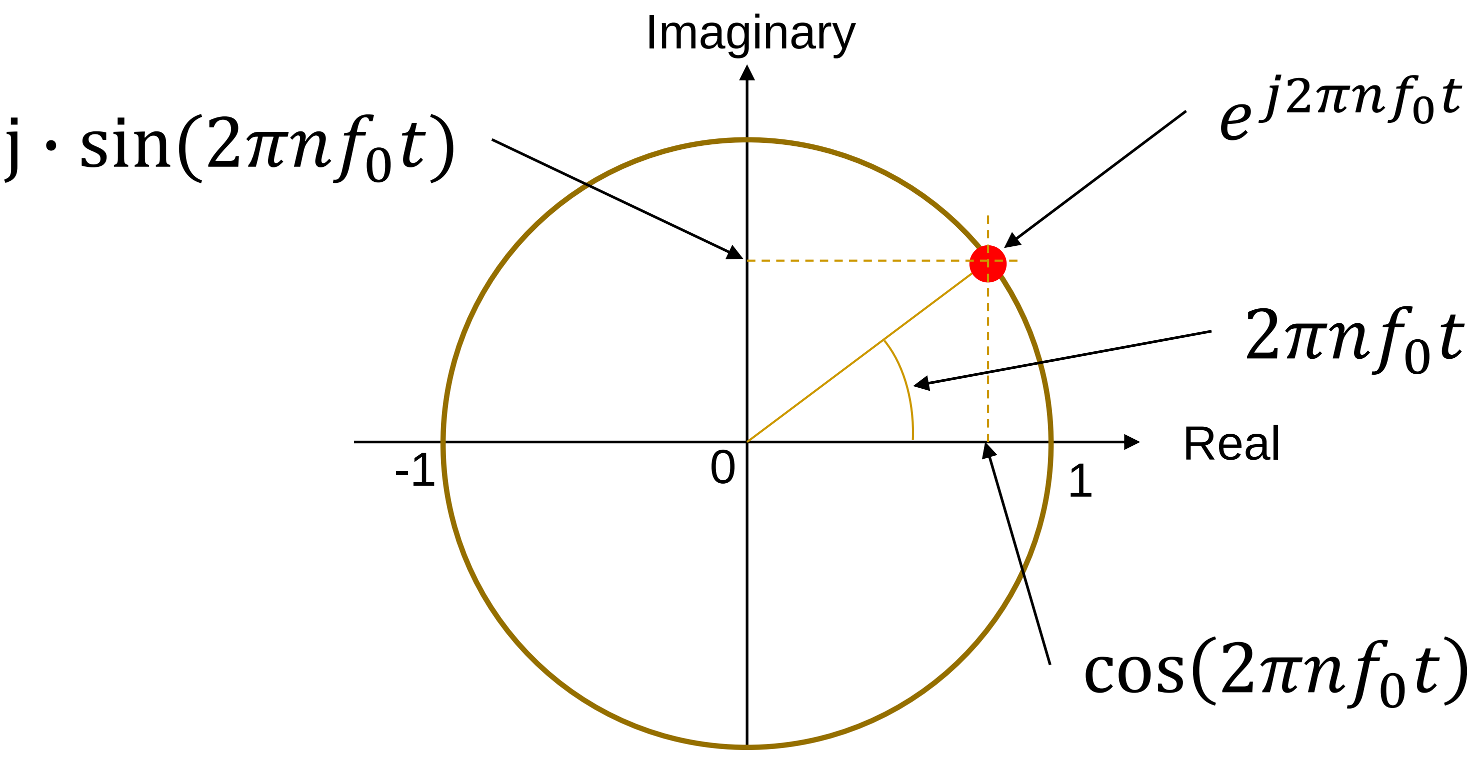
If x(t) has period of $T=1/f_0$,

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_0 t}$$

$$where \qquad C_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-j2\pi n f_0 t} dt$$

Euler's law

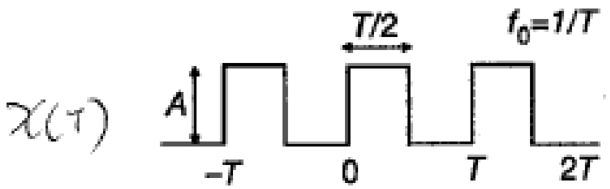
$$e^{j2\pi n f_0 t} = \cos(2\pi n f_0 t) + \mathrm{j} \cdot \sin(2\pi n f_0 t)$$



THIS IS ROTATION FUNCTION!

EXAMPLE 1.1

Find the **trigonometrical** Fourier series expansion for the following waveform:



Solution

$$x(t) = \begin{cases} A \rightarrow 0 < t < T/2 \\ 0 \rightarrow T/2 < t < T \end{cases}$$

Hence

$$a_0 = \frac{1}{T}\int_0^{T/2} A \cdot dt = \frac{At}{T}\Big|_0^{T/2} = \frac{A}{T}(T/2)$$

$$\Rightarrow a_0 = A/2$$

$$a_n = \frac{2}{T}\int_0^{T/2} A \cdot \cos 2\pi n f_0 t \cdot dt = \frac{2A}{2\pi n f_0 T} \cdot \sin 2\pi n f_0 T/2$$

Now,

$$f_0 = 1/T$$

$$\Rightarrow \therefore a_n = \frac{A}{\pi n} \sin \pi n = 0 \text{ for all } n$$

EXAMPLE 1.1 (2)

$$b_n = \frac{2}{T}\int_0^{T/2} A \sin 2\pi n f_0 t \cdot dt = \frac{-2A}{2\pi n f_0 T} \cdot (\cos(2\pi n f_0 T/2) - \cos 0)$$

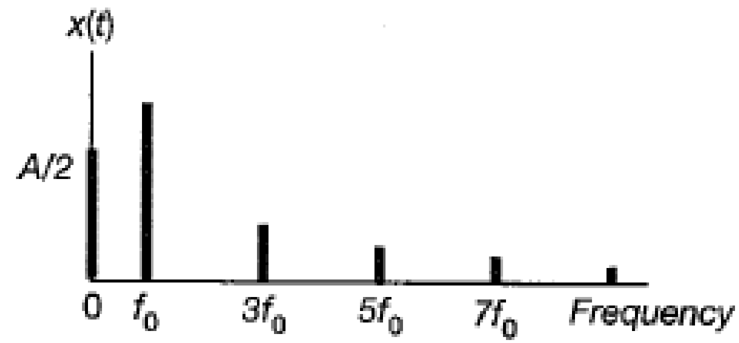
$$\Rightarrow b_n = \frac{A}{\pi n}(1 - \cos \pi n) = \frac{2A}{\pi n} \text{ for } n \text{ odd and } 0 \text{ for } n \text{ even}$$

The full Fourier series expansion can thus be written as:

$$\Rightarrow x(t) = A/2 + \sum_{n=1,3,5,\ldots}^{\infty} \frac{2A}{\pi n} \cdot \sin 2\pi n f_0 t$$

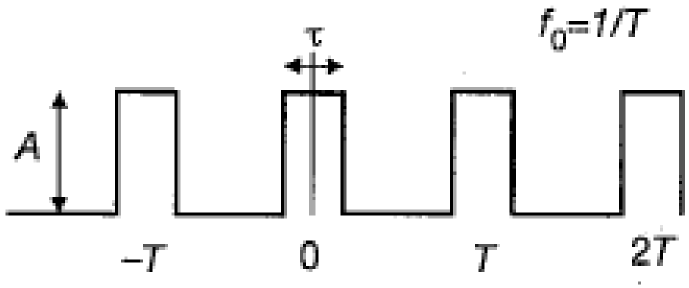
If the waveform is shifted by $T/4$, the Fourier series expansion would be represented by cosine terms as follows:

$$\Rightarrow x(t) = A/2 + \sum_{n=1,3,5,\ldots}^{\infty} \frac{2A}{\pi n} \cdot \cos 2\pi n f_0 t$$



EXAMPLE 1.2

Find the **trigonometrical** Fourier series expansion for the following waveform:

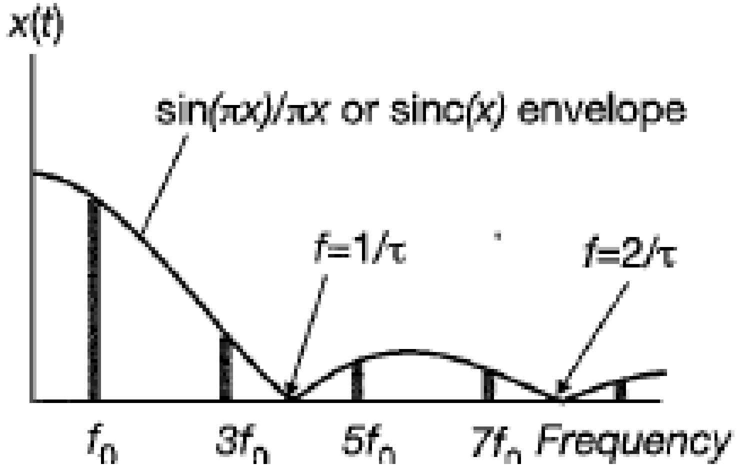


EXAMPLE 1.2 (2)

$$\Rightarrow x(t) = A\tau/T + \sum_{n=1,3,5,\ldots}^{\infty} \frac{2A\tau}{T} \cdot \frac{\sin(\pi n\tau/T)}{(\pi n\tau/T)} \cos(2\pi n f_0 t)$$

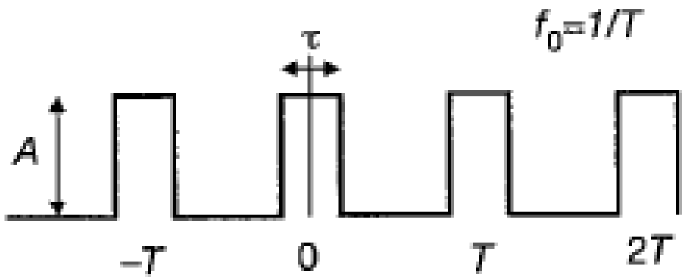
It is interesting to note that all of the Fourier series components for a train of pulses (data bits) with width τ (except for the dc component) are bounded by the

$$\frac{\sin(\pi n\tau/T)}{(\pi n\tau/T)} \quad \text{or} \quad \text{sinc}(n\tau/T) \text{ envelope.}$$



EXAMPLE 1.3

Find the complex Fourier series expansion for the following waveform:



Solution

The general expression for the complex Fourier series expansion of a time waveform $x(t)$ is given by:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t}$$

where

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t) \cdot e^{-j2\pi n f_0 t} \cdot dt$$

EXAMPLE 1.3 (2)

$$c_n = \frac{1}{T}\int_{-\tau/2}^{\tau/2} A \cdot e^{-j2\pi n f_0 t} \cdot dt = \frac{A}{T} \cdot \frac{e^{-j2\pi n f_0 t}}{-j2\pi n f_0}\Bigg|_{-\tau/2}^{\tau/2}$$

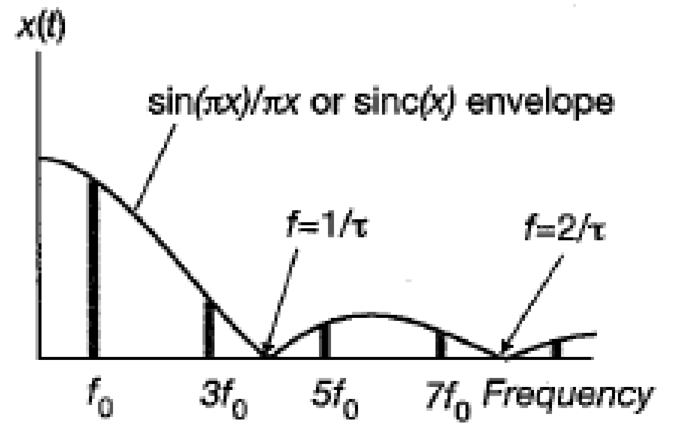
$$\therefore\ c_n = \frac{A(e^{j2\pi n f_0(\tau/2)} - e^{-j2\pi n f_0(\tau/2)})}{j2\pi n f_0 T} = \frac{A\tau}{T} \cdot \frac{\sin(\pi n f_0 \tau)}{\pi n f_0 \tau}$$

Now $f_0 = 1/T$, hence

$$c_n = \frac{A\tau}{T} \cdot \frac{\sin(\pi n \tau / T)}{(\pi n \tau T)}$$

The full complex Fourier series expansion thus becomes:

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{A\tau}{T} \cdot \frac{\sin(\pi n \tau / T)}{(\pi n \tau / T)} \cdot e^{j2\pi n f_0 t}$$

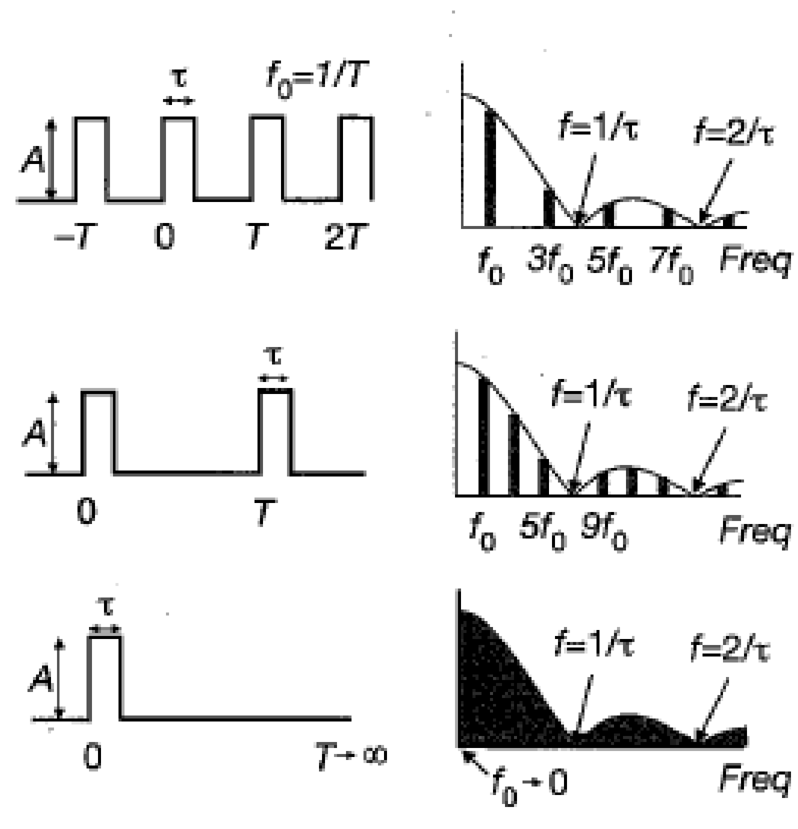


Spectrum of data pulse

As the fundamental period of the time waveform increases, the fundamental frequency of the Fourier series components making up the waveform decreases and the harmonics become more closely spaced.

In the limit, as the time between pulses approaches infinity, the harmonic spacing becomes infinitely small and the spectrum is in fact continuous and bounded by the *sinc* function as shown.

A single pulse is not of course a periodic time function and the spectrum cannot strictly be evaluated using the Fourier series expansion. Instead the more general Fourier transform should be used.

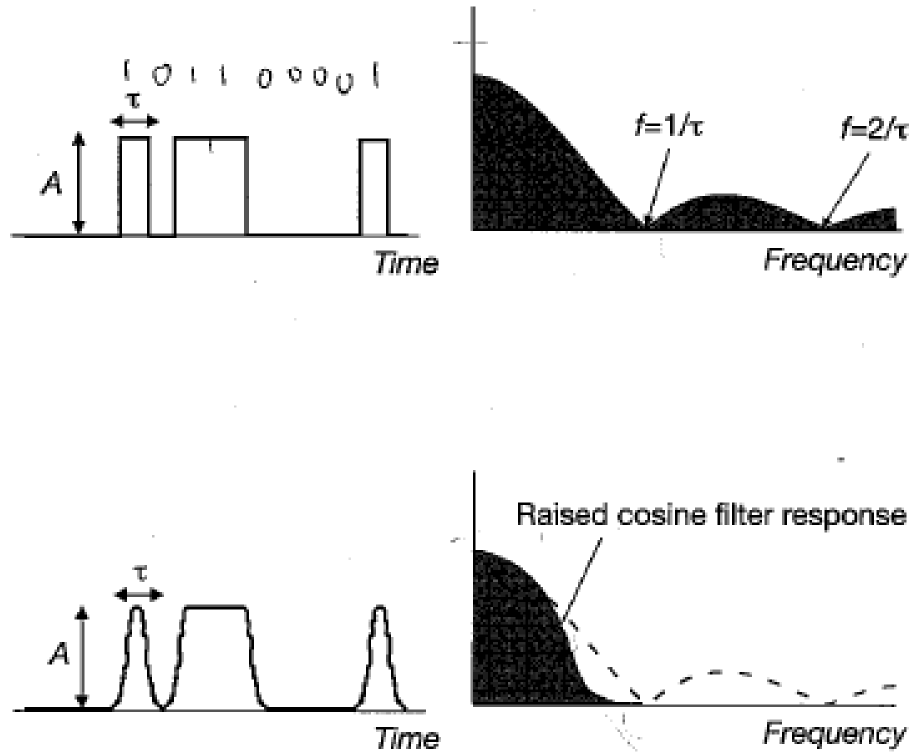


Smooth edges to reduce spectrum BW

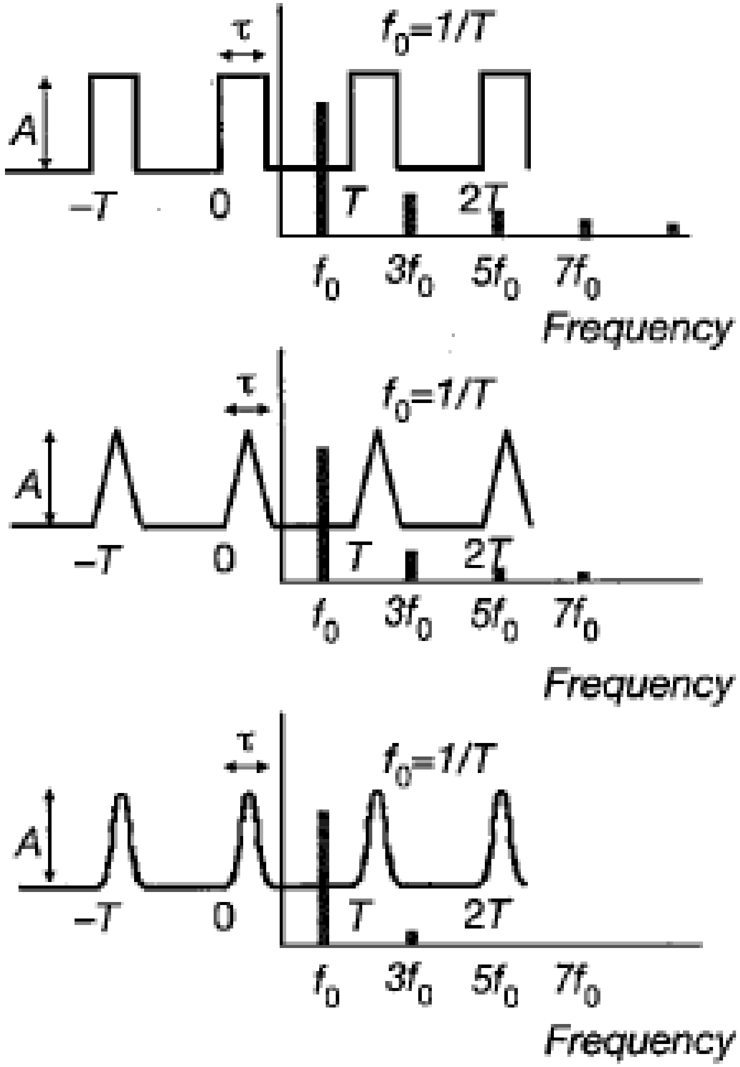
Spectrum of a baseband binary data stream

The spectrum (frequency domain representation) of a random data stream can be obtained by simply overlaying the instantaneous spectra for each individual pulse. We thus know that it will be bounded by the *sinc* envelope, and at any instance in time, the location and density of frequency components will depend on the particular pattern of data bits.

By shaping the data pulses, so that they have smooth edges, we would expect to reduce significantly the high frequency spectral content of the waveform. A commonly used pulse-shaping method is to pass the data stream through a low pass filter having a raised cosine response. The raised cosine filter belongs to a family of filters called Nyquist filters which have particularly useful properties in data communications (see Section 3.4).



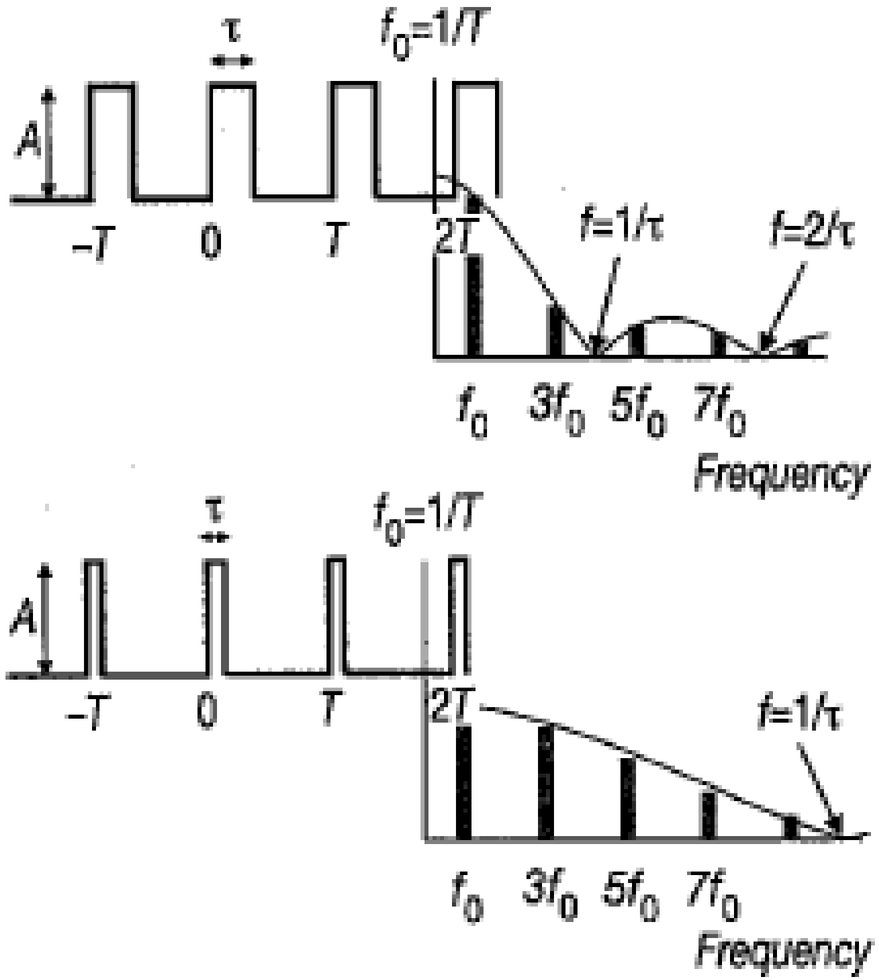
Smooth edges to reduce spectrum BW(2)



Short pulse increase higher harmonics

As can be seen here, reducing the width of the pulse but keeping the period of the waveform constant results in an increase in the level of the higher harmonics at the expense of the lower harmonic levels. Overall, the energy content in the waveform has also gone down and so the combined power of the harmonics must also be reduced.

In the limit, as the pulse width tends to zero (that is, a delta function), we can expect the amplitude of each harmonic to approach a constant yet diminishing value.



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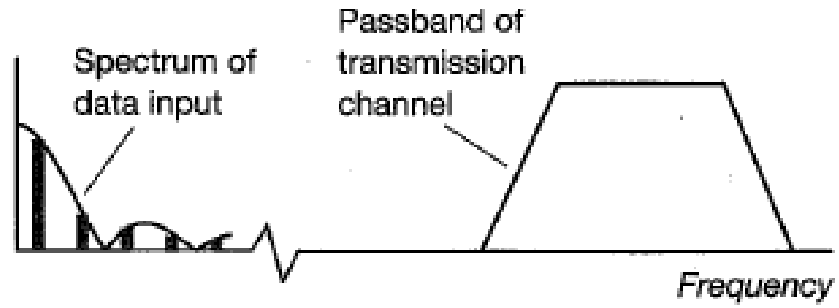
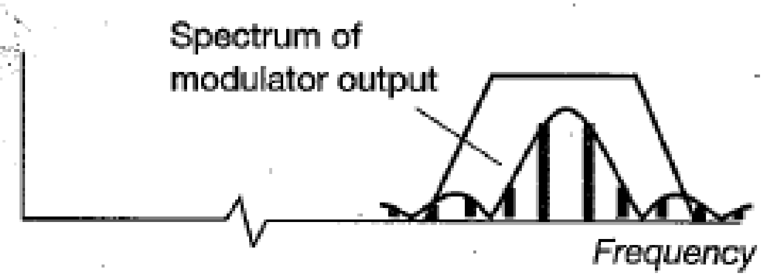
Frequency conversion

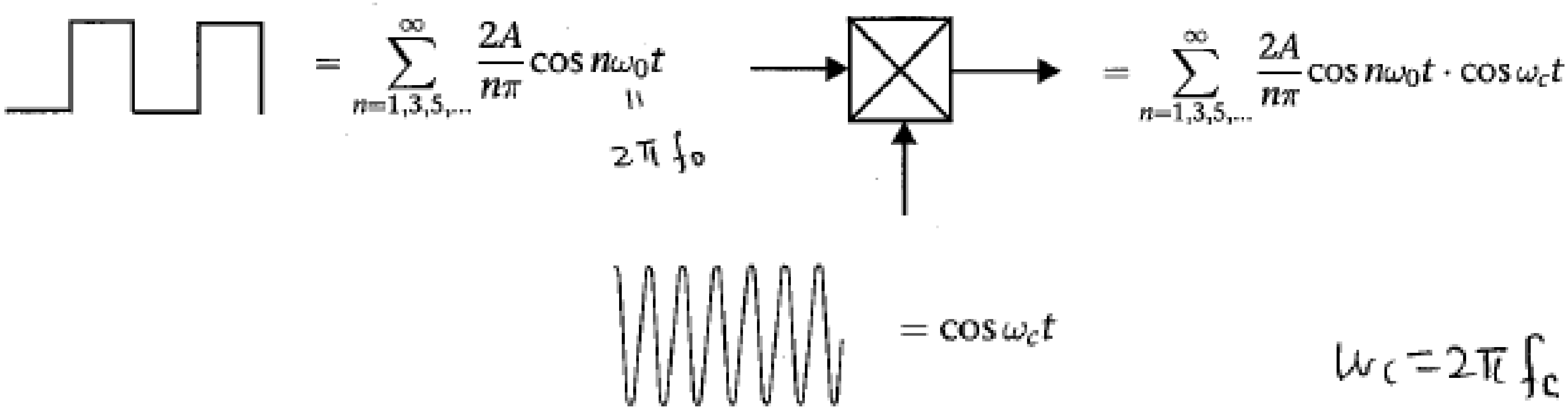
In order to translate the spectrum of the input signal to fit within the passband of a channel, a process of modulation is employed as described in Chapter 5. This process often involves mixing the input data signal with a high frequency sine or cosine term, called the carrier.



If we plot the spectrum of the modulated signal, we see that it is centred on the carrier frequency, and for this example, reproduces the components of the baseband data signal exactly mirrored on either side of the carrier. We thus have a method to translate spectral components to any frequency we choose using the mixing process.



Mixing process
- mix (analog)= multiply (digital)



$$Mixer\ output = \frac{1}{2} \cdot \frac{2A}{n\pi} \left(\sum_{n=1,3,5,\dots}^{\infty} \cos(\omega_c - n\omega_0)t + \sum_{n=1,3,5,\dots}^{\infty} \cos(\omega_c + n\omega_0)t \right)$$

EXAMPLE 1.4

A square wave with a frequency of 1 MHz is mixed in a receiver with a local oscillator sinusoidal at 7.5 MHz and the resulting signal passed through a brick-wall low pass filter with a cut-off of 700 kHz.

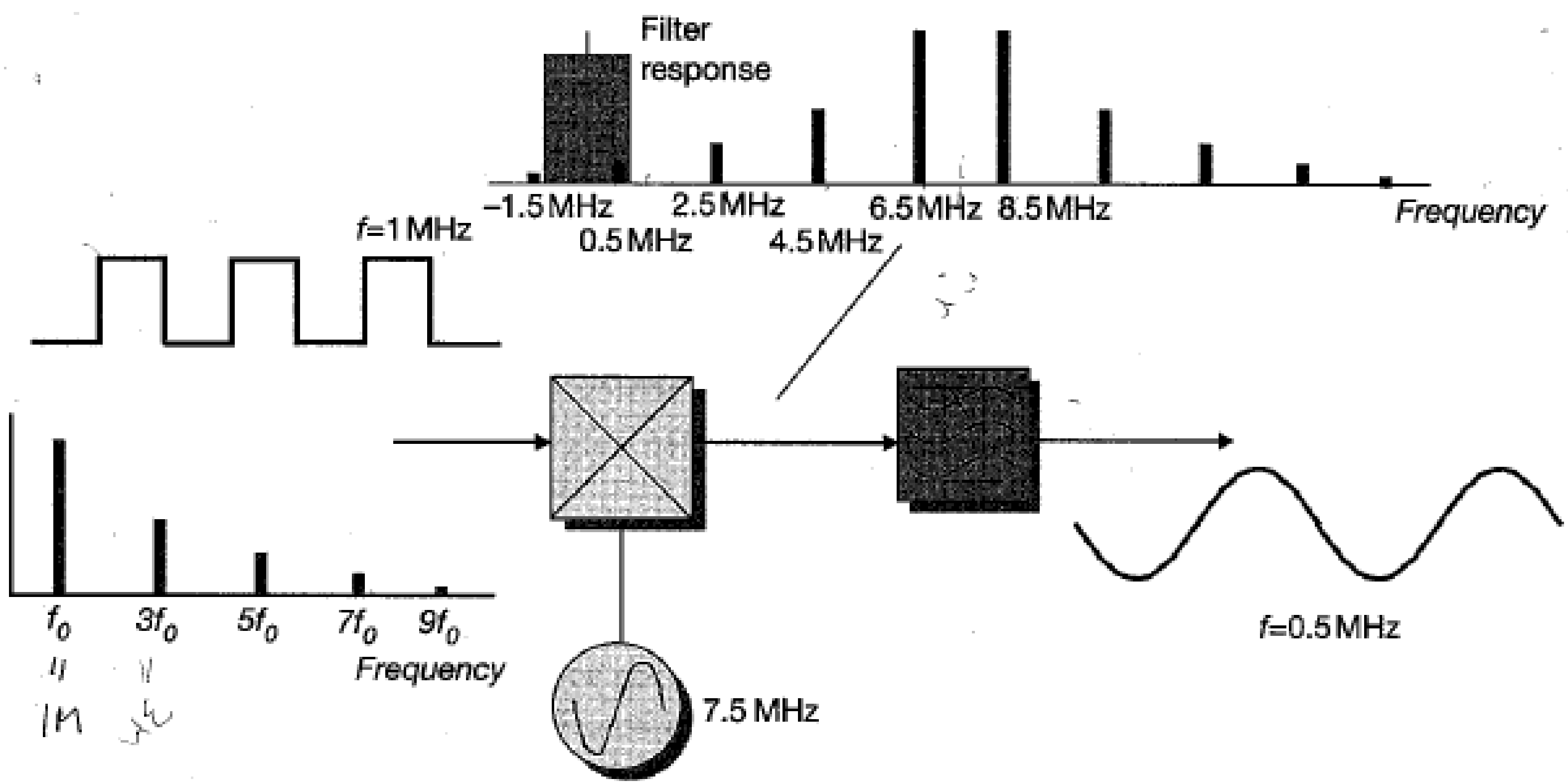
(a) What will appear at the output of the receiver?

(b) The output of the receiver is found to be too small for practical use. How can this output level be increased simply by altering the shape of the 1 MHz modulating component?

EXAMPLE 1.4 (2)

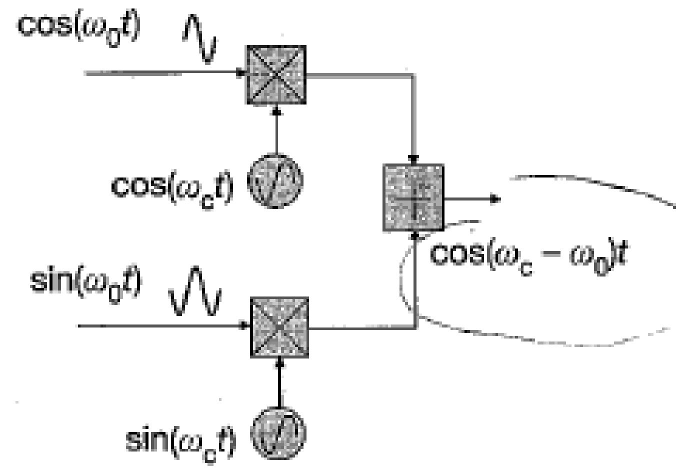
Solution

The square wave is made up of sinusoidal components given by the Fourier series as derived in Example 1.1. This signal, when mixed with the 7.5 MHz local oscillator, will give components at the sum and difference between each of the Fourier series components and the 7.5 MHz reference.



The Vector modulator

The arrangement of mixers and a combiner (summing device) shown here forms an extremely useful building block in digital communications systems. It achieves a linear frequency translation of all components in the input signal (represented by its in-phase and quadrature components) by a carrier frequency component (also represented by its in-phase and quadrature components). This building block is often referred to as a *vector modulator* or *quadrature modulator* and, as we shall see, can be used for both frequency *up-conversion* and *down-conversion*.



The output of the two mixing processes is given by:

$$\cos(\omega_0 t) \cdot \cos(\omega_c t) = 0.5\cos(\omega_c + \omega_0)t + 0.5\cos(\omega_c - \omega_0)t$$

and

$$\sin(\omega_0 t) \cdot \sin(\omega_c t) = -0.5\cos(\omega_c + \omega_0)t + 0.5\cos(\omega_c - \omega_0)t$$

which when subtracted from each other result in a single up-converted component:

$$\cos(\omega_c + \omega_0)t$$

and when summed give a down-converted component:

$$\cos(\omega_c - \omega_0)t$$

EXAMPLE 1.5

A vector modulator is fed with a perfect quadrature sinewave at the input, but there is a small phase error of 5° between the notional quadrature inputs of the carrier signal. What will be the ratio in dB between the sum and difference outputs of the vector modulator as a result of this phase error?

Solution

Let us write the inputs to the vector modulator as:

$$\cos(\omega_0)t, \quad \sin(\omega_0 t)$$

and the carrier inputs as:

$$\cos(\omega_c)t, \quad \sin(\omega_c t + \phi)$$

where ϕ is the phase error. Now:

$$\sin(\omega_c t + \phi) = \sin\omega_c t\cos\phi - \cos\omega_c t\sin\phi$$

and for small phase errors this can be approximated to:

$$\sin(\omega_c t + \phi) = \sin\omega_c t\cos\phi$$

The mixer outputs then become:

$$\cos(\omega_0 t)\cos(\omega_c t) = 0.5\cos(\omega_c + \omega_0)t + 0.5\cos(\omega_c - \omega_0)t$$

$$\sin(\omega_0 t)\sin(\omega_c t + \phi) = -0.5\cos(\omega_c + \omega_0)t\cos\phi + 0.5\cos(\omega_c - \omega_0)t\cos\phi$$

EXAMPLE 1.5 (2)

At the output of the summing device we get a wanted term at the difference frequency and an unwanted term (usually referred to as the image) at the sum frequency as follows:

Difference term:

$$0.5[1 + \cos\phi]\cos(\omega_c - \omega_0)t$$

Sum term:

$$0.5[1 - \cos\phi]\cos(\omega_c + \omega_0)t$$

The ratio of the amplitude of the wanted to unwanted terms is thus:

$$\textit{Amplitude ratio (image suppression)} = \frac{[1 + \cos\phi]}{[1 - \cos\phi]}$$

For a phase error of 5°, the amplitude ratio of wanted to unwanted signals is thus 525:1, or a relative power level of approximately 27 dB.

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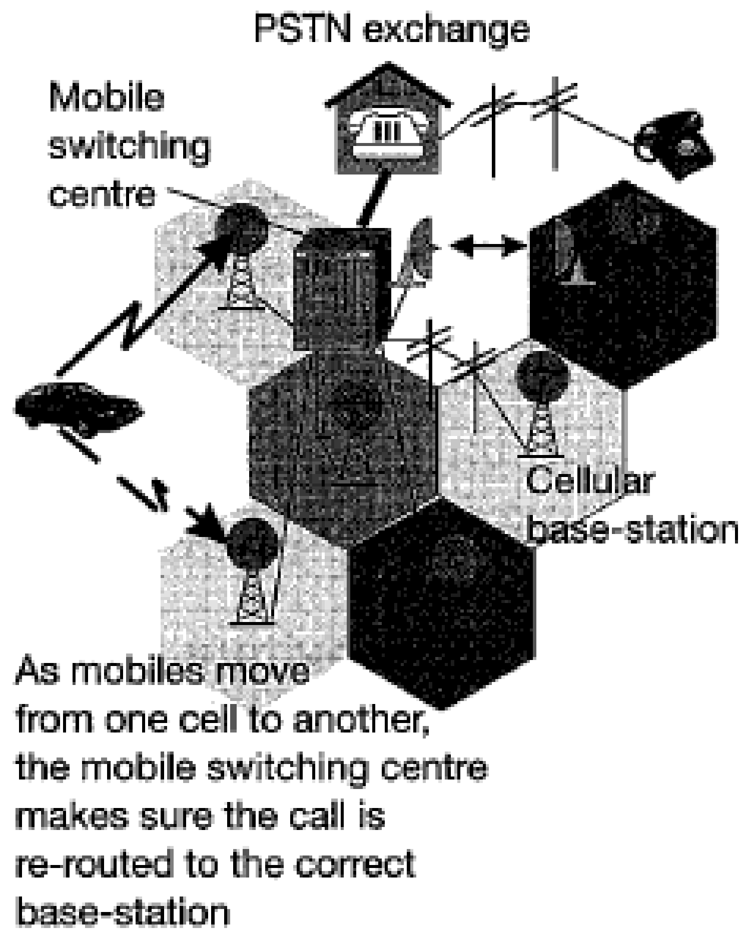
Cellular telephone example

A typical network configuration

Shown here is a typical configuration for a modern cellular telephone network. The wireless connection can be seen to be only a small part of a much larger network involving: a *mobile switching centre* to route calls from mobile to mobile or into the exchange; provision of private cable or *microwave radio links* for the interconnection of *base-station* sites to switching centres and between switching centres; interconnection to the *public switched telephone network (PSTN)* for the transfer of calls between mobile and domestic telephones.



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Elements of communication link

Transmitter

The transmitter (TX) element processes a message signal in order to produce a signal most likely to pass reliably and efficiently through the channel. This usually involves modulation of a carrier signal by the message signal (see Chapters 5 and 6), coding of the signal to help correct for transmission errors (see Chapter 7), filtering of the message or modulated signal to constrain the occupied bandwidth (see Chapter 3), and power amplification to overcome channel losses.

Transmission channel

Loosely defined as the electrical medium between source and destination, for example cable, optical fibre or free space, the channel is characterized by its loss/attenuation, bandwidth, noise/interference and distortion.

Receiver

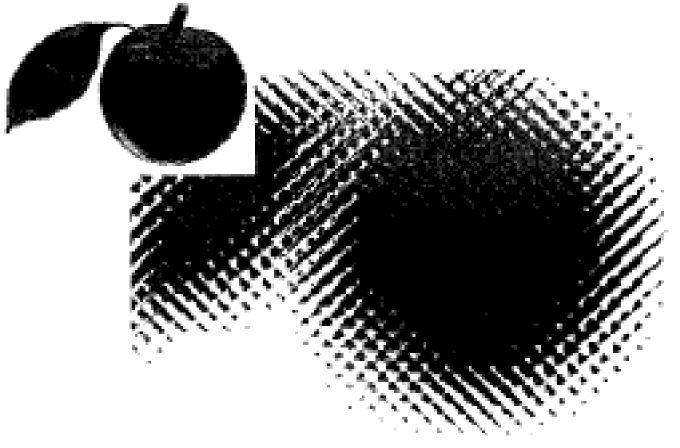
The receiver (RX) function is principally to *reverse* the modulation processing of the transmitter in order to recover the message signal, attempting to compensate for any signal degradation introduced by the channel. This will normally involve amplification, filtering, demodulation and decoding and in general is a more complex task than the transmit processing.

Distortion, Interference, Noise

Distortion

The common types of link distortion are:

- frequency-dependent phase shifts, giving rise to differential group delay (see Section 4.2)
- gain variations with frequency caused by the channel filtering effect (see Section 4.1)
- gain variations with time as experienced in a radio/infrared channel
- frequency offsets between transmitter and receiver due to Doppler shift or local oscillator errors (see Section 4.3).



Distortion can be introduced within the transmitter, the receiver and the channel. In some cases it can be corrected using channel equalizers (see Section 4.4), and gain and frequency control systems (see Section 4.2). Unlike noise and interference, distortion disappears when the signal is turned off.

Distortion, Interference, Noise

Interference

Interference arises owing to contamination of the channel by extraneous signals, for example from power lines, machinery, ignition systems, other channel users and so on. If the characteristics are known, then interference can often be suppressed by filtering or subtraction (for example, car suppressers).

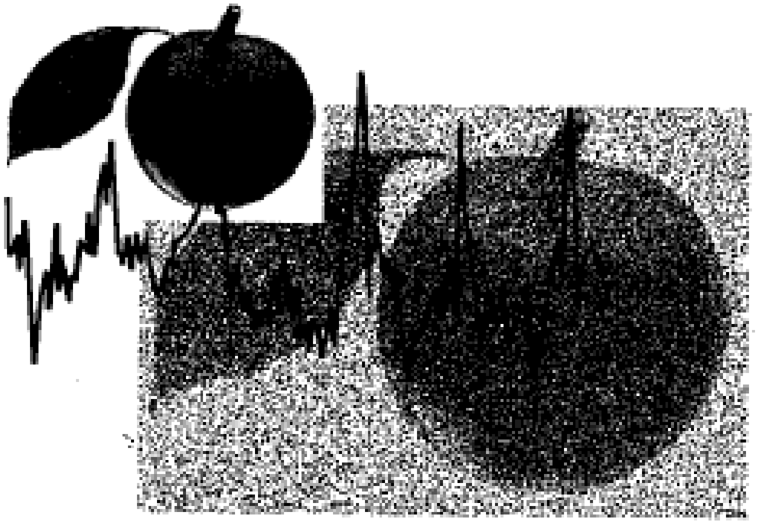
Interference is often impulse-like in nature and we know from our knowledge of the Fourier transform and Fourier series expansion (Section 1.1) that an impulse contains energy over a very wide bandwidth. In the case of ignition noise, the ignition system may be firing at only 4000 Hz (1000 rpm), yet significant high frequency energy will exist at frequencies of several MHz.

Distortion, Interference, Noise

Noise

Noise is charcterized as *random, unpredictable electrical signals from natural sources*, for example atmospheric noise, thermal noise, shot noise and so on.

Because of the multiplicity of noise sources in a communications link, it is difficult to define the properties (frequency range, level and instantaneous phase) of noise and hence equally difficult to reduce its effect on the communications link performance. For convenience, most textbooks and indeed practising engineers assume noise to fall predominantly into the class of Additive White Gaussian Noise (AWGN) which does indeed form an adequate classification for most cases. However, we should not forget that this is a *general simplification* of the whole noise issue.



Need two presenter

- Each presenter shall chose two problems from 1.4 to 1.10.
- Show your answer at the beginning of next lecture. Each has 5 minutes.