SYSTEM ARCHITECTURE
ADVANCED SYSTEM ARCHITECTURE
BATEMAN
Chapter6: Multi-level digital modulation

2013/Fall-Winter Term
Monday 12:50

Room# 1-322 or 5F Meeting Room

Instructor: Fire Tom Wada, Professor

Chapter6: Multi-level digital modulation

6.1 Introduction

6.2 M-ary Amplitude Shift Keying (M-ary ASK)

6.3 M-ary Frequency Shift Keying (M-ary FSK)

6.4 M-ary Phase Shift Keying (M-ary PSK)

6.5 Combined Amplitude and Phase Keying (QAM/APK)

6.6 Relative performance of multi-level bandpass modulation formats

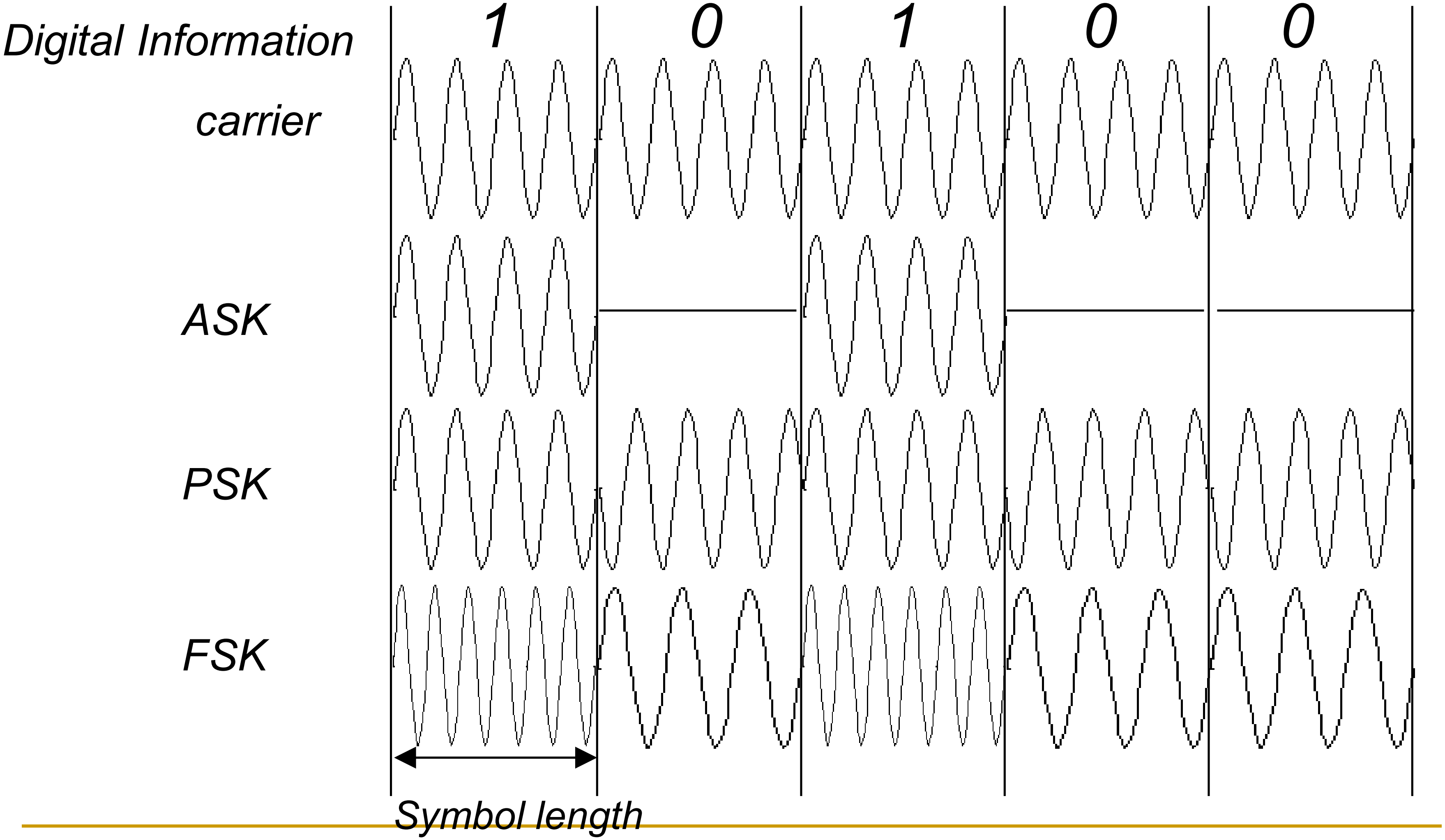
Previous Chapter 5
Digital modulation basics

- Digital modulation modulates three parameters of sinusoidal signal.
- A, θ_k fc,

$$s(t) = A \cdot \cos(2\pi \cdot f_c \cdot t + \theta_k)$$

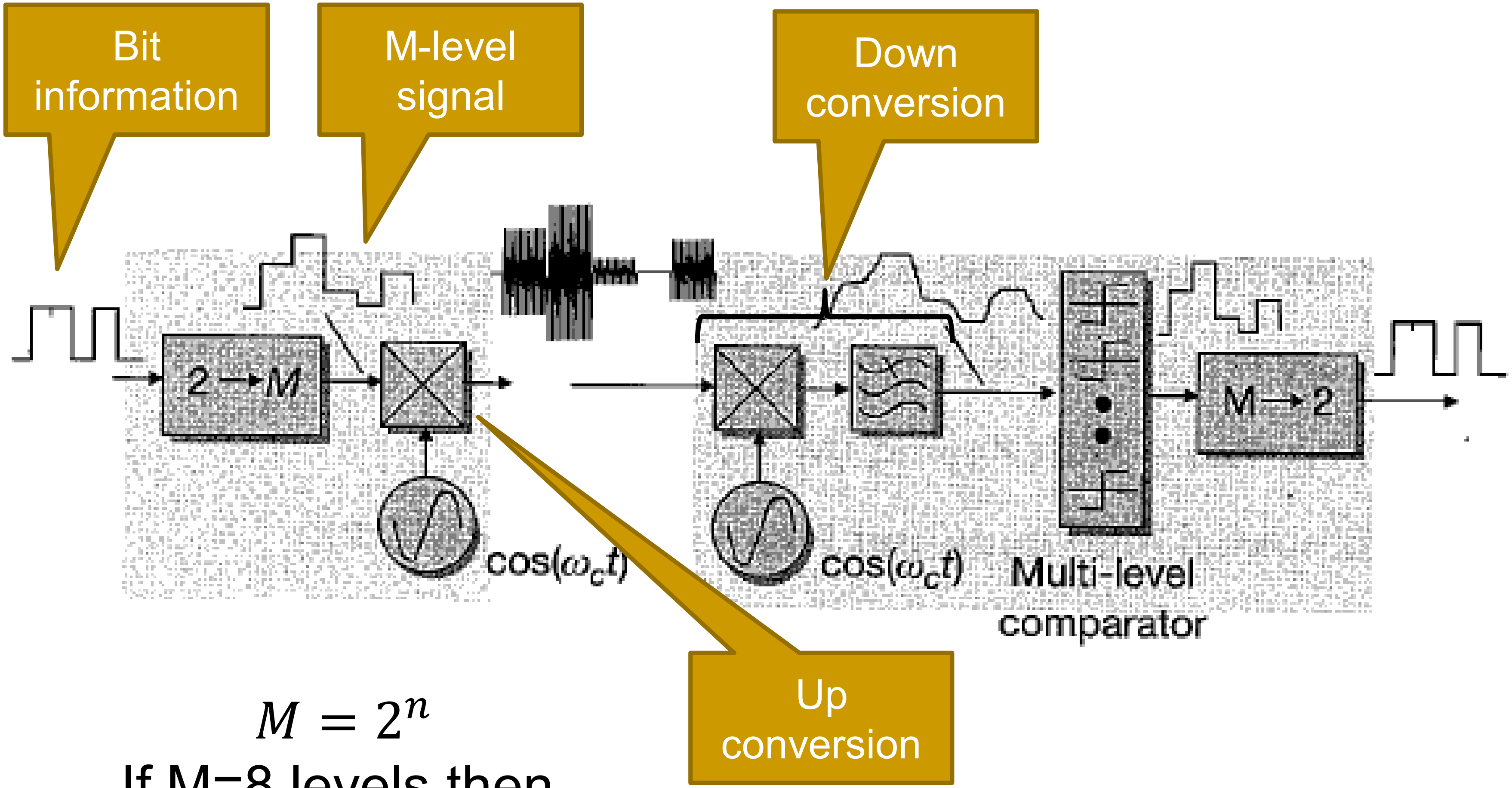
- Three type digital modulation:
  - ASK : Amplitude Shift Keying
  - PSK : Phase Shift Keying
  - FSK : Frequency Shift Keying

Symbol Waveform



Chapter6: Multi-level digital modulation

M-ary ASK



$$M = 2^n$$

If M=8 levels then

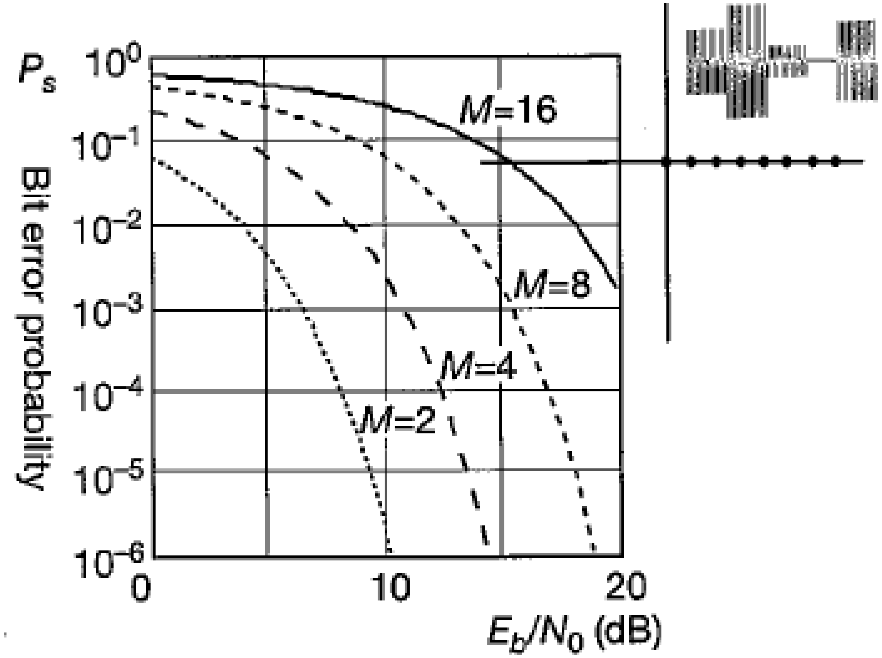
$$n = \log_2 M = \log_2 8 = 3\ bit$$

Performance of M-ary ASK

- When M is increased, number of bits transferred is increased. But need better SNR (Eb/N0).

M-ASK(symbol): $P_s = [(M-1)/M].erfc[\sqrt{\{3.(E_b/N_0)/(M^2-1)\}}]$

M-ASK(bit) ≈ M-ASK(symbol)/$\log_2 M$



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Quadrature Phase Shift Keying (QPSK)

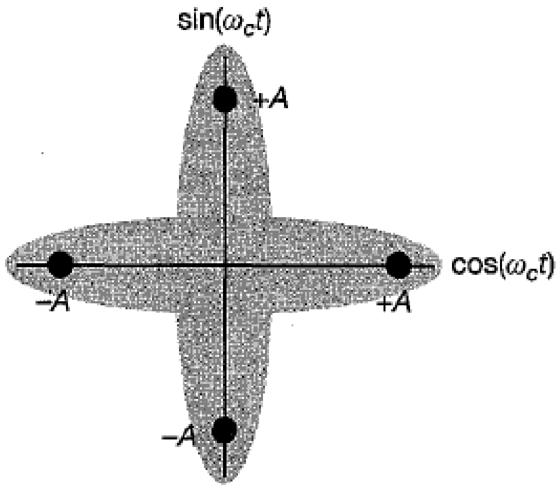
- 4 Orthogonal waves are used!

$$S_0(t) = A\cos\left(\omega_c t + \frac{0\pi}{2}\right) = +A\cos(\omega_c t)$$

$$S_1(t) = A\cos\left(\omega_c t + \frac{1\pi}{2}\right) = -A\sin(\omega_c t)$$

$$S_2(t) = A\cos\left(\omega_c t + \frac{2\pi}{2}\right) = -A\cos(\omega_c t)$$

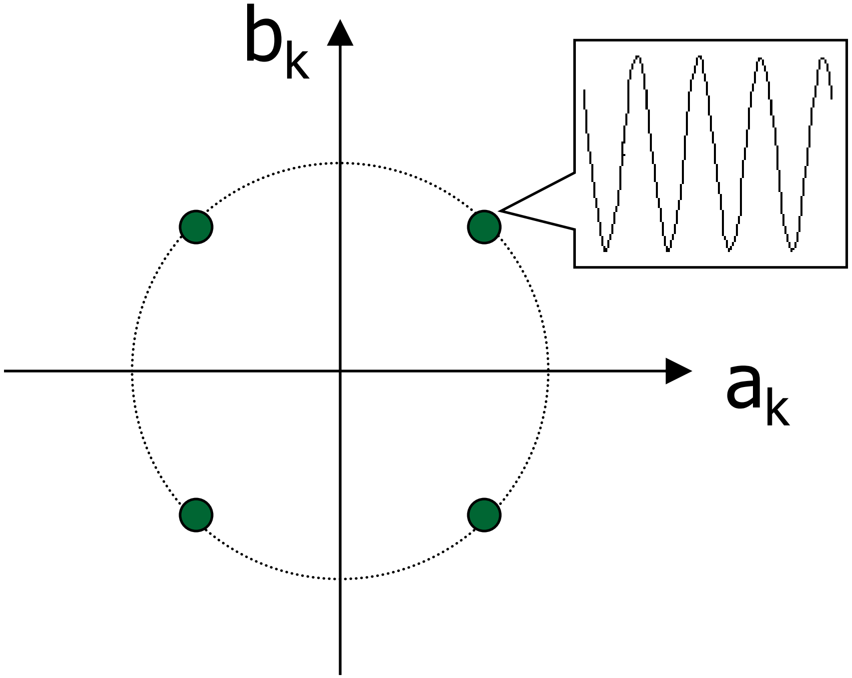
$$S_3(t) = A\cos\left(\omega_c t + \frac{3\pi}{2}\right) = +A\sin(\omega_c t)$$



Another Quadrature Phase Shift Keying (QPSK)

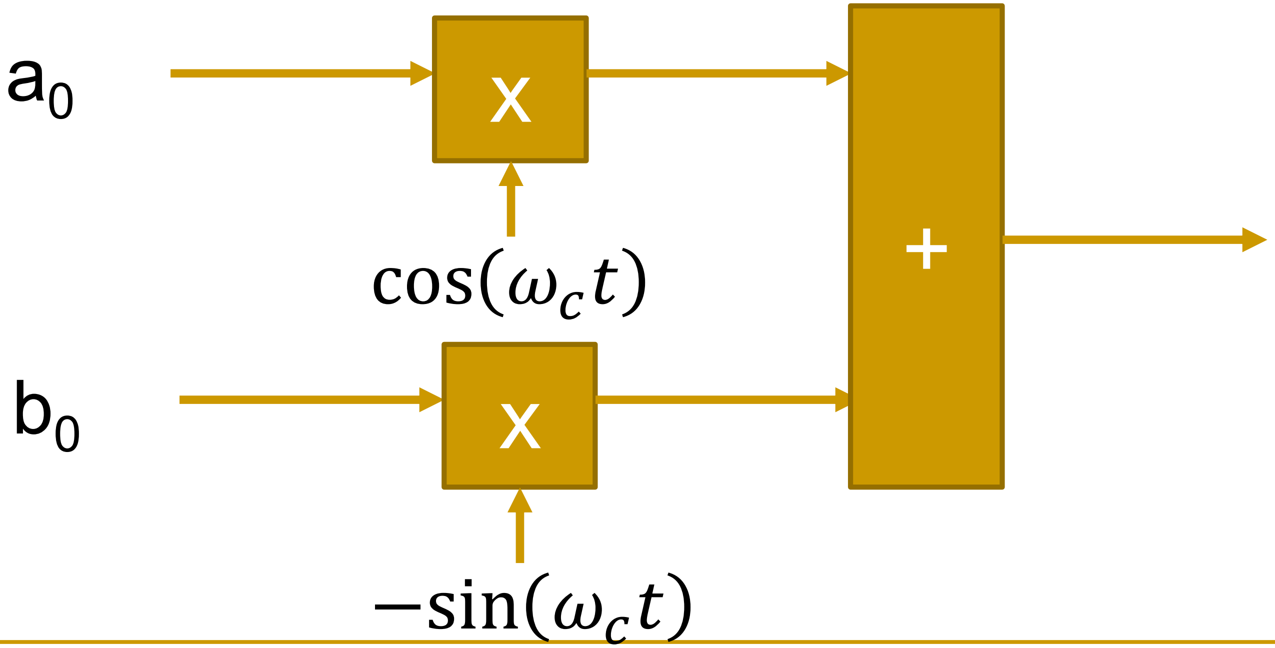
$$S_k(t) = \cos(\omega_c t + \theta_k)$$

data	θk	a_k	b_k
00	π/4	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
01	3π /4	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
11	5π /4	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
10	7π /4	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$



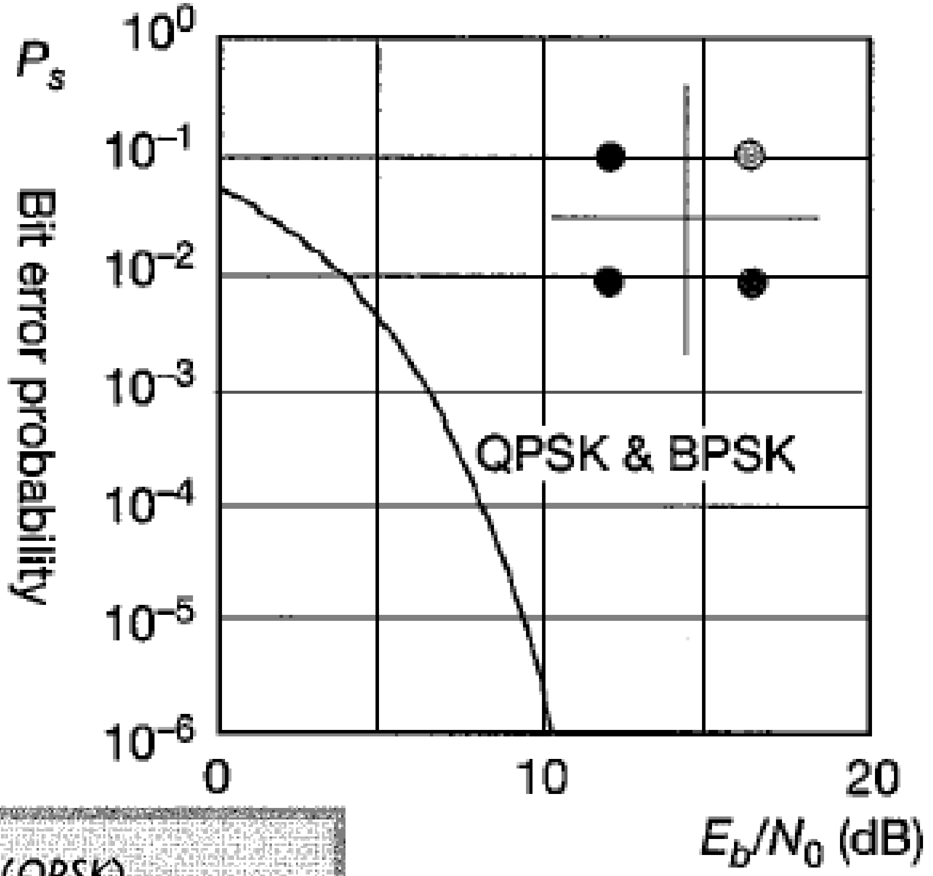
QPSK modulator

$$S_0(t) = \cos\left(\omega_c t + \frac{\pi}{4}\right)$$
$$= \cos(\omega_c t)\cos\left(\frac{\pi}{4}\right) - \sin(\omega_c t)\sin\left(\frac{\pi}{4}\right)$$
$$= a_0\cos(\omega_c t) - b_0\sin(\omega_c t)$$



Bit error performance of QPSK

QPSK(bit) = BPSK(bit): $P_s = 0.5 erfc[\sqrt{(E_b/N_0)}]$



Maximum bandwidth efficiency (QPSK)
= 2 bits/second/Hz

Symbol error performance of QPSK

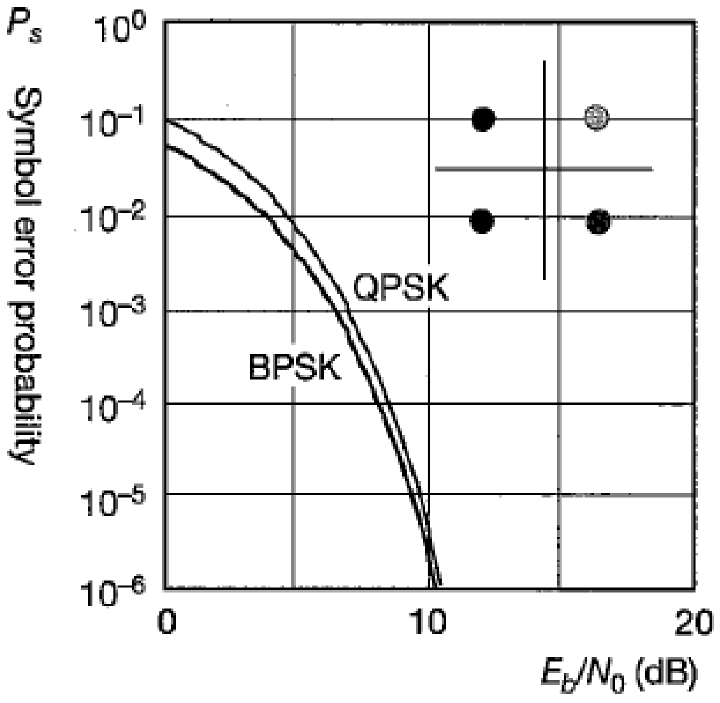
Many textbooks present results for the *symbol* error rate for QPSK which indicate a worse performance than for BPSK. This is to be expected since the symbol states are closer together. It must be remembered, however, that QPSK is conveying *two bits of information for every symbol* and the likelihood of both bits being detected in error is much smaller than only one bit being in error (assuming Gray coding (see Section 3.5) of the symbol states).

Taking this into account, the *bit* error probability will be less than the *symbol* error probability for QPSK, and as we already know is the same as the bit error rate for BPSK.

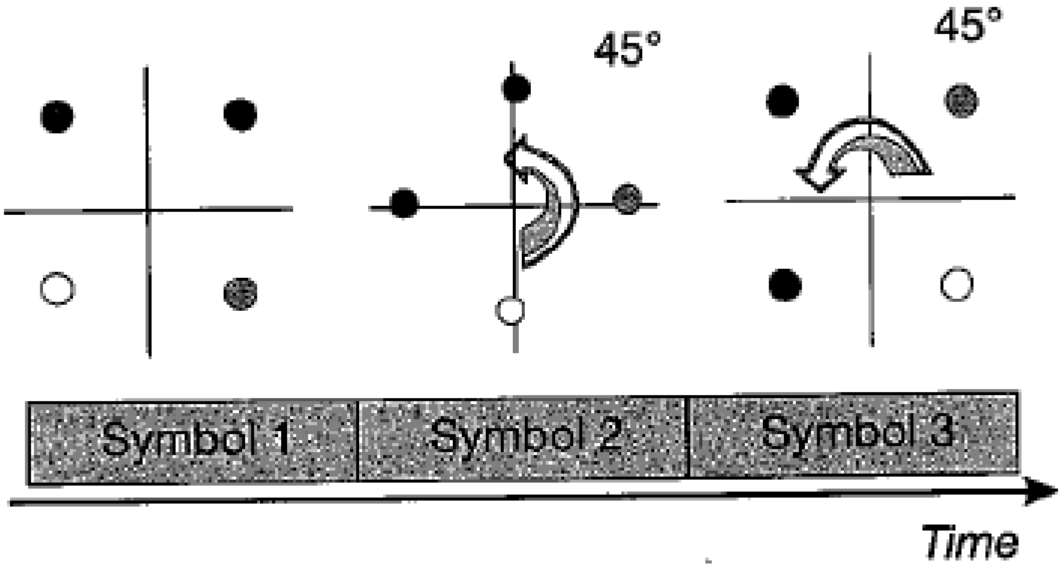
Because of the good bandwidth efficiency and noise performance, QPSK and its variants are currently the most widely used modulation types in both wired and wireless modems.

QPSK(symbol): $P_s = erfc[\sqrt{(E_b/N_0)}]\{1-0.25erfc[\sqrt{(E_b/N_0)}]\}$

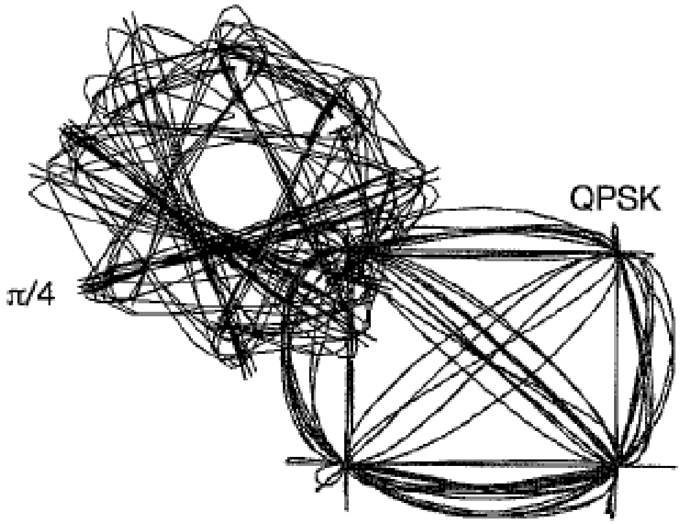
QPSK(bit) = BPSK(bit): $P_s = 0.5erfc[\sqrt{(E_b/N_0)}]$



π/4 QPSK



π/4 rotating symbol set



Performance of M-ary PSK

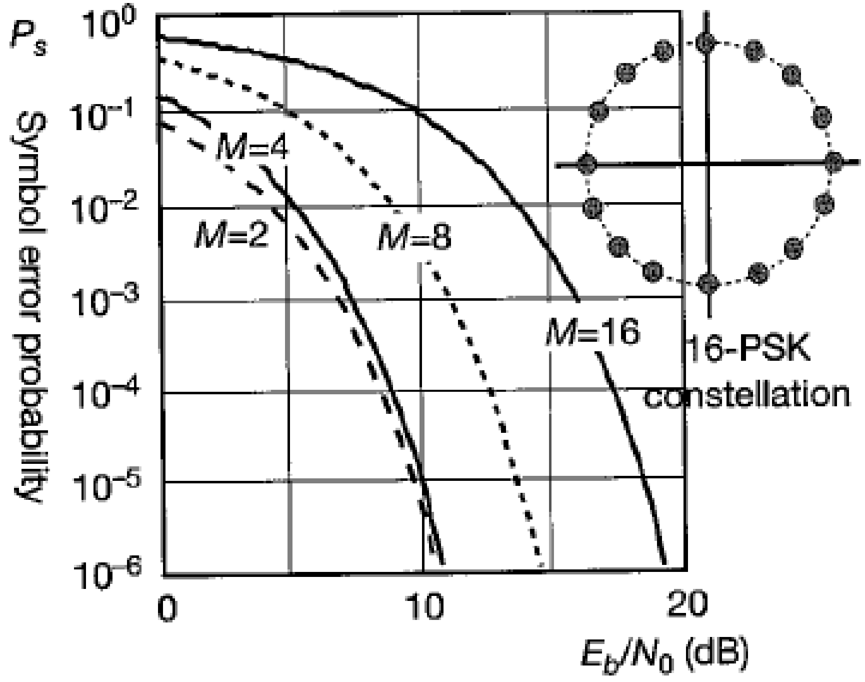
Increasing the number of symbol states for M-ary PSK beyond four allows further improvements in bandwidth efficiency, but the additional symbol states are no longer orthogonal (they do not lie on the sine or cosine axis of the constellation diagram). The result is that the performance in noise for $M > 4$ degrades rapidly as M increases.

The bandwidth efficiency for M-ary PSK is:

Maximum bandwidth efficiency (M-ary PSK) = $\log_2 M$ bits/second/Hz

M-PSK(symbol): P_s (approx.) = $erfc[\sqrt{(\log_2 M.E_b/N_0)}.\sin(\pi/M$

M-PSK(bit) ≈ M-PSK(symbol)/$\log_2$M



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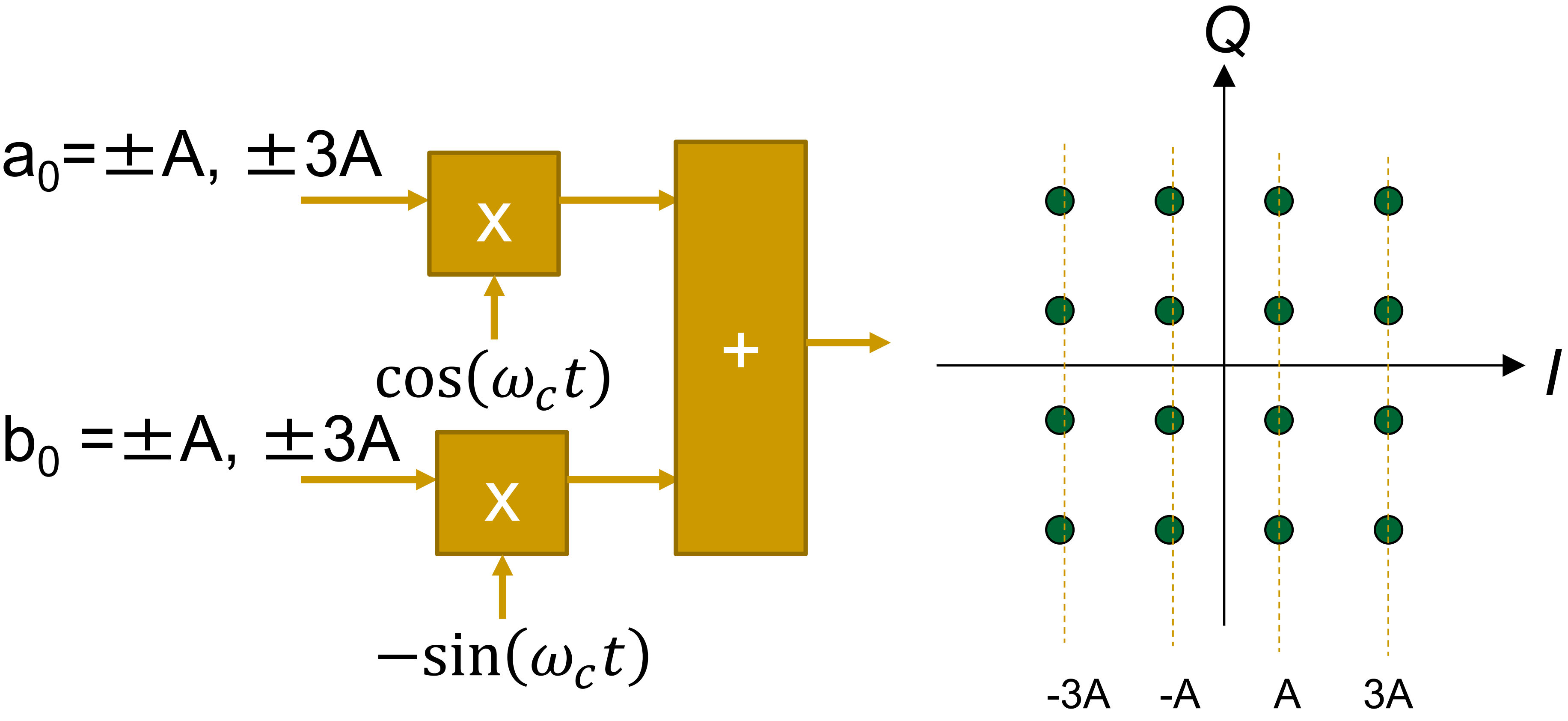
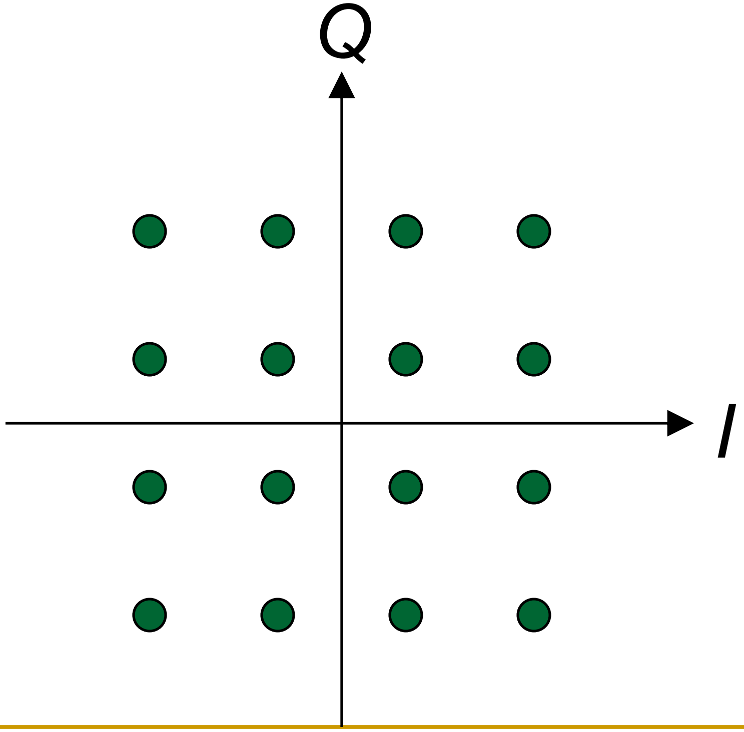
QAM generation

16QAM

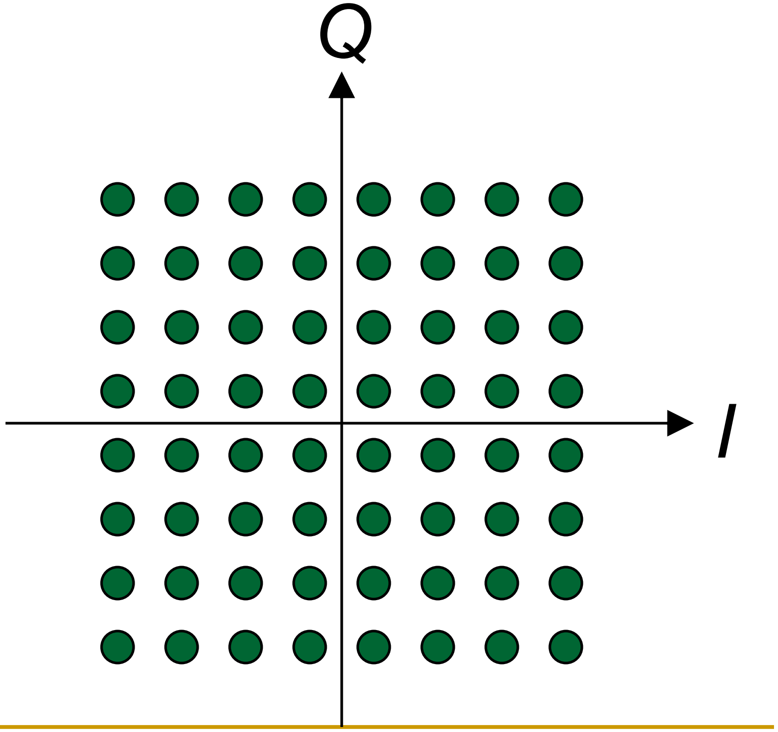


Quadrature Amplitude Modulation (QAM)

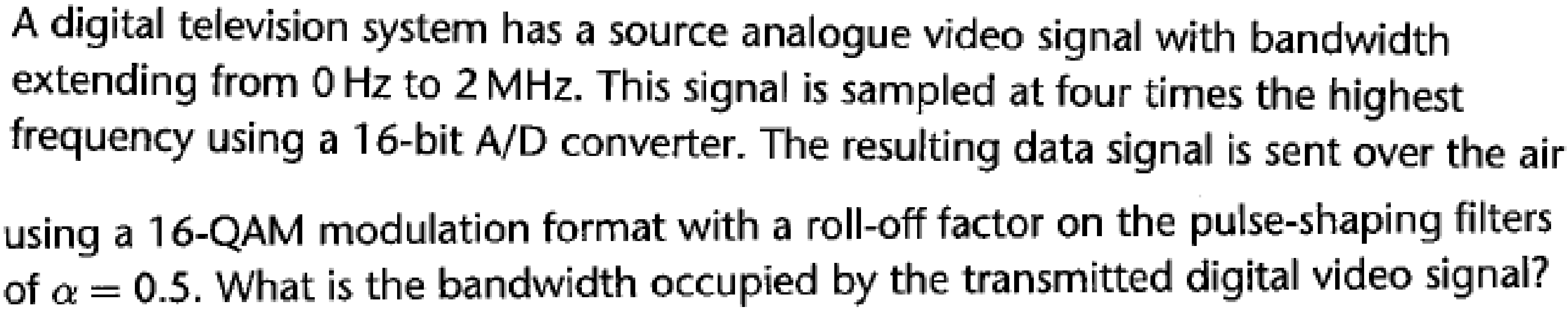
16QAM



64QAM



Example 6.1



A digital television system has a source analogue video signal with bandwidth extending from 0 Hz to 2 MHz. This signal is sampled at four times the highest frequency using a 16-bit A/D converter. The resulting data signal is sent over the air using a 16-QAM modulation format with a roll-off factor on the pulse-shaping filters of $\alpha = 0.5$. What is the bandwidth occupied by the transmitted digital video signal?

Raised cosine filtering

3.4 Raised cosine filtering

Raised cosine filter family

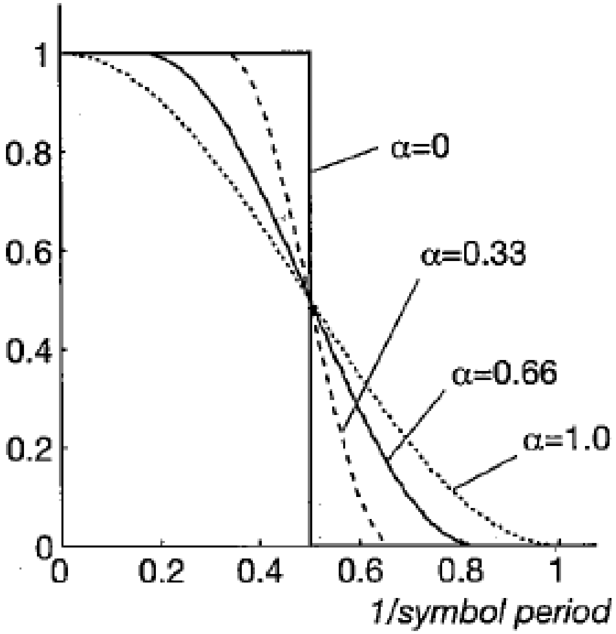
MAT LAB

A commonly used realization of the Nyquist filter is a *raised cosine filter*, so called because the transition band (the zone between passband and stopband) is shaped like part of a *cosine* waveform.

The sharpness of the filter is controlled by the parameter α, *the filter roll-off factor*. When $\alpha = 0$ this conforms to an ideal brick-wall filter.

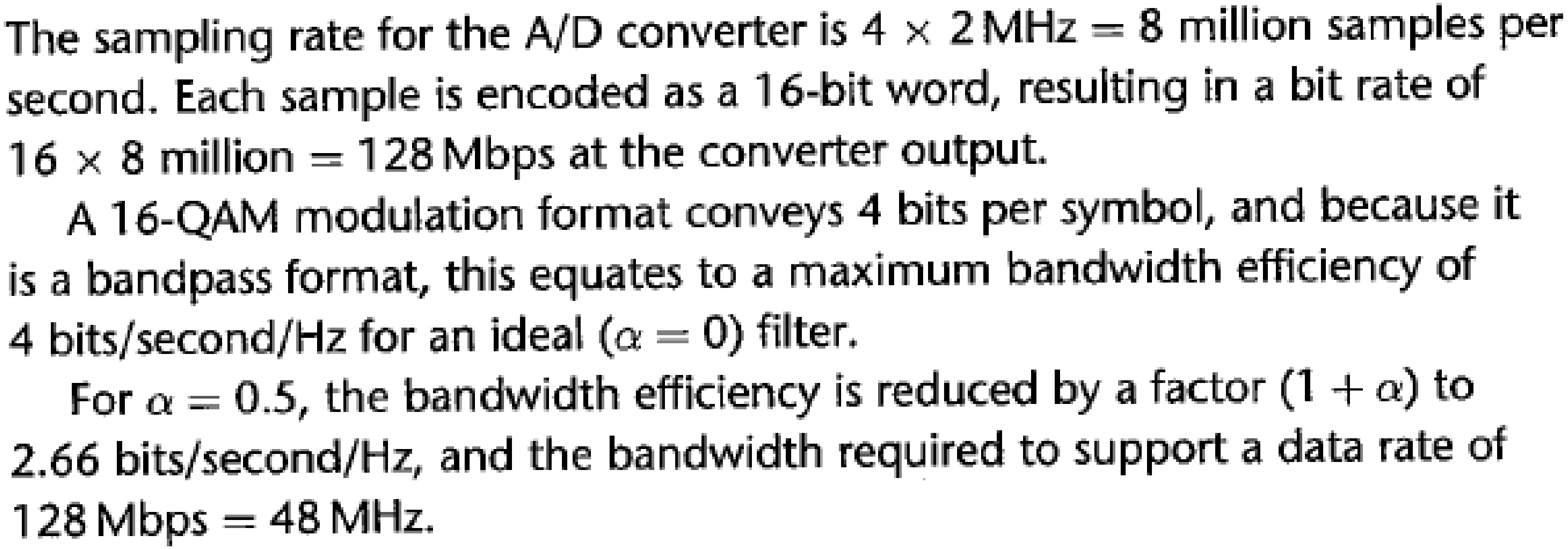
The bandwidth B occupied by a raised cosine filtered data signal is thus *increased* from its minimum value, $B_{\text{min}} = 0.5 \times 1/T_s$, to:

Actual modulation bandwidth,
$B = B_{\text{min}}(1 + \alpha)$



Frequency response for
raised cosine filter family

Solution 6.1



The sampling rate for the A/D converter is 4×2 MHz $=$ 8 million samples per second. Each sample is encoded as a 16-bit word, resulting in a bit rate of 16×8 million $=$ 128 Mbps at the converter output.

A 16-QAM modulation format conveys 4 bits per symbol, and because it is a bandpass format, this equates to a maximum bandwidth efficiency of 4 bits/second/Hz for an ideal ($\alpha = 0$) filter.

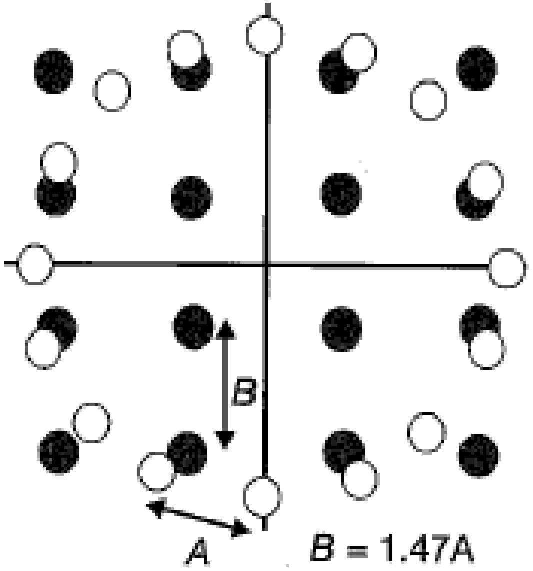
For $\alpha = 0.5$, the bandwidth efficiency is reduced by a factor $(1 + \alpha)$ to 2.66 bits/second/Hz, and the bandwidth required to support a data rate of 128 Mbps $=$ 48 MHz.

16-PSK vs 16-QAM

Comparing the constellation diagrams of M-ary QAM with M-ary PSK we can see that the spacing between symbol states for QAM is greater than that for PSK which is restricted to having symbol states of equal amplitude and thus on a circle equidistant from the origin.

The constellation diagrams shown here have been drawn to scale for *equal average symbol power* for both QAM and PSK systems, and the larger spacing between symbols for QAM means that the detection process should be less susceptible to noise.

The *peak* power for QAM under these conditions is, however, greater than that for M-ary PSK and this must be taken into account if the transmission process is peak power limited.



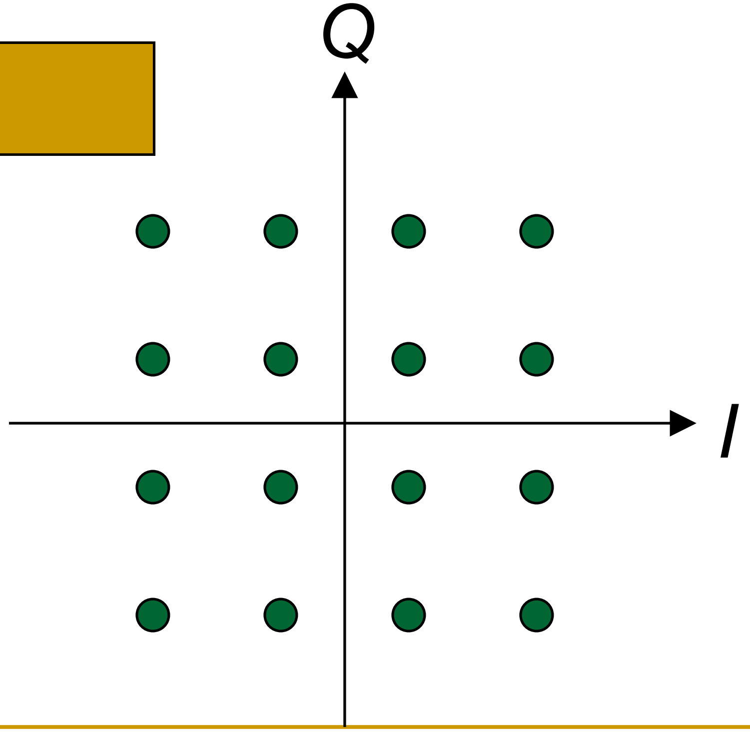
Comparison of 16-PSK and 16-QAM for equal average symbol power

Example 6.2

Draw the constellation diagrams for square 16-QAM.

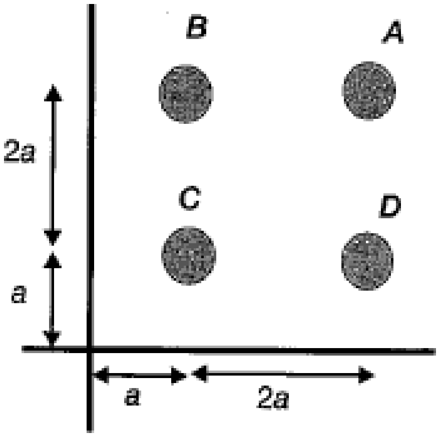
If the maximum vector length in a square 16-QAM constellation is 100 V rms, determine the long-term average power that would be delivered into a 50-ohm antenna load if each point in the constellation has an equal probability of transmission.

16QAM



Solution 6.2

With reference to one quadrant of the 16-QAM constellation, the average power developed by each of the vectors ***A, B, C, D*** is as follows:



$$A^2 = (3a)^2 + (3a)^2 = 18a^2$$

$$B^2 = D^2 = (3a)^2 + (a)^2 = 10a^2$$

$$C^2 = (a)^2 + (a)^2 = 2a^2$$

$$\text{Average power} = \frac{18a^2 + 2 \times 10a^2 + 2a^2}{4R}$$

The maximum vector length,

$$A = 100\,\text{mV} = \sqrt{18a^2}$$

Therefore

$$a = \sqrt{\frac{(100\,\text{mV})^2}{18}} = 23.6\,\text{mV}$$

Therefore the average power for all symbol states is:

$$\text{Average power} = 10a^2/R = 111\,\text{W}$$ 111 uW

Example 6.3

A transmitter for a digital radio system is peak power limited to 150 W. Determine the average power that can be supported for both 16-PSK and square 16-QAM transmission.

Solution 6.3

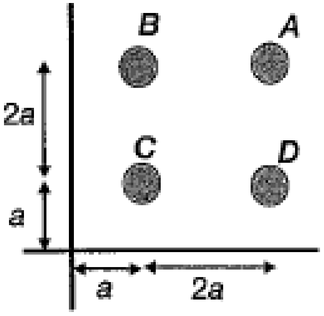
With reference to one quadrant of the 16-QAM constellation, the average power developed by each of the vectors *A, B, C, D* is as follows:

$$A^2 = (3a)^2 + (3a)^2 = 18a^2$$

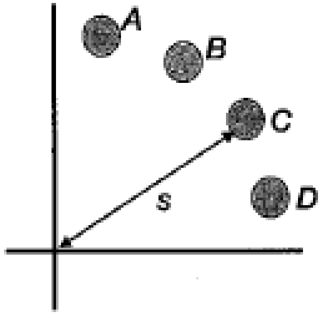
$$B^2 = D^2 = (3a)^2 + (a)^2 = 10a^2$$

$$C^2 = (a)^2 + (a)^2 = 2a^2$$

$$\text{Average power} = \frac{18a^2 + 2 \times 10a^2 + 2a^2}{4R}$$



16-QAM



16-PSK

The maximum vector power is given as 150 W, therefore

$$A^2/R = 18a^2/R = 150$$

Therefore

$$a = \sqrt{\frac{150 \times 50}{18}} = 20.4\,\text{W}$$

Therefore the average power for all symbol states is:

$$\text{Average power}_{\text{QAM}} = 10a^2/R = 83.33\,\text{W}$$

The average power for 16-PSK is the same for all symbol states and is equal to the peak symbol power since unfiltered PSK is a constant envelope modulation format. Thus:

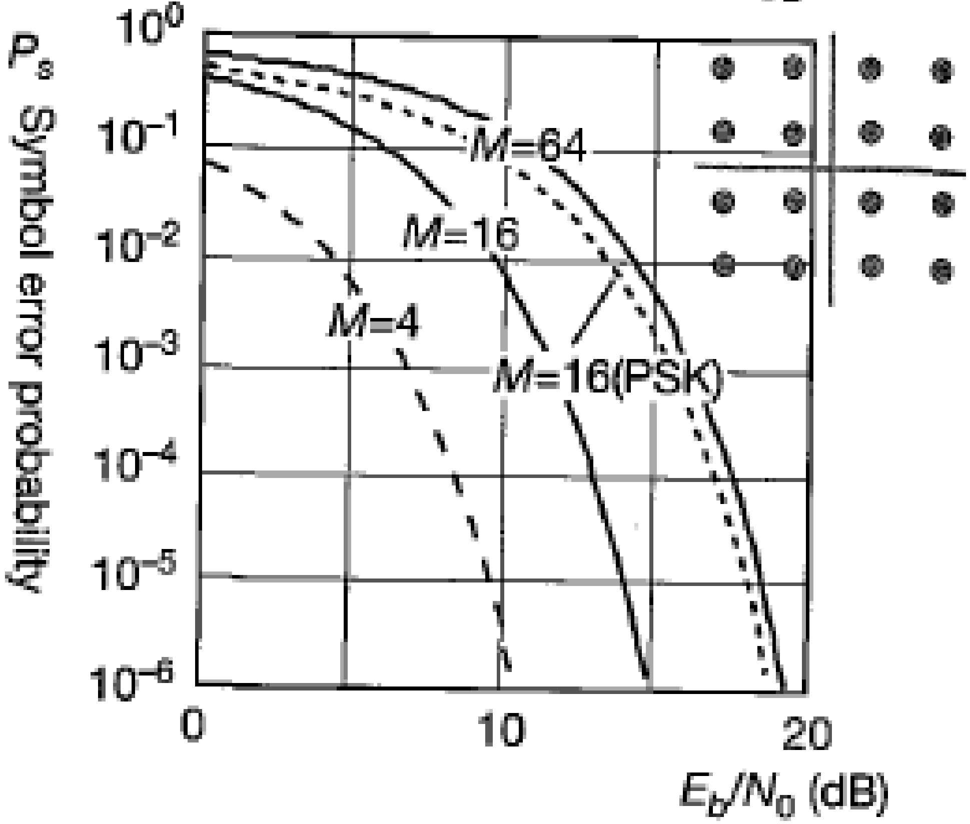
$$\text{Average power}_{\text{PSK}} = s^2/R = 150\,\text{W}$$

BER performance of QAM

M-QAM(symbol): $P_s = 1-(1-P_m)^2$

$P_m=(1-1/\sqrt{(M)})\mathrm{erfc}(\sqrt{\{3.k.(E_b/N_0)/(2.(M-1))\}})$

M-QAM(bit) ≈ M-QAM(symbol)/$\log_2 M$



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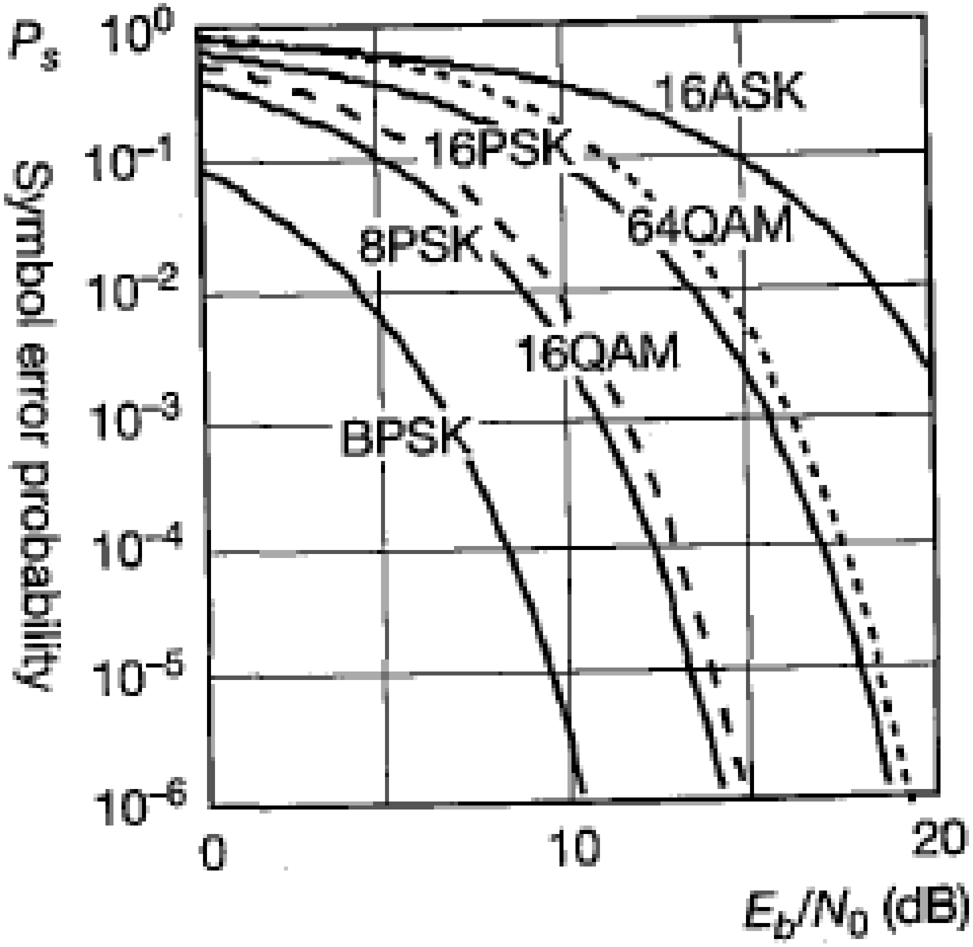
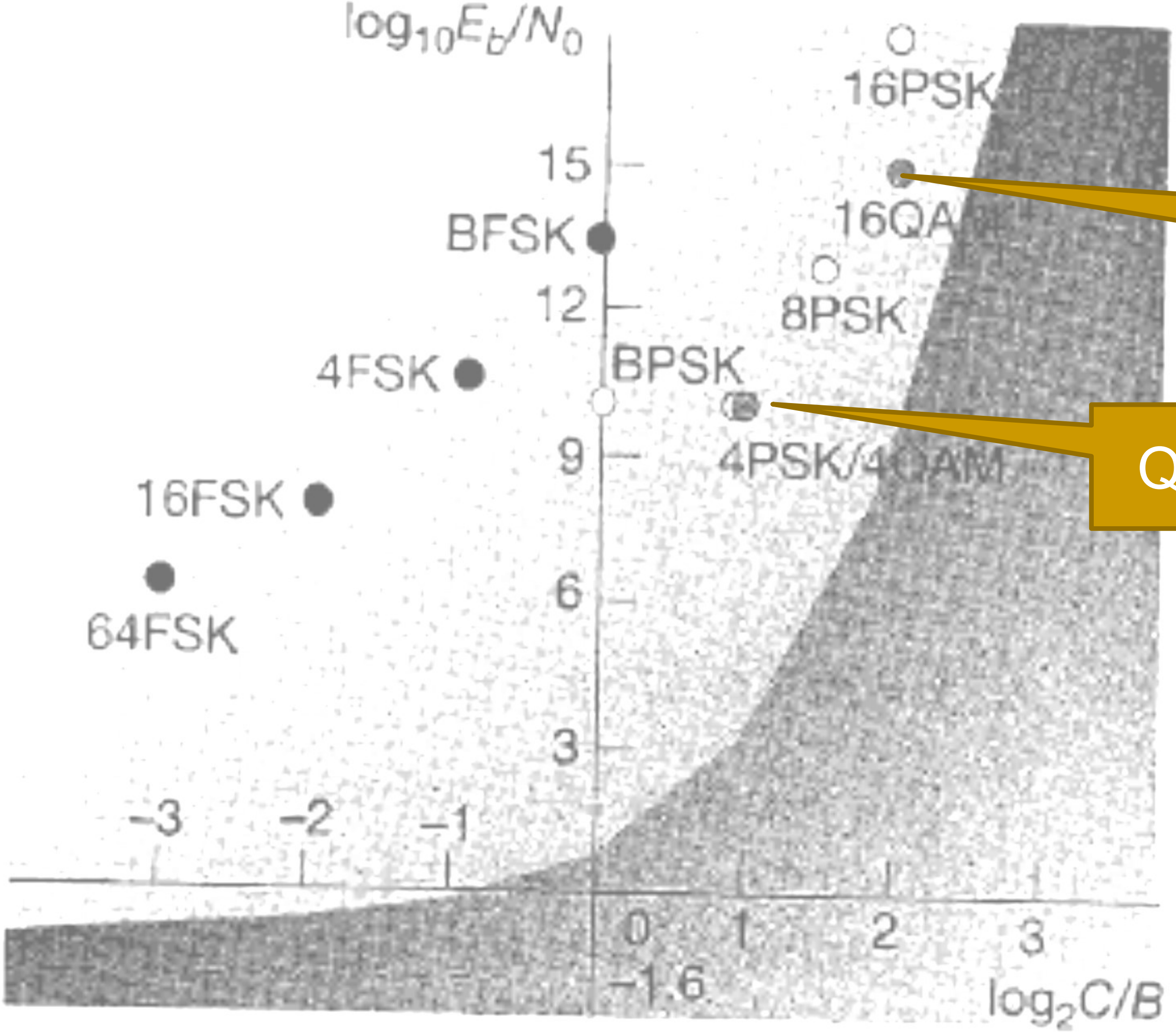
BER comparison



Comparison of M-ary data systems

Close to Shannon limit



16QAM

QPSK

Need two presenter

- Each presenter shall chose two problems from 6.1 to 6.10. The one for even and the other for odd problems.
- Show your answer at the beginning of next lecture. Each has 5 minutes.
- Next lecture will be 12/9 (mon)

- Dec/14th Saturday,1pm to 5pm is Matlab lab at computer center, please prepare your login ID