OFDM(2)
Matrix Based Calculation

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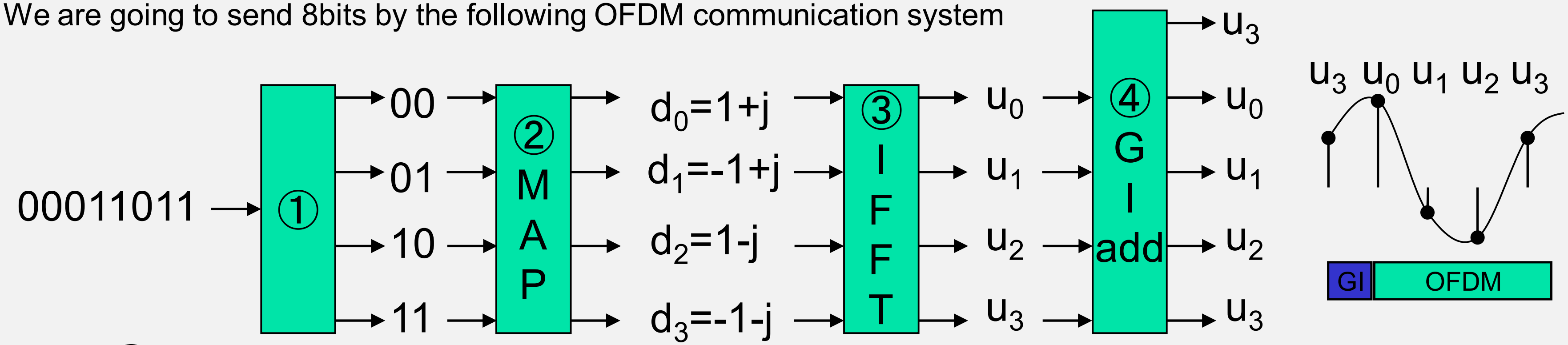
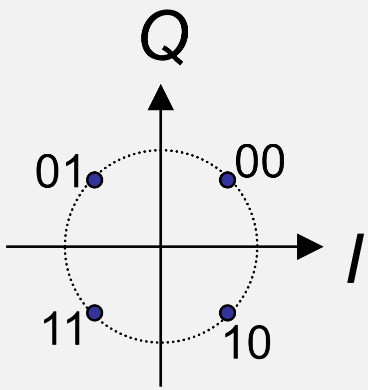
OFDM digital communication WORK SHEET

We are going to send 8bits by the following OFDM communication system





$$u_k = \frac{1}{4}\sum_{n=0}^{3} d_n \cdot \left(e^{j\frac{2\pi}{4}}\right)^{nk} = IFFT(d_n) \quad (k = 0,1,2,\cdots,3)$$

$$u_0 = \frac{1}{4}(d_0 + d_1 + d_2 + d_3)$$

$$u_1 = \frac{1}{4}(d_0 + d_1 \cdot (j) + d_2 \cdot (-1) + d_3 \cdot (-j))$$

$$u_2 = \frac{1}{4}(d_0 + d_1 \cdot (-1) + d_2 + d_3 \cdot (-1))$$

$$u_3 = \frac{1}{4}(d_0 + d_1 \cdot (-j) + d_2 \cdot (-1) + d_3 \cdot (j))$$

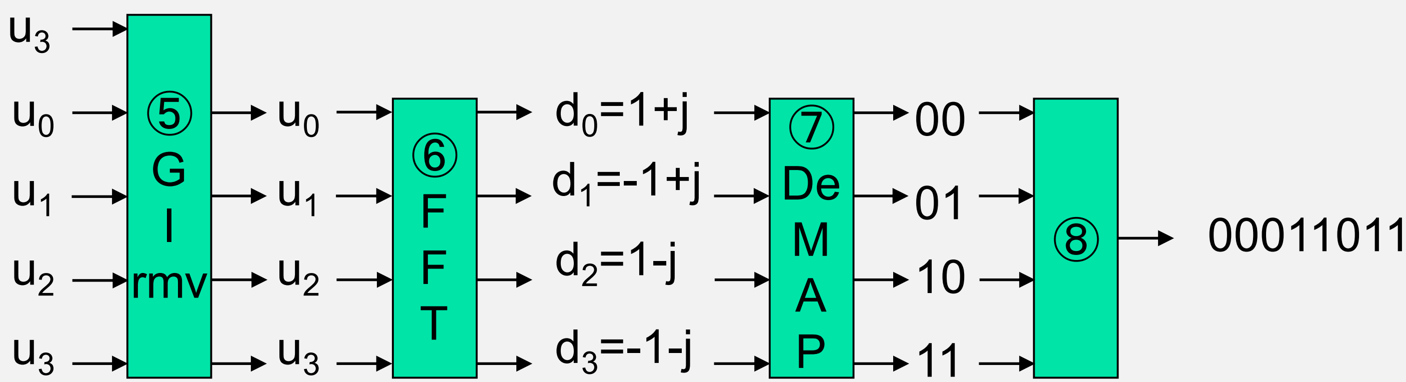
$$d_n = \sum_{k=0}^{3} u_k \cdot \left(e^{-j\frac{2\pi}{4}}\right)^{nk} = FFT(u_k) \quad (n = 0,1,2,\cdots,3)$$

$$d_0 = u_0 + u_1 + u_2 + u_3$$

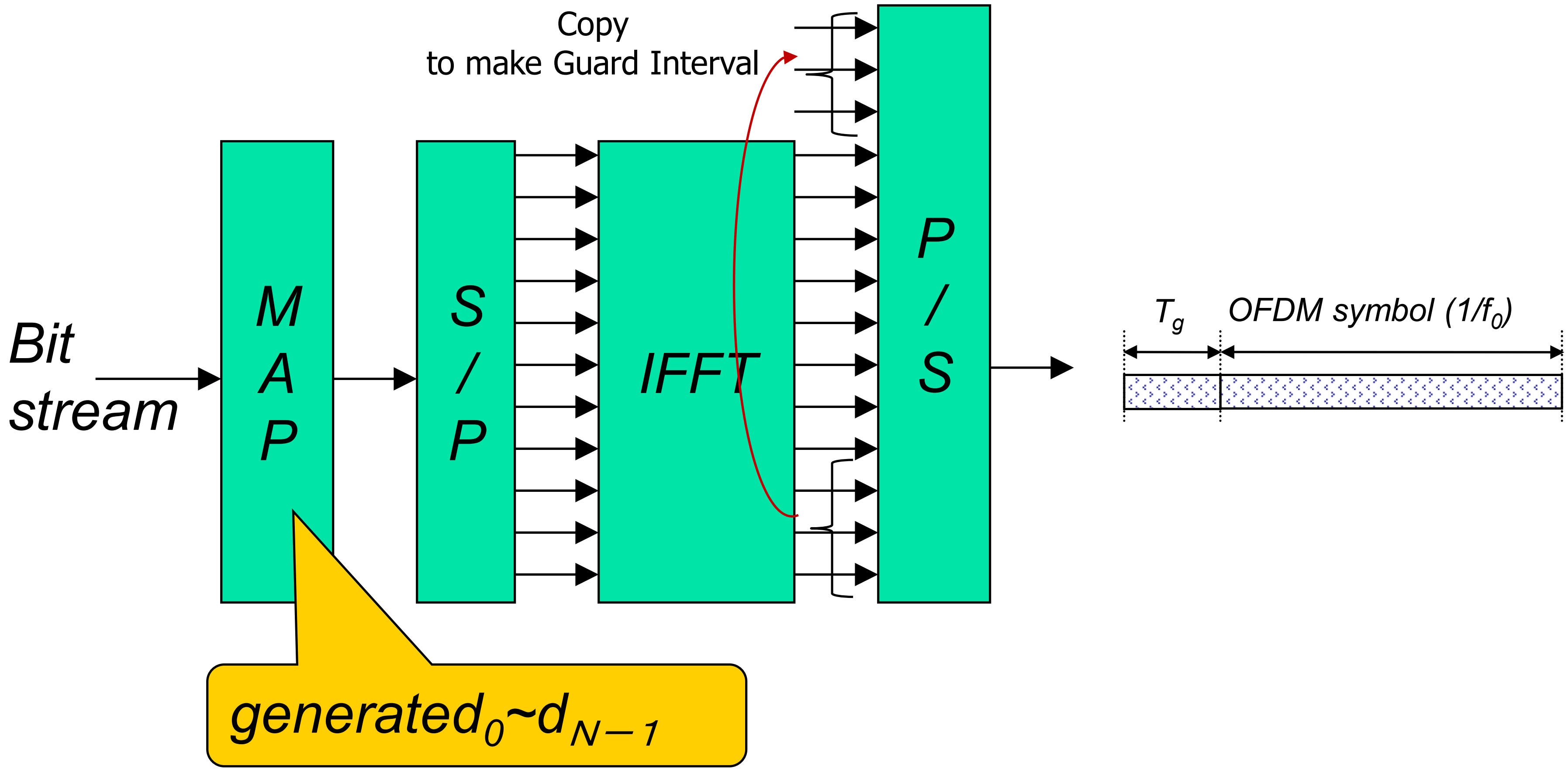
$$d_1 = u_0 + u_1 \cdot (-j) + u_2 \cdot (-1) + u_3 \cdot (+j)$$

$$d_2 = u_0 + u_1 \cdot (-1) + u_2 + u_3 \cdot (-1)$$

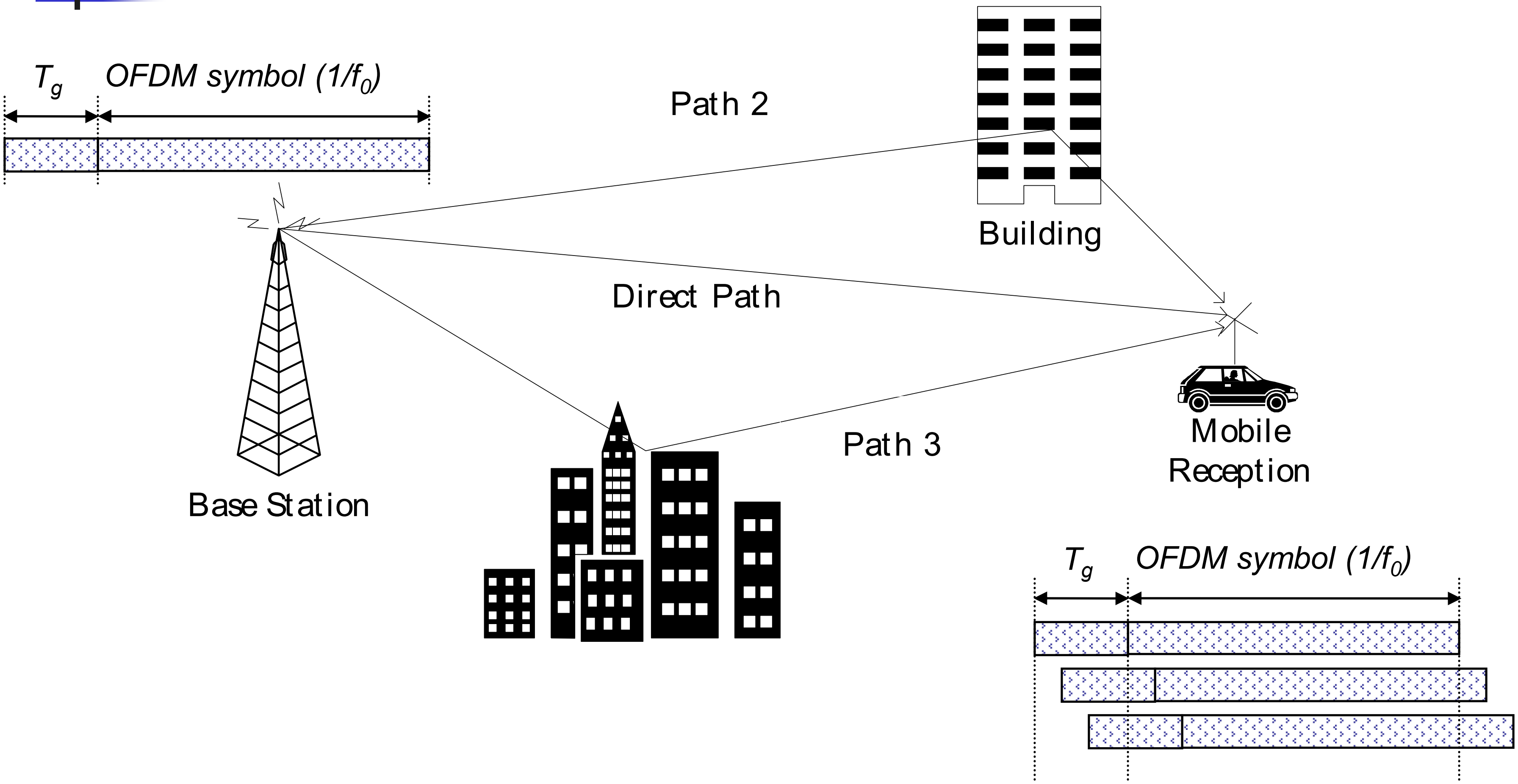
$$d_3 = u_0 + u_1 \cdot (j) + u_2 \cdot (-1) + u_3 \cdot (-j)$$



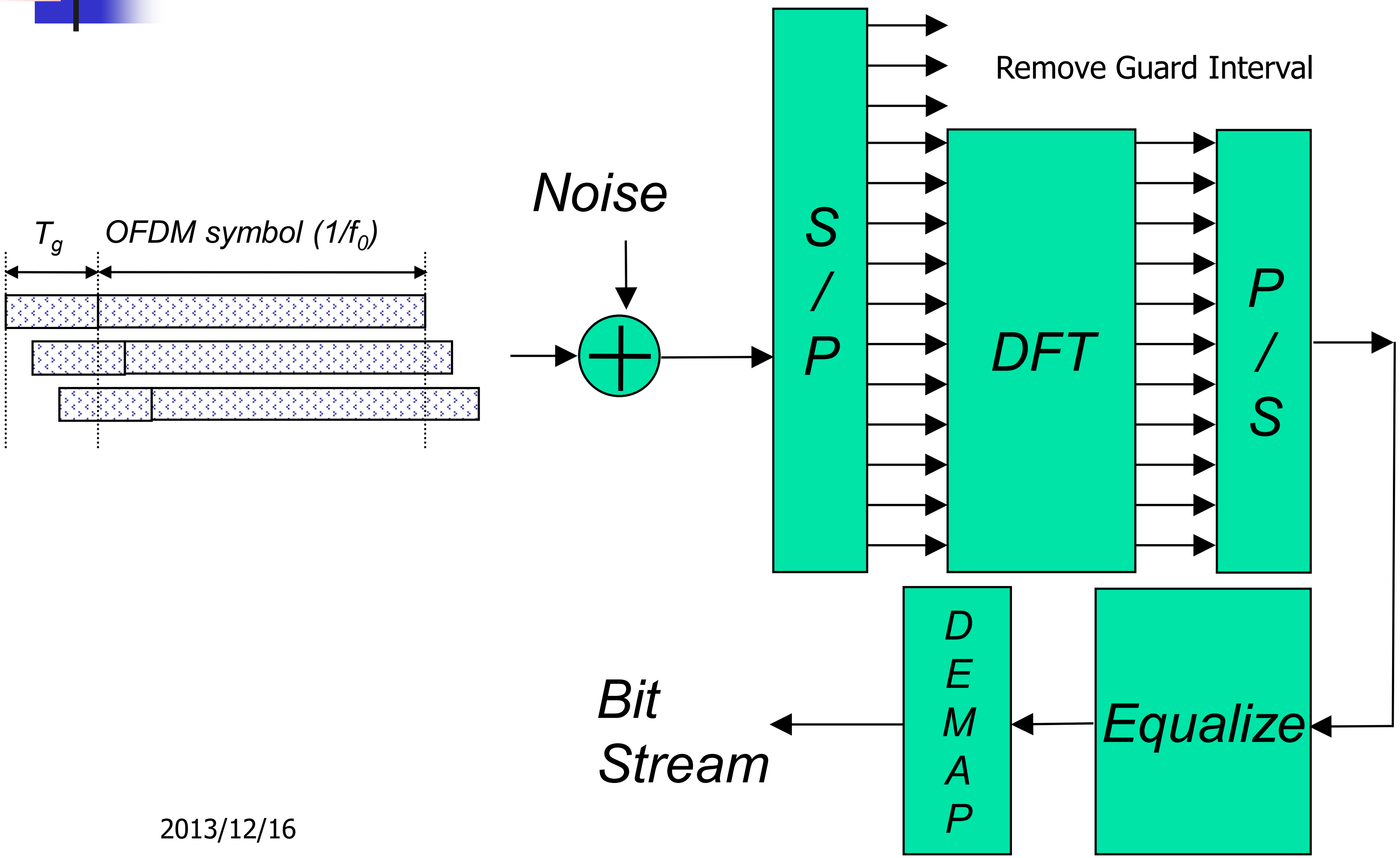
OFDM Modulator



Multi-path channel



OFDM Demodulator



IFFT matrix

$$\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix} = IFFT\begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix} = \frac{1}{M}\left[\omega^{(k-1)*(l-1)}; k-row, l-column\right]\begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix}$$

$$\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix} = \frac{1}{\sqrt{M}}\begin{pmatrix} \omega^0 & \omega^0 & \cdots & \omega^0 \\ \omega^0 & \omega^1 & \cdots & \omega^{(M-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^0 & \omega^{(M-1)} & \cdots & \omega^{(M-1)*(M-1)} \end{pmatrix}\begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix}$$

$$Here, \omega = e^{j\frac{2\pi}{M}}$$

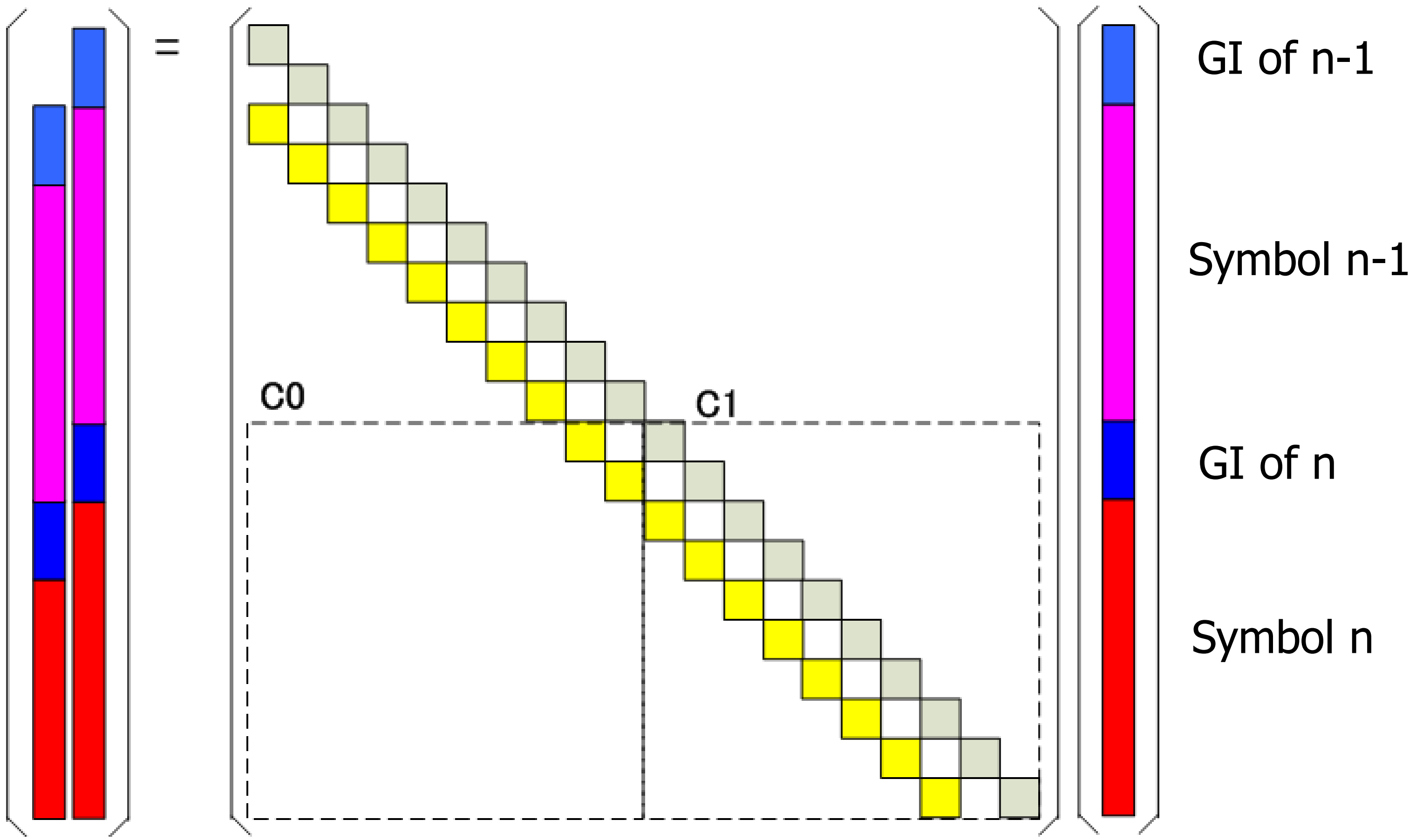
FFT matrix

$$\begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix} = FFT\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix} = \left[\omega^{-(k-1)*(l-1)}; k-row, l-column\right]\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix}$$

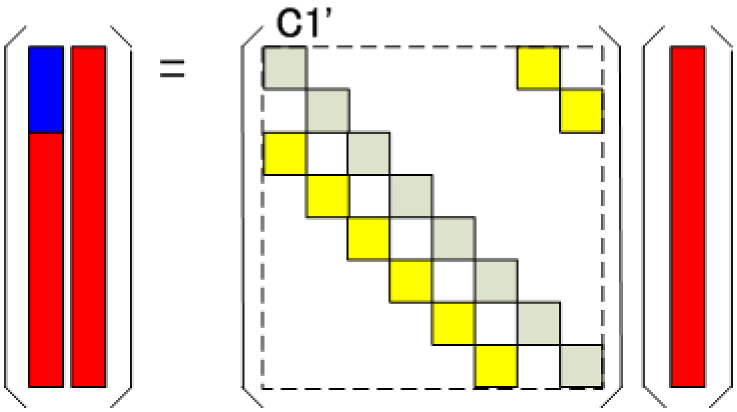
$$\begin{pmatrix} Y(0) \\ Y(1) \\ \vdots \\ Y(M-1) \end{pmatrix} = \begin{pmatrix} \omega^0 & \omega^0 & \cdots & \omega^0 \\ \omega^0 & \omega^{-1} & \cdots & \omega^{-(M-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^0 & \omega^{-(M-1)} & \cdots & \omega^{-(M-1)*(M-1)} \end{pmatrix}\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(M-1) \end{pmatrix}$$

$$Here, \omega = e^{j\frac{2\pi}{M}}$$

Multi-path channel in Matrix

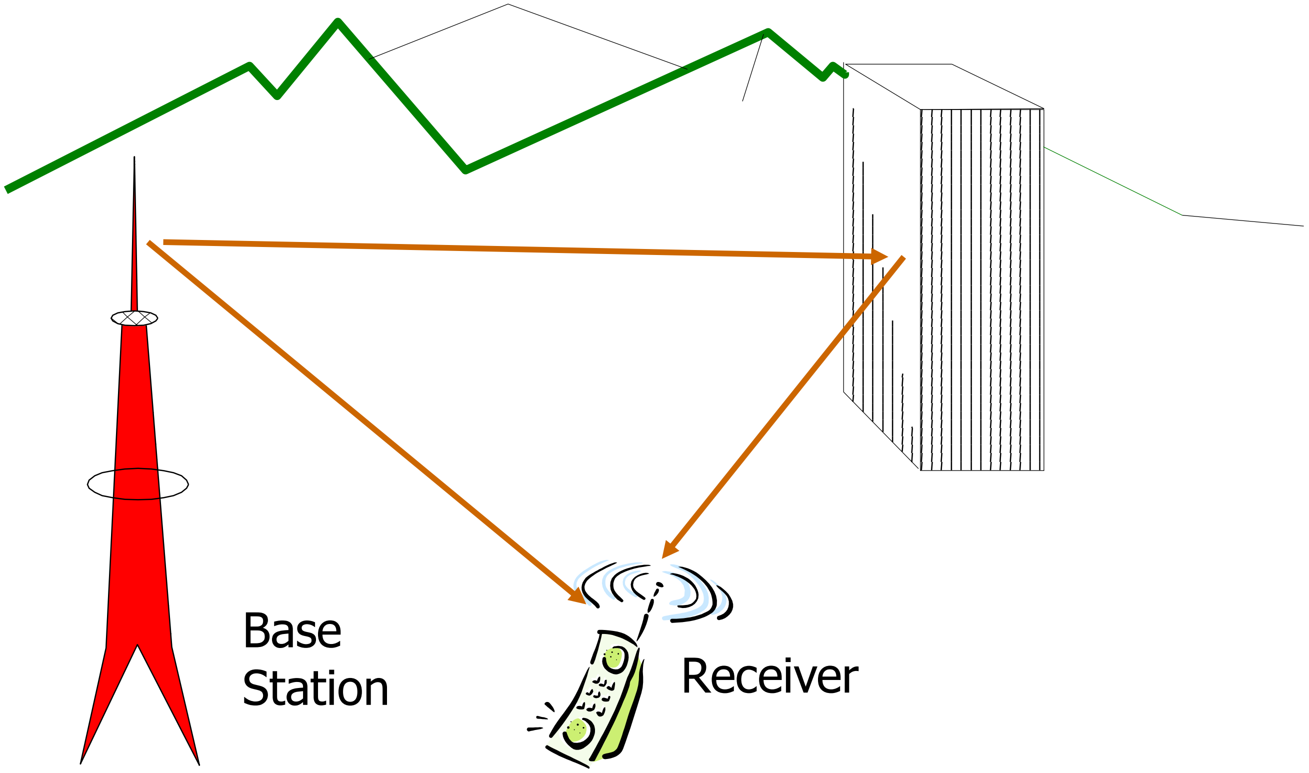


If Multi-path delay is small than GI length



- Channel Matrix is Cyclic Matrix!

Two path Multi path Channel Example



Channel Impulse Response = [1, 0.5 , 0, 0]

Two path Multi path Channel Example

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = FFT * Channel * IFFT * \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix}$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \omega^{-3} \\ 1 & \omega^{-2} & \omega^{-4} & \omega^{-6} \\ 1 & \omega^{-3} & \omega^{-6} & \omega^{-9} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0.5 \\ 0.5 & 1 & 0 & 0 \\ 0 & 0.5 & 1 & 0 \\ 0 & 0 & 0.5 & 1 \end{pmatrix} \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{1} & \omega^{2} & \omega^{3} \\ 1 & \omega^{2} & \omega^{4} & \omega^{6} \\ 1 & \omega^{3} & \omega^{6} & \omega^{9} \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix}$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = \begin{pmatrix} H(0) & 0 & 0 & 0 \\ 0 & H(1) & 0 & 0 \\ 0 & 0 & H(2) & 0 \\ 0 & 0 & 0 & H(3) \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix}$$

If time domain channel matrix is cyclic, Frequency Domain Channel Matrix is diagonal!

Additive Noise

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = FFT * \left[Channel * IFFT * \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} + \begin{pmatrix} noise(0) \\ noise(1) \\ noise(2) \\ noise(3) \end{pmatrix} \right]$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \omega^{-3} \\ 1 & \omega^{-2} & \omega^{-4} & \omega^{-6} \\ 1 & \omega^{-3} & \omega^{-6} & \omega^{-9} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0.5 \\ 0.5 & 1 & 0 & 0 \\ 0 & 0.5 & 1 & 0 \\ 0 & 0 & 0.5 & 1 \end{pmatrix} \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{1} & \omega^{2} & \omega^{3} \\ 1 & \omega^{2} & \omega^{4} & \omega^{6} \\ 1 & \omega^{3} & \omega^{6} & \omega^{9} \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} + \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \omega^{-3} \\ 1 & \omega^{-2} & \omega^{-4} & \omega^{-6} \\ 1 & \omega^{-3} & \omega^{-6} & \omega^{-9} \end{pmatrix} \begin{pmatrix} noise(0) \\ noise(1) \\ noise(2) \\ noise(3) \end{pmatrix}$$

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = \begin{pmatrix} H(0) & 0 & 0 & 0 \\ 0 & H(1) & 0 & 0 \\ 0 & 0 & H(2) & 0 \\ 0 & 0 & 0 & H(3) \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} + \begin{pmatrix} N(0) \\ N(1) \\ N(2) \\ N(3) \end{pmatrix}$$

How to recover sending signal from receiver signal. - EQUALIZE -

$$\begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix} = FFT * Channel * IFFT * \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} + \begin{pmatrix} N(0) \\ N(1) \\ N(2) \\ N(3) \end{pmatrix} = \begin{pmatrix} H(0) & 0 & 0 & 0 \\ 0 & H(1) & 0 & 0 \\ 0 & 0 & H(2) & 0 \\ 0 & 0 & 0 & H(3) \end{pmatrix} \begin{pmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{pmatrix} + \begin{pmatrix} N(0) \\ N(1) \\ N(2) \\ N(3) \end{pmatrix}$$

Then

$$\begin{pmatrix} \hat{X}(0) \\ \hat{X}(1) \\ \hat{X}(2) \\ \hat{X}(3) \end{pmatrix} = \begin{pmatrix} \frac{1}{H(0)} & 0 & 0 & 0 \\ 0 & \frac{1}{H(1)} & 0 & 0 \\ 0 & 0 & \frac{1}{H(2)} & 0 \\ 0 & 0 & 0 & \frac{1}{H(3)} \end{pmatrix} \begin{pmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{pmatrix}$$